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STATISTICAL ENERGY ANALYSIS AT LOW FREQUENCIES

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INTRODUCTION

Statistical Energy Analysis (SEA) is a useful tool for studying sound transmission in buildings but suffers from the disadvantage that at low frequencies there are few resonant modes in the various elements or subsystems that make up the building. Since SEA requires that the response of the building elements be determined by resonant modes this places a lower limit on the frequency range where SEA can be applied.

In an earlier paper [1] it was shown, from experimental data, that the lower frequency limit depends only on the the properties of the receiving subsystem. It was suggested that SEA could be used where the modal overlap was greater than 0.3 or where the mode count was at least 1 mode /band. In this paper the analysis of sound transmission between walls and floors in buildings is continued and upper and lower error limits are derived. This not only allows the lower frequency limit to be assessed more accurately but it allows an estimation of the potential error should multimodal conditions not be met.

COUPLED RESONATOR MODEL

In order to derive upper and lower error limits for SEA it is useful to consider coupling between two simple resonators as shown in Fig 1.

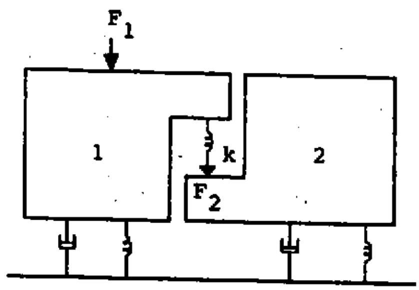

Fig 1. Two coupled resonators.

For each resonator the power, W, entering the resonator will be given by [2]

$$W = F^2 \, Re(Y) \tag{1}$$

where F is the exciting force and Y is the mobility of the resonator and is defined as the ratio of the velocity to force v/F. In this paper only the real part of the mobility is used so this is abbreviated to Y.

For the coupled resonators the force acting on the second resonator is equal to the compression of the spring times the spring stiffness, k. This can then be written in terms of the velocity of the resonators as

$$F_2 = (v_1 - v_2) \, k/\omega \tag{2}$$

If the velocity of the second resonator is small compared to the velocity of the first resonator then the force can be approximated to $F_2 = v_1 \, k/\omega$. This gives the power input into the second resonator as

$$W_{12} = F_2^{\,2} \, Y_2 \propto v_1^{\,2} \, Y_2 \tag{3}$$

This model of two resonators is very simple and makes assumptions about the relative vibration levels but nevertheless it leads to two important results that are observed in practise. Firstly, the power flow is determined by the vibration level of the source resonator but does not actually require that the resonator be vibrating at resonance. This agrees with the measurements made in buildings.

The second important result is that the power flow is proportional to the mobility of the receiving subsystem. As the frequency decreases the number of modes decreases so the mobility fluctuates about the mean position. The power flow will fluctuate in the same manner.

In SEA notation the power flow between two subsystems is defined as

$$W_{12} = E_1 \omega \eta_{12} \tag{4}$$

where E is the energy in a subsystem and η_{12} is the coupling loss factor. Since (for walls and floors) $E = mv^2$ it follows that the coupling loss factor must be proportional to the mobility of the receiving subsystem. Therefore an analysis of the mobility of subsystems can be used to predict coupling between subsystems and hence to give the limits of SEA.

The coupling loss factors (CLFs) are known for most cases where the subsystems are large and multimodal. Since the CLF is proportional to the mobility the CLF at lower frequencies, where multimodal conditions do not apply, can be given as

$$\eta'_{12} = \eta^{\infty}_{12} \times Y_2 / Y_{\infty 2} \qquad (5)$$

where η^{∞}_{12} is the usual SEA CLF prediction for multimodal subsystems and η' is the CLF where multimodal conditions do not exist. Thus the CLF is proportional to the mobility and is equal to the infinite plate value at high frequencies.

The ratio of the CLFs η' to η_∞ is a measure of the error in the SEA prediction. This is equal to the ratio of Y/Y_∞

$$\frac{\eta'}{\eta_\infty} = \frac{Y}{Y_\infty} \qquad (6)$$

As the frequency is reduced so the ratio of Y/Y_∞ will fluctuate causing a fluctuation in the CLF. However the fluctuations in the ratio of mobilities can be readily determined.

For measurements made in 1/3 octave bands the upper limit to the ratio Y/Y_∞ is given in dB by [3]

$$\text{Upper limit} = 10 \text{ Log } (1/N) \qquad (7)$$

where N is the number of modes in a band (statistical value not actual value). This is valid where $N < 1$.

The lower limit to the ratio Y/Y_∞ occurs when there are no resonant modes in a band and is given by [3]

$$\text{Lower limit} = 10 \text{ Log}(4\, M/\pi) \qquad (8)$$

M is the modal overlap and is given by

$$M = n\, f\, \eta \tag{9}$$

where n is the modal density of the plate and η is the damping.

