Proceedings of The Institute of Acoustics

FLUCTUATIONS OF SIGNALS AND NOISE IN THE SEA AND THEIR EFFECT ON SONAR TARGET DETECTION

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INTRODUCTION

It has been said that the most constant characteristic of sound in the sea is the presence of fluctuations. Indeed, everything fluctuates; nothing is ever constant, except variability itself. The basic cause of fluctuation is motion, of the sea itself and of the source and/or the receiver. If the sea were frozen solid, there would be no fluctuations.

In this paper, models for the fluctuation of transmitted signals and the ambient noise background will be described and the effect of fluctuation on the detection of sonar targets will be examined in theory and in field exercises.

Like most aspects of sound in the sea, the fluctuations of transmitted sound were first studied in a scientific way during World War II [1][2]. The subject then had practical importance because surface ships, when echo ranging on a submarine contact, often found the echoes to disappear just when they were needed to prosecute a depth-charge attack. Attention to fluctuation continued in later years, so that by 1973, one bibliography, complete with annotations [3], contained some 132 items. Attention continues unabated, as is evidenced by the existence of the present conference.

An illustration of the effect of fluctuations on sonar detection is given in Fig. 1. The solid curve shows the probability of detection of a target as it was observed during a field excercise; the dashed curve shows the result of a calculation using the sonar equation if fluctuations are ignored. We see that fluctuations cause a target to be detected better at long ranges during signal surges when the target is momentarily strong, and to be detected more poorly at short ranges during signal fades when the target is momentarily weak.

TYPES OF FLUCTUATION

There are two basic types of fluctuations. One is deterministic and is predictable - at least to some extent - by means of our understanding of underwater sound. This type of fluctuation has periods of minutes or longer. Examples are of the well-known diurnal and annual variations of transmitted sound caused by changing thermal gradients in the upper part of the sea. Another example is shown in Fig. 2, where we see fluctuations of 15 db or more associated with the tidal cycle; these can be attributed in mode theory to interference between the lowest two normal modes of transmission.

The other type of fluctuation is unpredictable or stochastic, and can be described only in a statistical way. These fluctuations are generally of short period having periods of a fraction of a second up to a few minutes, such as those caused by the moving sea or those caused by interference of the propagation paths from a moving source. As an example of the latter kind of fluctuation, Fig. 3 shows the signal from a 142 Hz continuous sound source

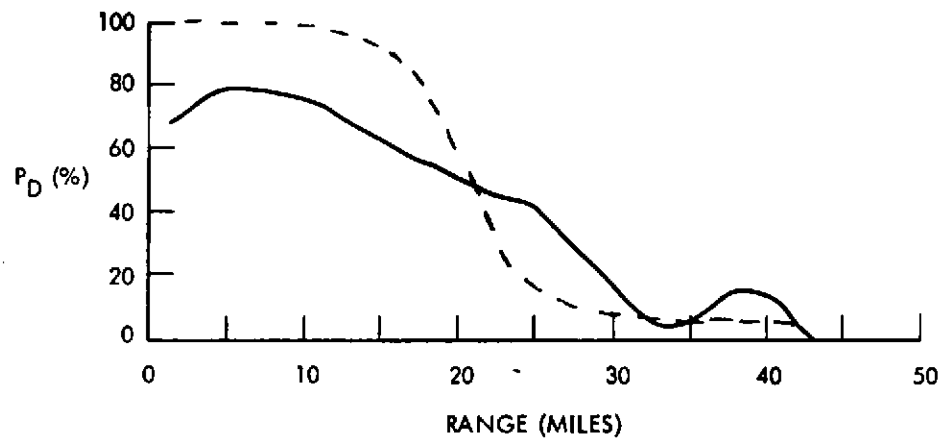


Fig. 1 Detection probability versus range as observed (solid curve) and as calculated from the sonar equations (dashed curve). The lump in the observed detection curve at 39 miles is due to the convergence zone, not included in the calculations.

