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SOUND TRANSMISSION PATHS THROUGH LARGE STRUCTURES

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INTRODUCTION

One of the reasons why statistical techniques (such as statistical energy analysis) were introduced in the early 60s was that classical techniques such as modal analysis could not be used with any great confidence on, large structures with many thousands of modes and irregular boundary conditions. Since then statistical energy analysis has been used for increasingly complex problems.

Total energy is usually assumed to be only one unknown variable for each room, wall or floor so that for small structures the number of variables is manageable and the overall performance of a structure can be understood. In principle there is no reason why large structures cannot be studied since a building will probably have no more that a few hundred walls, floors and rooms (or subsystems). However, whilst there is no problem for a computer to determine the solution for a particular design, generalising the effects of design changes is not easy.

In this paper alternative methods of determining the performance of large structures are outlined based on the concept of sound transmission along specific paths. Some of the general properties of these paths are then described.

TRANSMISSION BY PATHS

If two subsystems are connected together as shown in Figure 1 then the power balance for subsystem 2 is

$$E_2/E_1 = \eta_{12}/\eta_2 = s_{12} \tag{1}$$

The ratio η_{12}/η_2 determines the energy ratio of the two subsystems and since it occurs frequently is given the symbol s denoting the step from 1 to 2.

In buildings it is found that the values of the coupling loss factors (CLFs) for coupling between structural elements are very similar for almost all types of joint found in domestic buildings. Since the total loss factor is the sum of the coupling loss factors plus the internal loss factor a quick approximation to the value of the step, s, for structural coupling is 1/(number of connections). Thus for the system in Figure 2, which shows two rooms separated by a common wall (but excludes the floors and ceilings) there are about 4 structural connections to each element and therefore s is approximately 1/4.

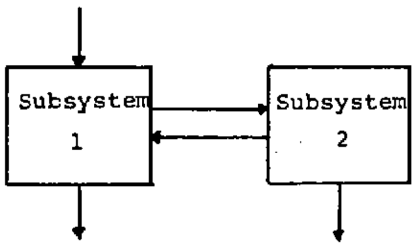

Figure 1 Two coupled subsystems.

In practise including the internal losses will decrease the value of s. For real buildings the value of s is about 1/10 so that as a rule of thumb the structural attenuation is 10dB for each joint.

It can be shown that for any path through the system from subsystem 1 to n the attenuation along the path given by E_n/E_1 can be given as

$$E_n/E_1 = s_{12}\ s_{23}\ s_{34}\ s_{45}\ \cdots\cdots\quad s_{n-1\ n} \tag{2}$$

and is therefore simply the product of all the steps in the path.

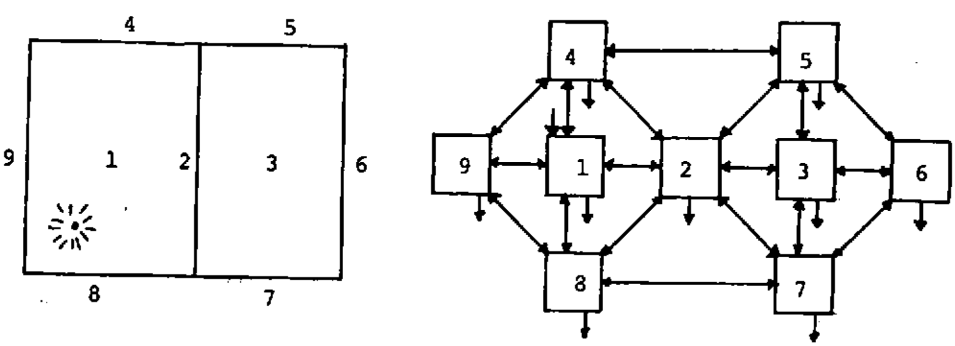

Figure 2 Model of two rooms separated by a common wall (ignoring the floors and ceilings).

The valid paths through the system are any path that begins with the source subsystem and ends in the receiving subsystem and does not include the source subsystem except as the starting point. All other subsystems may be included any number of times. Thus for sound transmission from room 1 to room 3 in the model shown in Figure 2 the path 1 2 3 2 3 2 3 5 3 is a valid path.

For the system in Figure 2 (again for transmission from room 1 to room 3) the shortest path is path 123. Paths involving 1 structural joint are 1453, 1423, 1253, 1873, 1273, 1823. There is therefore 1 path of length 3 subsystems and 6 with 4 subsystems. The number of paths for longer path lengths is given in Table 1.