These limits derived for mobility apply equally to the coupling loss factor.

RESULTS

Sound transmission between pairs of walls and floors was measured in a building [1]. It was found that, where the source subsystem had no resonant modes, the measured coupling loss factor agreed well with the predicted CLF. This is consistent with the simple model of two resonators.

Results for the case where transmission was from a large wall to a small wall are given in Fig 2.

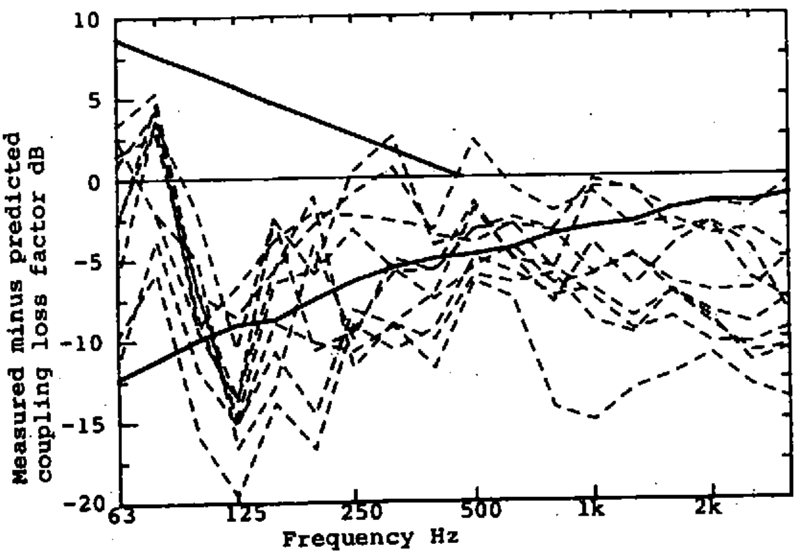

Fig 2 Sound transmission from a large wall to a small wall. ____ individual results; ----- upper and lower error limit.

Ten measured examples of the construction were measured and all are given in the figure. It can be seen that at mid frequencies the measured results are less than predicted by about 5dB and this is probably due to a limitation of the theory for coupling between large plates. Therefore all the measured results should be increased by 5dB. More importantly at the lower frequencies the results tend to lie between the proposed limits. At 80Hz there is the first resonant mode in the receiving subsystem and so there is a peak in the CLF. At 125Hz there is no mode in the band and so the CLF approaches the lower error limit.

Poor agreement at the high frequencies is due to the limitations of thin plate bending wave theory which was used for all predictions.

Similar results for transmission from one small wall to another small wall are given in Fig 3. Again ten measured results are given. In this case the agreement at the mid frequencies is better and again at the lower frequencies the results tend to lie inside the proposed error limits.

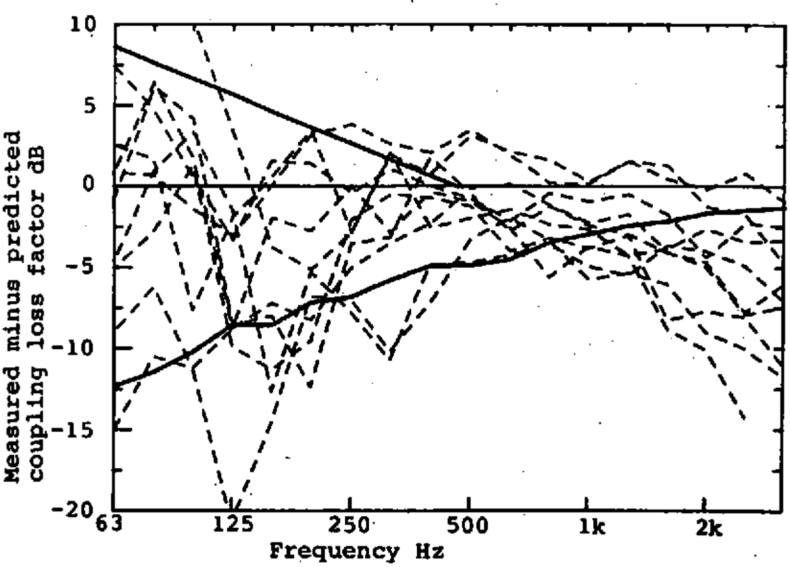

Fig 3. Sound transmission from a small wall to a small wall. _____ individual results; ----- upper and lower error limit.

CONCLUSIONS

An analysis of coupled resonators has shown that under some special circumstances the coupling between subsystems will not depend on whether the vibrations of that subsystem are due to resonant modes. This agrees with experimental results.

The analysis also shows that the coupling is proportional to the mobility of the receiving subsystem. Upper and lower error limit derived for mobility are found to fit well with data for coupling loss factors.

ACKNOWLEDGEMENTS

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