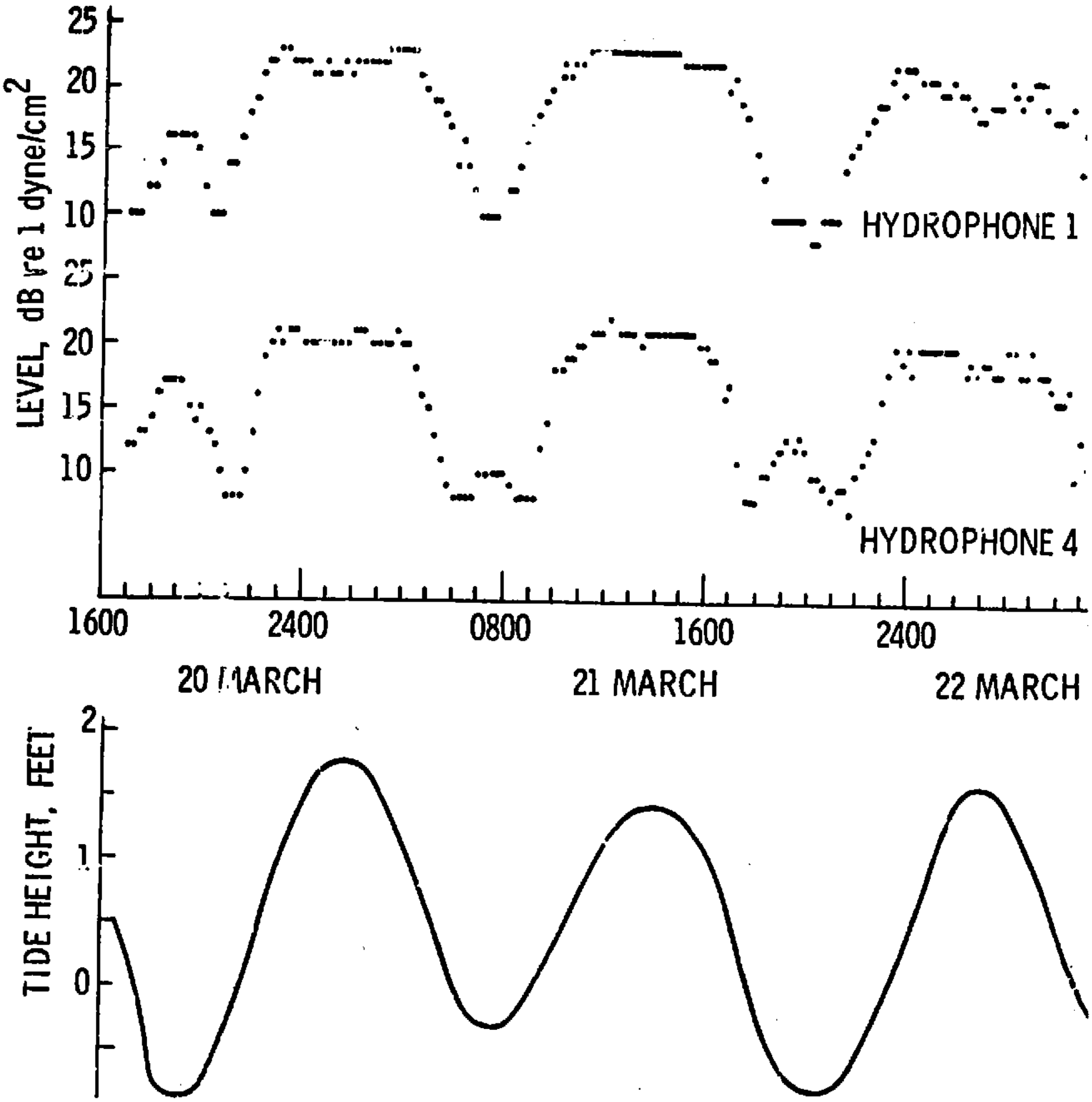


Fig. 2 Comparison of the received signal from a steady sinusoidal 1120 Hz source received by a hydrophone 5000 feet away in 60 feet of water. The upper figure shows the received level averaged over 15 minute intervals while the lower figure shows the height of the tide as taken from tide tables.

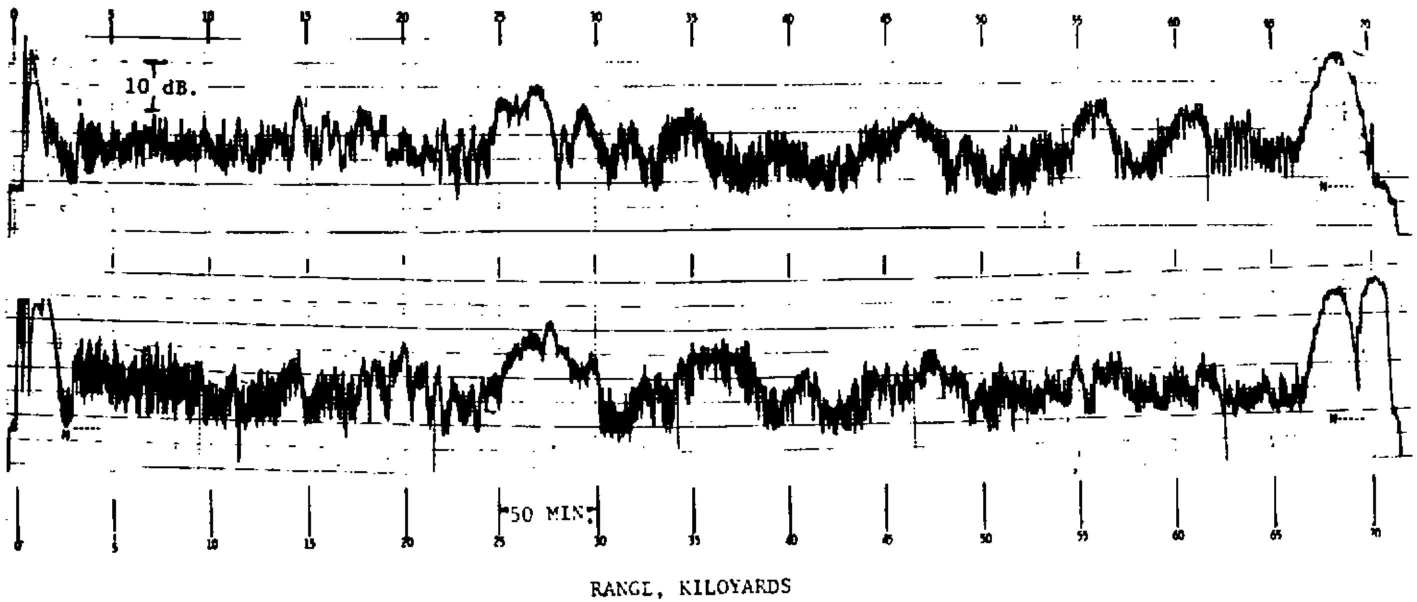


Fig. 3 The signal from a steady source at 142 Hz received at two depths (90 and 300 ft) as the source was towed outward in range at a speed of 2.7 knots.

as it was towed from outward as far as the first convergence zone from receivers at depths of 90 and 300 feet. The fast unresolved fluctuations are caused by the motion of the sea surface, while the slower fluctuations are doubtless caused by the changing interferences between the bottom and surface reflections as the source changes range. In the following section we will describe a mathematical model for these fast changes giving the fraction of the time that the received signal level is greater or less than its mean level.

FLUCTUATION MODEL

The model rests on the postulate that the signal received from a distant steady source consists of two components: a steady component and a random component [4]. The latter is caused by random processes such as scattering, diffraction, microstructure and multipath interferences. It increases with range at the expense of the steady, invariant component, until, at long ranges, the steady component disappears and the signal is entirely random. The amount of fluctuation depends on the fraction of the received power that is random - a fraction which may be called the randomicity, to be designated by T.

The mathematics of this physical model was investigated during the World War II years by S.O. Rice in a classic paper [5] that gives the distribution of the envelope of a sinusoidal signal in narrow-band noise. In Fig. 4, let the steady component be represented by the rotating vector P and the random component, representing the sum of many random vectors, be R. The vector for the received signal is the sum of P and R, and is denoted by V. The x and y components of V are taken to be Gaussian time-variates of zero mean and with variances σ_x^2 and σ_y^2. Rice showed that when σ_x and σ_y are arbitrarily set equal to unity, the probability density of V is given by

$$p(V) = V \exp\left[-\left(\frac{V^2+P^2}{2}\right)\right] \cdot I_0(PV)$$

where $I_0(PV)$ is the modified Bessel function of argument PV, for which tables are available [6]. This is sometimes called the "Rician" distribution. For this distribution, it can be shown that the randomicity T equals $2/(P^2+2)$. Figure 5 shows the resulting cumulative distribution curves for the probability that the signal has a level relative to the mean equal to or less than that given by the horizontal scale, with the randomicity as a parameter. For T = 1, the signal is completely random and the distribution is Rayleigh; for T = 0, the signal is completely steady and there is no fluctuation.