Number of subsystems in path	Number of Paths
2	0
3	1
4	6
5	24
6	96
7	360
8	1368
9	5112

Table 1 Number of paths for the system shown in Figure 2.

The number of paths increases at a rate equal to the average number of connections between subsystems. Thus for the system shown in Figure 2 there are approximately 4 connections to each subsystem and so the number of paths increases by a factor of about 4 each time the path length increases by one.

There is an infinite number of paths through the system. As the paths become longer they are also more numerous. However, since the importance of a path becomes smaller at a faster rate than the number of paths increases the overall effect is that long paths are less important.

If the internal loss factors were zero then the rate at which path attenuation decreased would exactly match the rate at which the number of paths increased.

The importance of the internal losses can be seen by considering a system in which every subsystem is connected to every other subsystem and which for convenience all the CLFs are equal. For this type of system the number of paths with n subsystems is

$$\text{Number of paths} = (m-2)^{(n-2)} \qquad (3)$$

where m is the number of subsystems in the model. The magnitude of each step can be given by

$$s = 1/(r(m-1)) \quad (4)$$

where m is the number of subsystems in the model and r is the ratio of the total loss factor to the sum of the coupling loss factors.

For a path with n subsystems the magnitude of the path is

$$E_i/E_1 = s^{(n-1)} = 1/(r(m-1))^{(n-1)} \quad (5)$$

and since the number of paths is as given in equation (3) the sum of all paths of length n is

$$E_i/E_1 = (m-2)^{(n-2)} / (r(m-1))^{(n-1)} \quad (6)$$

Summing over all paths (letting n vary from 2 to infinity in equation (6)) gives the energy due to all paths as

$$E_i/E_1 = 1/(n(r-1) - r + 2) \quad (7)$$

Subsystems in path	number of paths	percentage sound transmission due to paths $r = 1.1$	$r = 2$
2	1	19.2	55.6
3	8	15.5	24.7
4	64	12.5	11.0
5	512	10.1	4.9
6	4096	8.2	2.2
7	32768	6.2	1.0
8	262144	5.3	0.4
9	2097152	4.3	0.2
10	16777216	3.5	0.1
sum of 10	19173961	85.3	99.9
sum of all paths (percentage)		100.0	100.0
sum of all paths ($10Log(E_i/E_1)$)		-2.8dB	-10.0dB

TABLE 2 Contribution of paths of a particular length.

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For most real buildings, the sum of the coupling loss factors can be approximated to $f^{-1/2}$ and the internal loss factor will usually be approximately 0.01 to 0.02. Thus at low frequencies at 100Hz the value of r lies between 1.1 and 1.2. At high frequencies say at 3150 Hz will be approximately 2.

The effect of changing the value of r from 1.1 to 2 can be seen in Table 2 which shows the contribution of all paths of a particular length for a system with 10 subsystems all of which are interconnected

The difference between high and low frequency sound transmission can be clearly seen in the way in which the sum of the contributions of the paths converges on the final answer. At high frequencies sound transmission is dominated by transmission along short paths. Despite the large number of connections over half the sound is transmitted by the direct path.

In contrast at low frequencies only 20% of the sound is transmitted by the direct path which is 7dB less than the total sound transmitted. Even accounting for the first 20 million paths only accounts for 85% of the sound. The resultant energy level difference between each subsystem and the source subsystem is also much less so that the attenuation changes from 10dB to only 2.8dB.

IMPLICATIONS FOR BUILDING DESIGN

So far as structure-borne sound is concerned it is unfortunate that there is little scope for major changes in performance for sound transmission over long distances. If the value of 'r' can be increased then attenuation will be more rapid and the resultant energy will be less. However this is difficult to achieve. The internal loss factor which dissipates the energy is usually within the range 0.01 - 0.02 and unless drastic measures, such as coating all surfaces with bitumen, are taken then it cannot be significantly changed.

Decreasing the average coupling loss factor is another way of changing the the value of 'r' but this too is difficult. Making all the walls twice as thick will actually increase the CLF by 1.5dB making things worse. It will however have the effect that forces acting on the walls and floors will input 6dB less power into the structure thus reducing the overall transmitted sound. This is true both for airborne sound and structure-borne sound. The fraction of sound radiated will also be reduced (by 3dB) giving further reductions in sound transmission.

Another way the sound can be reduced in a noise sensitive area is to transmit the sound to another part of the building where an increase in not important. However, here again it is difficult to achieve significant reductions in sound.