This model has been validated recourse to a variety of field data. An example is shown in Fig. 6. Fig. 6a shows the geometry. Fig. 6b shows paper playouts of the level of a sinusoidal signal received at a range of 5 miles at two depths. The paper playouts were read off at 100 equal intervals, and the cumulative distribution of the 100 values was found (Fig. 6c). It is seen that the data points are fitted reasonably well by the Rician curves for T = 1 at 8000 feet and by T = 0.2 at 1000 feet. This result is consistent with our understanding of the propagation from the source to the two receivers: the receiver at 1000 feet - below the surface duct - receives only the completely random sound diffracted and scattered out of the duct, so that T = 1, while

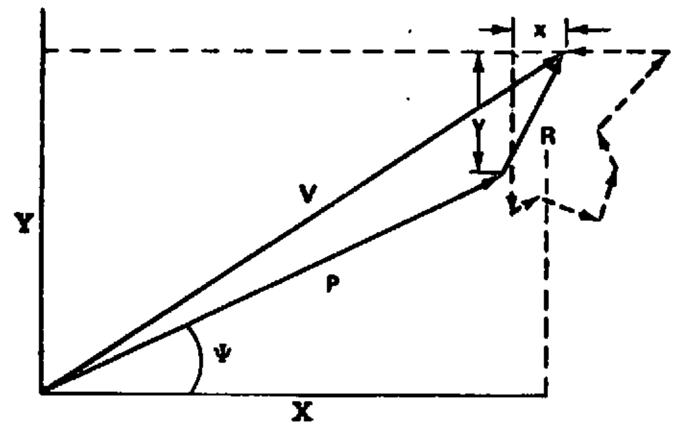


Fig. 4 The steady component P plus a random component R give the fluctuating resultant V.

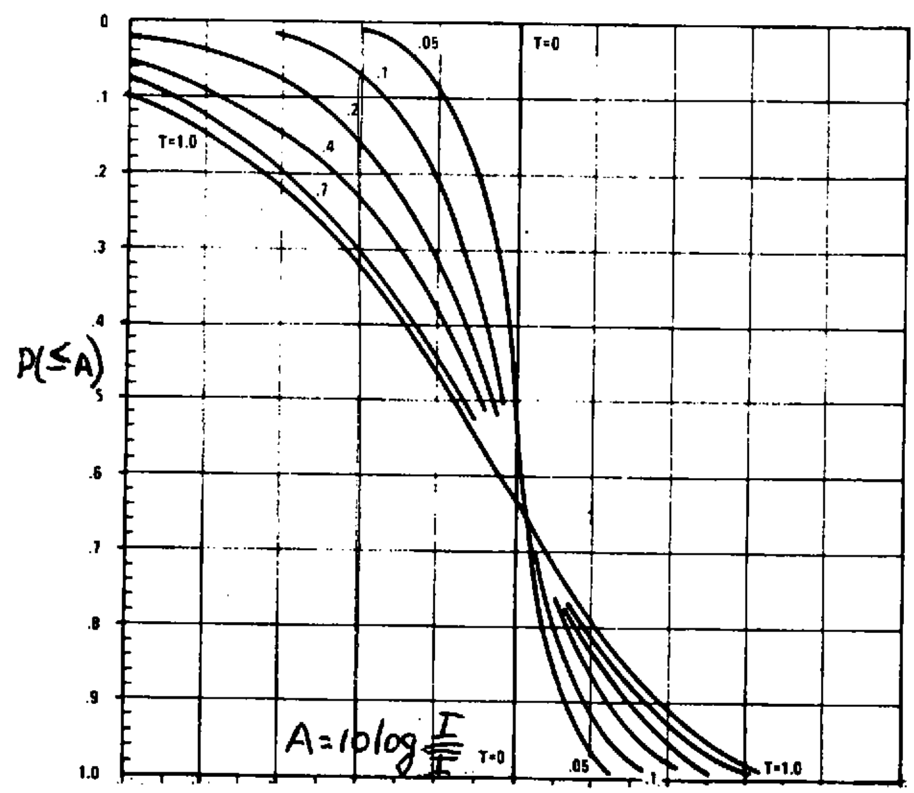


Fig. 5 Cumulative distribution curves of the level of a received sinusoidal signal. Vertical scale is the fraction of signal samples equal to or less than the number of decibels relative to the mean, as abscissa. T is the randomicity, or fraction of random power in the received signal.

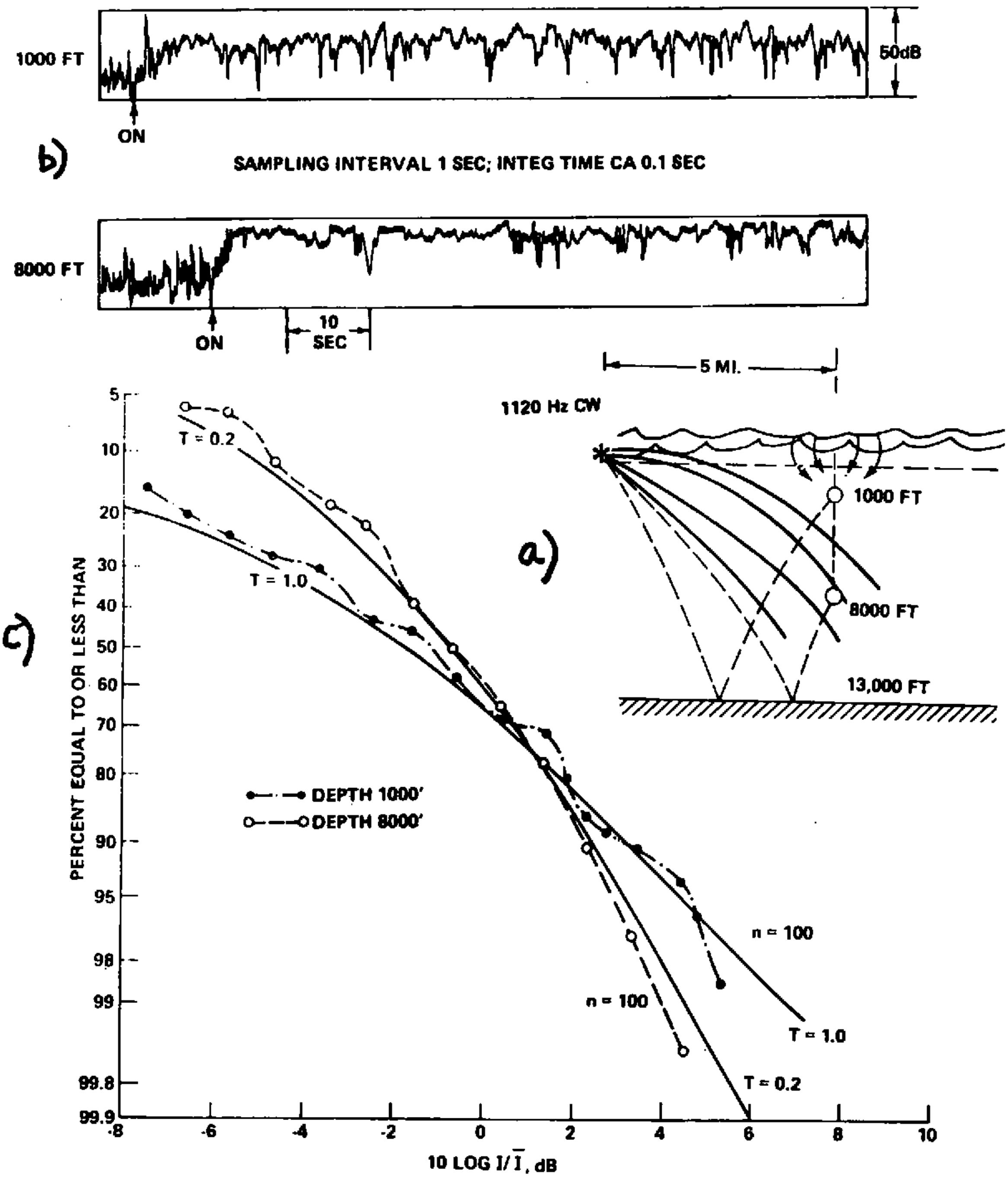


Fig. 6 Model validation. a) shows a field measurement of transmission from a source in a surface duct to receivers at two depths 5 miles away. b) gives paper playouts with a db scale of the received signal samples at 100 equal intervals. c) shows the cumulative distribution of the samples in db relative to the mean intensity, with two of the curves of Fig. 5 superposed.

at 8000 feet the receiver receives principally the direct sound from the source, that is not only stronger but is more steady. Only 20% of the power received at 8000 feet is random power.

Other data, taken from the literature, such as the sound received over a distrance of 700 miles (from the Bahamas to Bermuda), and sound transmitted within the surface duct, confirm the model, as does a number of other comparisons with real world data [7].

AMBIENT NOISE MODEL

For the ambient background we consider the fluctuation of noise at the output of a processor consisting of 1) a narrow-band filter of bandwidth w, 2) a square-law detector and 3) an integrator, or low-pass filter, of integration time t. In this conventional processor, the voltage at the output of a narrow band filter is squared by the detector, giving an output proportional to power and is then integrated so as to yield energy. Hence such a processor is sometimes said to be an "energy" detector. The input to the square-law detector can be described by means of 2w samples per second of the output of the pre-detector filter. These 2w samples are then squared and then integrated for a period of t seconds. There is a theorem in statistics that says [8] that the sum of n samples of the square of Gaussian variate has chi-square distribution with n degrees of freedom. In our case, n = 2wt. Accordingly, samples of the output of an energy processor should have a chi-square distribution with 2wt degrees of freedom, if the ambient noise in the sea has Gaussian statistics. That this is so was observed many years ago [9].

Figure 7 shows cumulative chi-square distribution curves having 2wt as a parameter. On comparing with the distribution curves for sinusoidal signals, we observe that they are identical at both large values of 2wt and at small values of T. At these extremes both the chi-square and the Rician distributions become normal or Gaussian, with 2wt = 1/T. The distributions are also identical for 2wt = 1 and T = 1; the chi-square distribution for 2wt = 1 becomes Rayleigh when T = 1.

That the chi-square distribution applies for real-world ambient noise has been shown by analyses of noise recordings obtained with a long-line towed array. In the analysis, the recordings were filtered in narrow bands centered at 300 Hz, and distribution curves obtained for different 2wt products using different bandwidths w and integration times t. Results are shown in Fig. 8. The upper part A is a playout of slightly over 2 hours of recorded noise showing that the noise was essentially, but not completely, stationary over this length of time. The lower part B shows that distribution of samples of the input, along with the corresponding chi-square curves. The agreement is surprisingly good, except for 2wt = 2, (possibly as a result of system noise) and for 2wt = 64 and 128 possibly as a result of non-stationarity over the long analysis times - 80 and 160 minutes - required for data analysis.

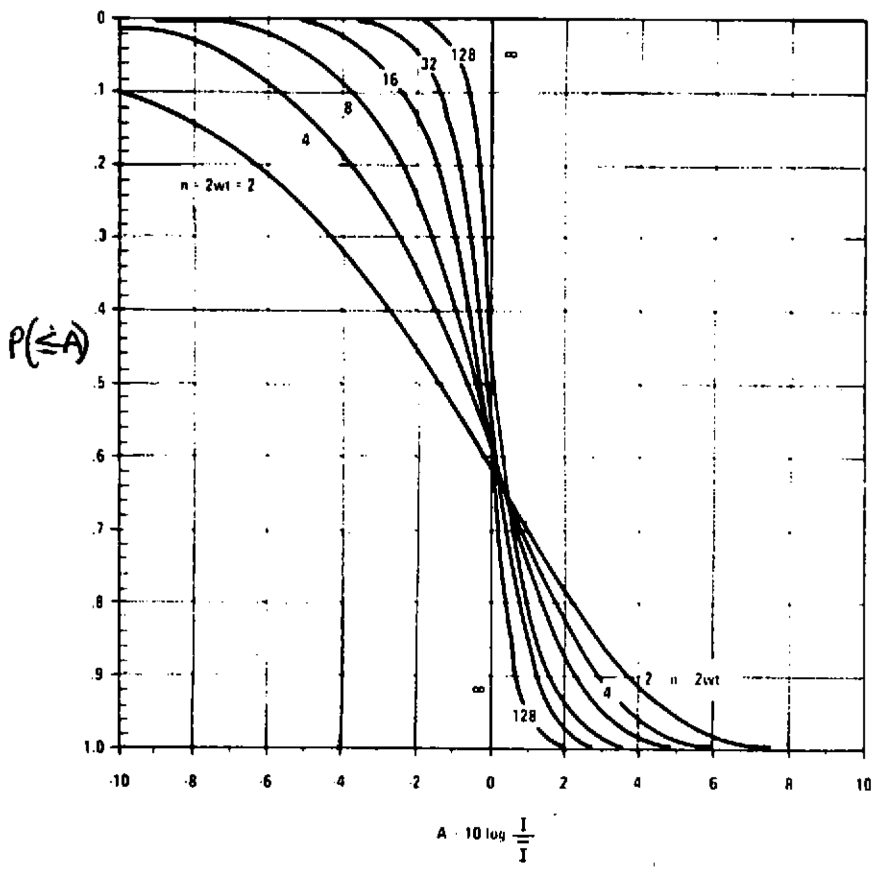
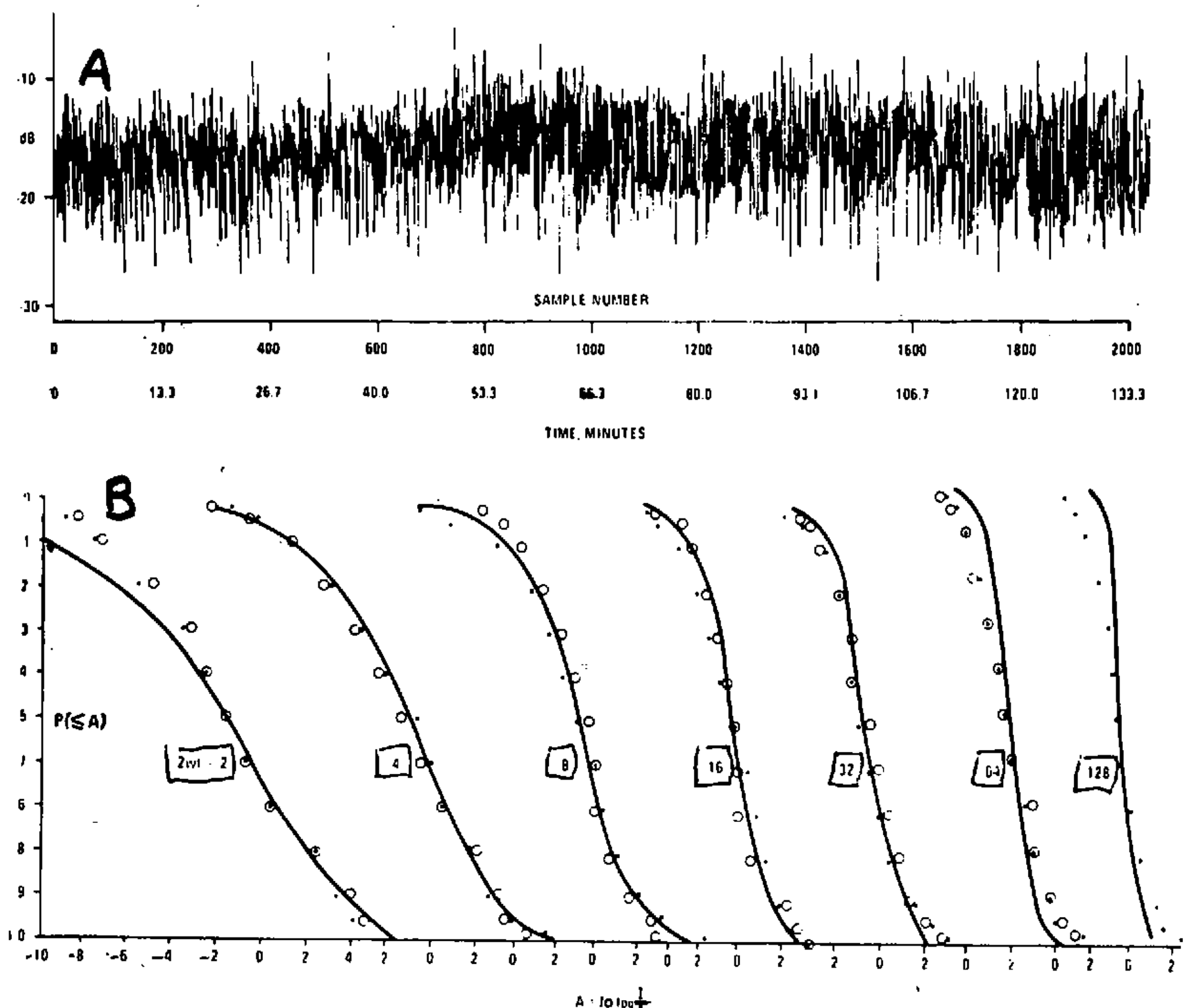


Fig. 7 Cumulative distribution curves of the level of Gaussian noise at the output of a processor of bandwidth w and integration time t.



Symbol	2wt	W, Hz	t, sec	Analysis Time, min
•	2	.25	4	10
○	"	.25	4	
•	4	.5	4	10
○	"	.5	4	
•	8	1	4	10
○	"	.25	16	40
•	16	2	4	10
○	"	.25	32	8
•	32	.25	64	160
○	"	1	16	40
•	64	1	32	80
○	"	1	32	80
•	128	1	64	160

Fig. 8 The upper figure (A) shows a 133.3 minute sample of noise filtered in a 0.25 band at 300 Hz with a 4 second integration time. (B) gives the cumulative distribution of the noise for different 2wt products compared with theoretical chi-square curves from Fig. 7. The table below shows the w, t and the analysis time for the various 2wt products above.

From all of this, we conclude that the narrow-band sound from a steady source fluctuates because of the transmission and scattering characteristics of the ocean medium. On the other hand, stationary narrow-band ambient noise, originating at the sea surface essentially over the receiver - as it is known to do over a wide range of frequency [10] - has fluctuation characteristics determined by the processor used, rather than by the characteristics of the ocean medium.

SIGNAL EXCESS AND TRANSITION CURVES

To determine the effect of fluctuation on detection, we turn to the concept of signal excess, abbreviated SE. This is the difference between the signal-to-noise level (in dB) required for detection with a probability of detection P(D) and that required for a probability of detection of 0.5, at a constant value of false alarm probability P(FA). That is, SE = 0 for P(D) = 0.5. In auditory acoustics a curve of P(D) versus SE is known as a transition curve. Fig. 9 shows transition curves for different fluctuation functions at a value of P(FA) equal to 10^{-4}. These curves were obtained by expressing P(D) as an integral over a product of two factors:

$$P(D) = \int_{-\infty}^{\infty} P(M, M_E) \cdot P_D(T, M)\, dM,$$

where $P(M, M_e)$ is the probability density of a sample mean signal + noise amplitude M about the ensemble mean M_e, and $P_D(T, M)$ is the probability of detection of M at threshold setting T determined by the selected P(FA). In terms of SE the above expression becomes

$$P(D) = \int_{-\infty}^{\infty} P(SE, \overline{SE}) \cdot P_D(T, SE)\, d(SE),$$

where $P(SE, \overline{SE})$ is the probability of occurrence of a given value of SE about its mean $\overline{SE}$, and $P_D(T, \overline{SE})$ is the probability of detection at that value of SE with a threshold setting T. This expression was evaluated on a computer for four different fluctuation functions: 1) no fluctuations with $V, \sigma = 0$, 2) Rayleigh fluctuations, 3) amplitude-normal fluctuations for a coefficient of variation V equal to 0.4 and 1.0 and 4) log-normal, (sometimes called db-normal) fluctuations of standard deviation σ equal to 0.2 and 0.6. The resulting cumulative distribution curves are shown in Fig. 9.

From Fig. 9 we may observe that at high values of P(D), where SE is positive, the effect of fluctuations is to reduce P(D); for small P(D) where SE is negative, the effect of fluctuations is to increase P(D) and so to cause better detection.

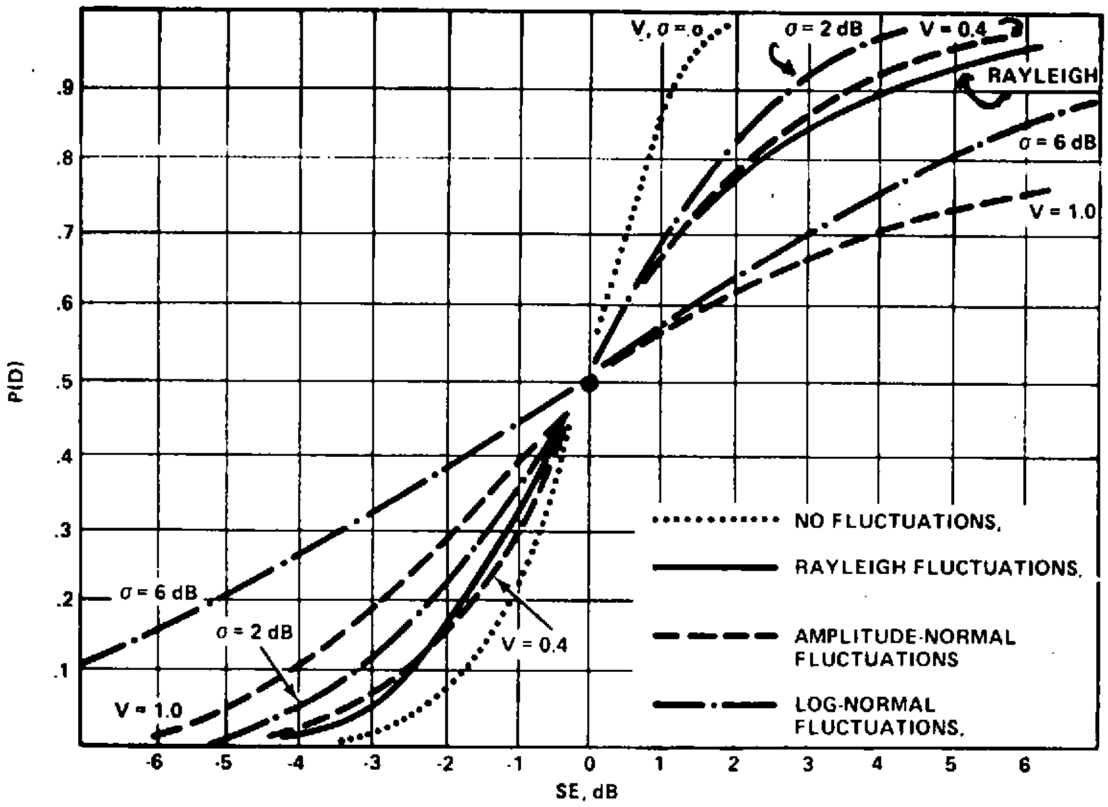


Fig. 9 Transition curves for various fluctuation functions for P(FA) equal to 10^{-4}.

FIELD VALIDATION

The question now is, which of these fluctuation functions and therefore which transition curve, if any, applies best in the real world. This question can only be answered by using operational detection data obtained from naval excercises, that is, from data on P(D) versus range as found during detection field trials. For this purpose a number of excercise data were examined [11]. One is shown in Fig. 10B for P(D) versus range for a particular active sonar against a submarine within and below a surface layer. Conversion to P(D) versus SE was done by means of curves of transmission loss versus range given in Fig. 10A, and the resulting data points for the exercise are plotted in Fig. 10C. Superposed on Fig. 10C is the computed transiton curve for log-normal fluctuations with σ = 8 db. This process was gone through for a number of excercises, and when averaged in 2-db steps, the resulting average P(D) vs SE are plotted in Fig. 11. There was no appreciable difference between passive and active sonars - amid the scatter and the paucity of detection data. Superposed are transition curves for log-normal fluctuations of SE for σ = 6, 8 and 10 db.

The cause of the apparent existence of log-normal fluctuations may well be the Central Limit Theorem applied to the sonar equations expressed as the sum of the sonar parameters in decibels. All of these parameters fluctuate in some manner. In this paper we have carefully evaluated the fluctuation of received signals caused by a fluctuation transmission loss, and the fluctuation of ambient noise at the output of all of the detection system. The fluctuation of sonar parameters, when summed, results in a normal distribution of SE. An example of the fluctuation of submarine echoes is presented in Fig. 12 which shows a sequence of echoes 1 second apart from a submarine at an oblique aspect angle and at a short range of a few hundred yards, where propagation fluctuations are negligible. We see here the changes of echoes and of the parameter target strength that are likely to be caused by small changes in aspect angle of the approaching submarine as the helmsman attempts to steer a steady course.

CONCLUSION

In this paper we have considered models for the fluctuations of narrow-band signals received at a distance from a steady source, and of the ambient background of noise in the sea at the output of an energy detector. The fluctuations of transmitted sound were found to have a Rician distribution with the randomicity T as a parameter, while those of stationary Gaussian ambient noise were found to have a chi-square distribution with twice the bandwidth-time product 2wt as a parameter. However, all of the other quantities that affect the detection of a target fluctuate as well. The result is a log-normal distribution with a standard deviation of 6-8 db for the probability of detection against signal excess, and therefore against range - a distribution that is most convenient in practical calculations. In any event, fluctuations of underwater sound cannot be ingnored in making sonar performance evaluations.

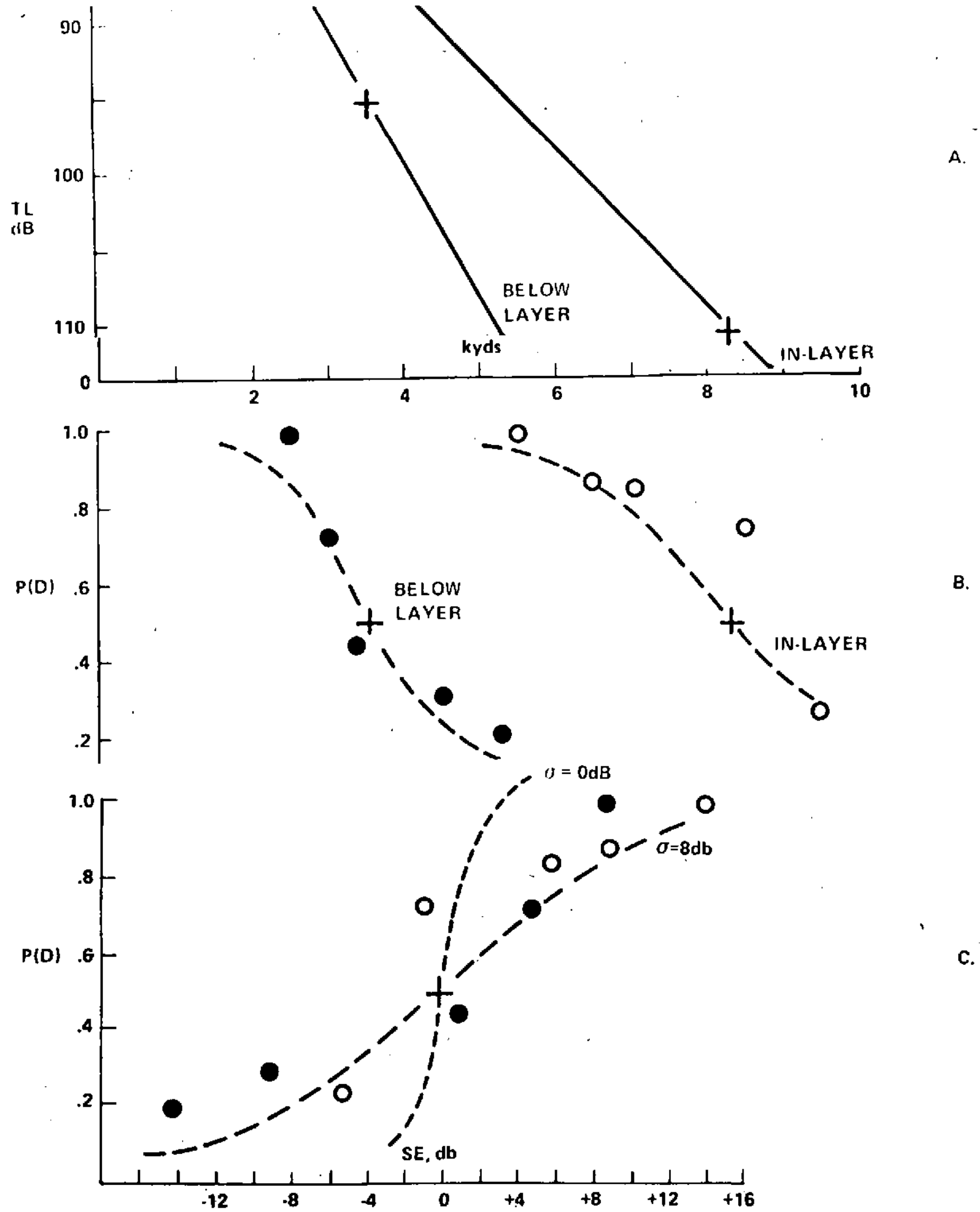


Fig. 10 Detection data of a surface-ship sonar echo-ranging on a submarine target. In A is shown the transmission loss. B shows the observed detection probability plotted against range in kiloyards for a below-layer and in-layer submarine. C is the signal excess as computed.

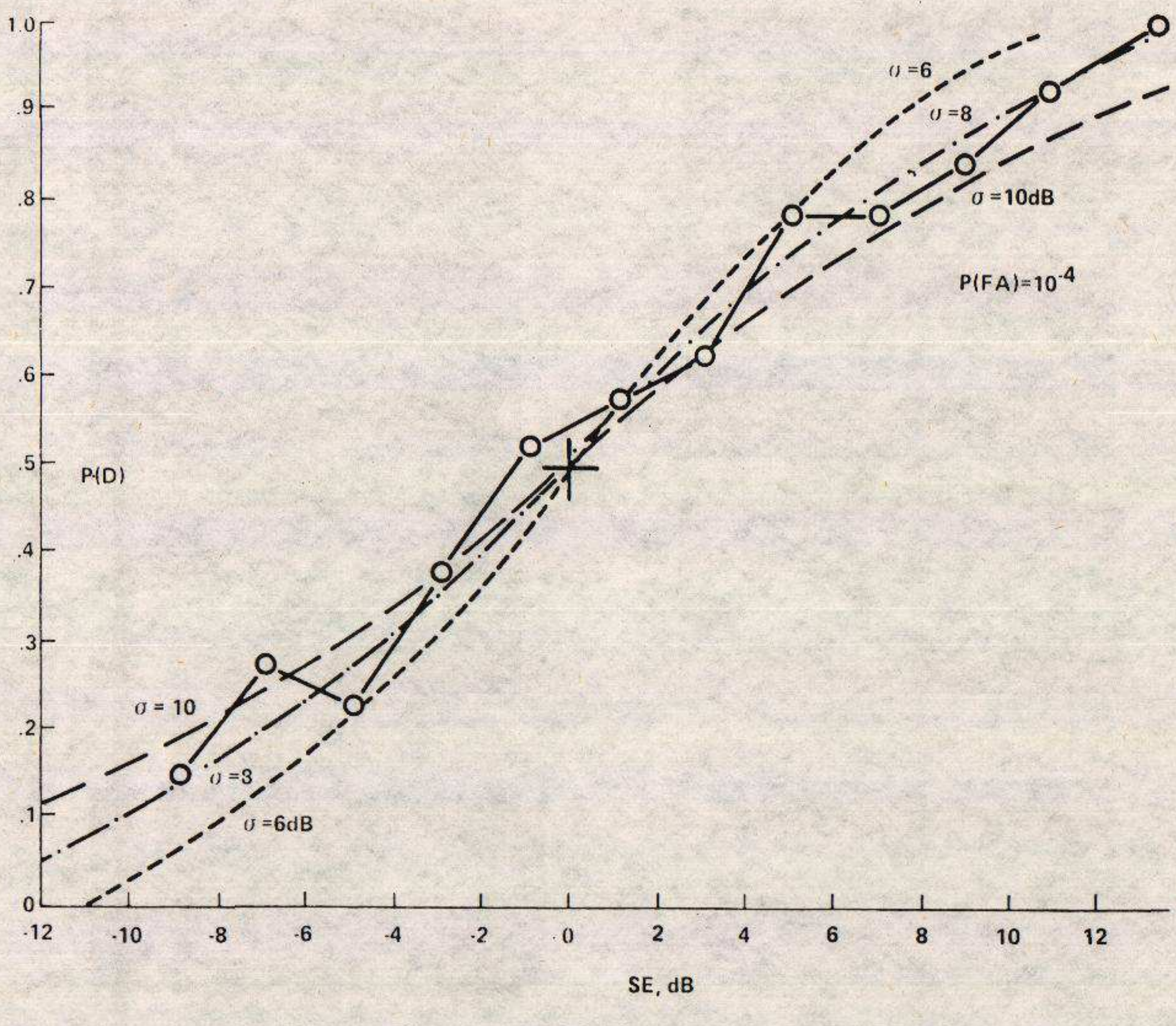


Fig. 11 Composite plot of detection probability P(D) vs. signal excess SE from the examples. The plotted points are averages in 2-dB intervals.

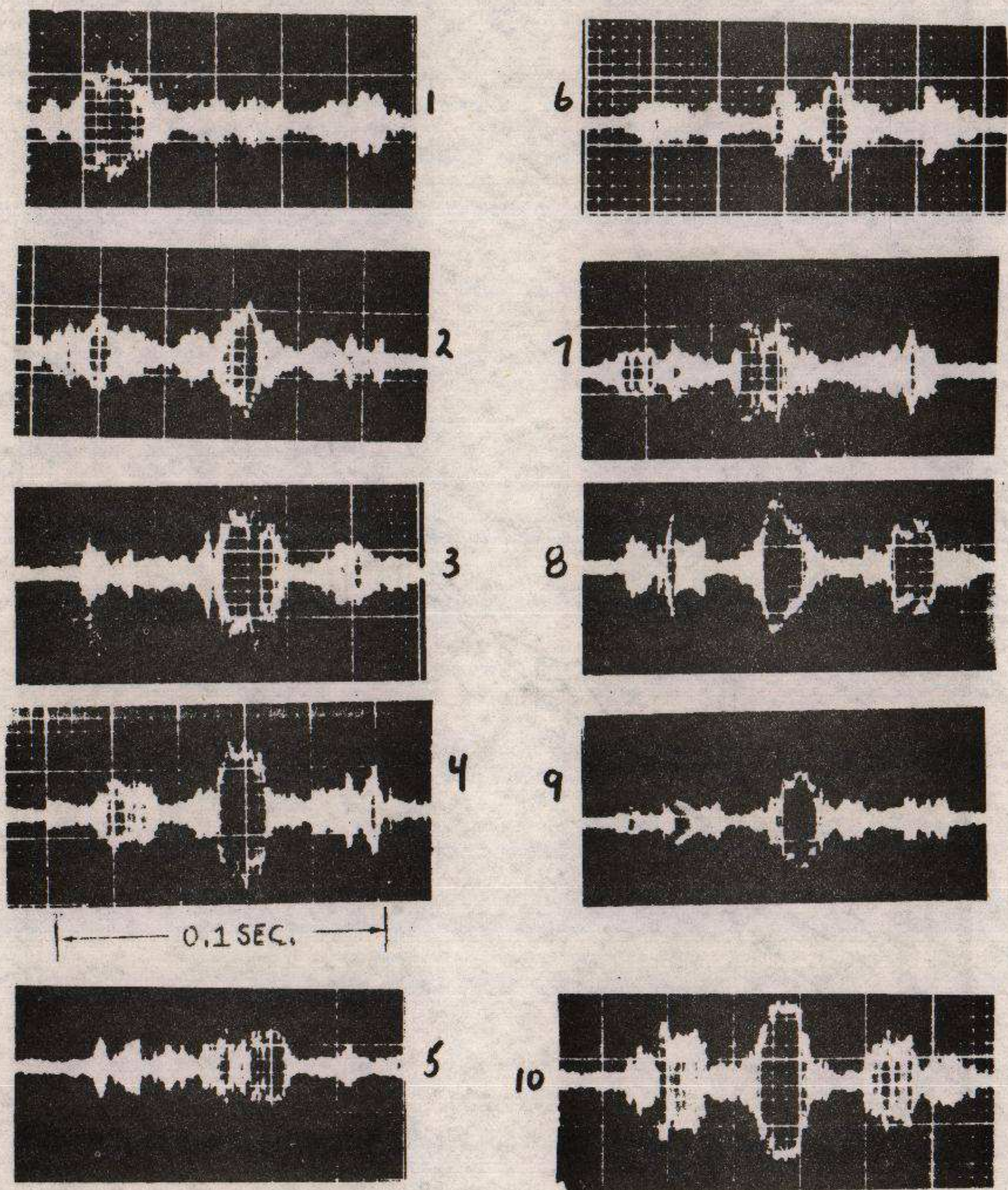


Fig. 12 A series of echoes at 1 second intervals from an approaching short range submarine at an oblique aspect angle. The fluctuation from echo-to-echo is caused by small changes of aspect angle needed to steer a constant course. The sonar pulse length was 50 ms.

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