Resolution Performance Of A Broadband Time Domain Algorithm.

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1. INTRODUCTION

Successful application of adaptive beamforming algorithms (ABF) to passive data from sonar arrays has obvious implications in the commercial and defence industries, thus making it a long established topic for research. To date many methods have resulted, most of which are narrow band or complex broadband algorithms which are computationally expensive to use in a realistic environment. The problems involved include :

a) The large bandwidth; typical frequency ranges are of 5-8 octaves. Hence for a fully optimised system with a frequency domain ABF, an optimal solution would be required for every FFT cell, resulting in a large computational workload. The alternative would be the use of suboptimal techniques.

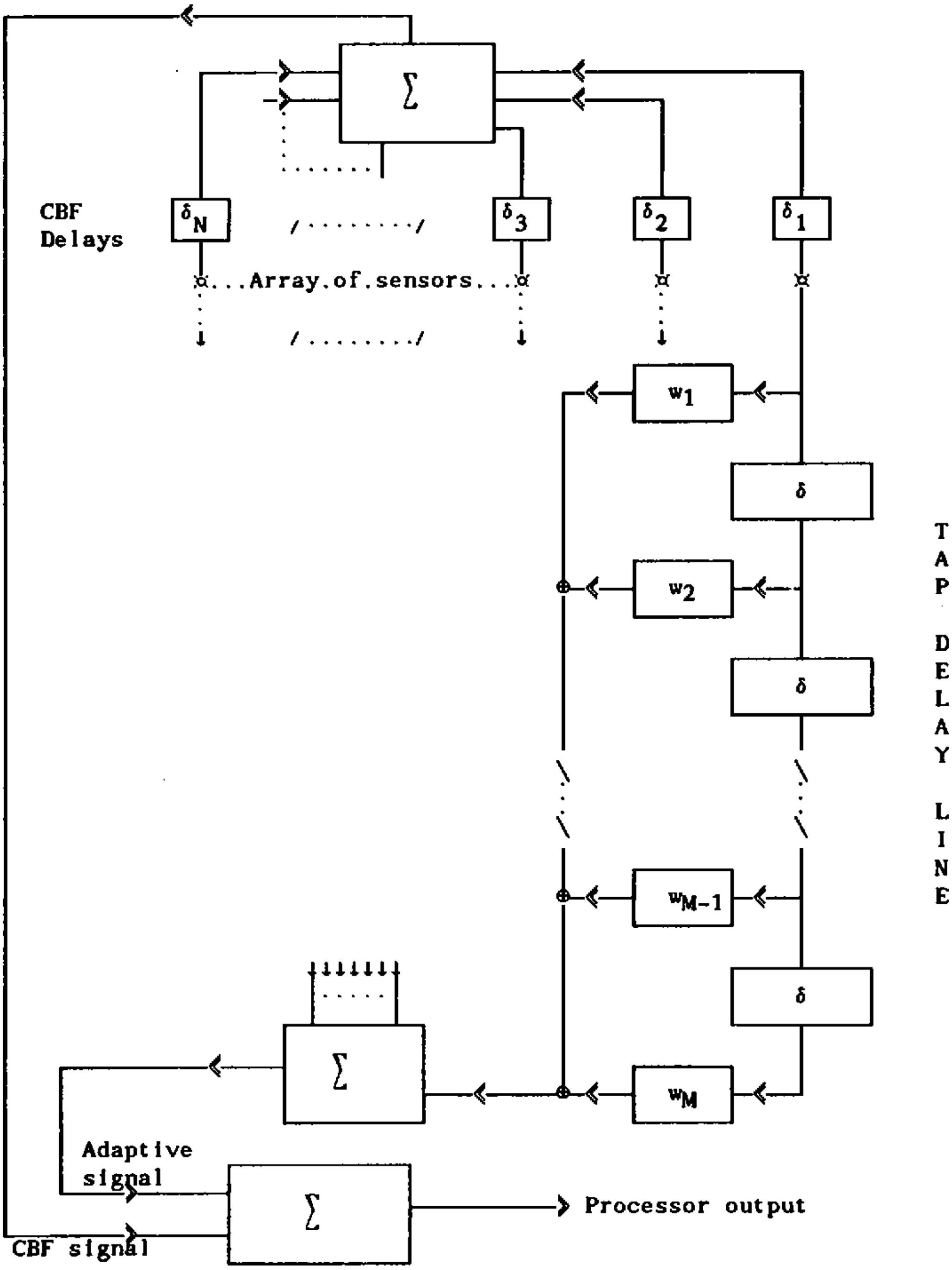
b) Random amplitude and phase errors at the sensors give rise to rapid degradation of the algorithm, as the system will then be rejecting the wanted signal as well as the interfering noise.

c) Isotropic noise, sensor positional errors, and sensor self noise also cause significant damage.

The processor to be presented uses a sub-optimal time time domain adaptive algorithm, designed to be robust against noise and errors. The algorithm is of the the LMS type and is a continuation of work done by Nunn[2][3] following the works of Frost[4] and Griffiths[5]. The one octave bandwidth together with the robust sub-optimal nature of the algorithm imply low computational costs, making the system a practical choice for real time implimentation.

2. PROCESSOR CONFIGURATION

For processing purposes the array is divided into two. The main array and the adaptive array. The main array usually consists of most of the available sensors and is used to form a complete set of fixed beams of desired beamwidth. Conventional beamforming is used in preference to time shifting and Chebyshev weighting. The adaptive array embodies a combination of sensors taken to cover the whole of the aperture. As can be seen from figure 1, the processor output is obtained from the sum of the adaptive and main beams, where the adaptive beam is formed from the weighted sum of the tap / sensor signals.



Figre 1 : The Processor Architecture.

AN ADAPTIVE TIME DOMAIN PROCESSOR

3. LIMITATIONS

Before going on to describe the processor in detail it is important to note its limitations, and the conditions essential for its successful operation.

This processor has far fewer variables per octave than its corresponding frequency domain processor [6], and so represents a coarse optimisation. Thus only a limited number of strong spectral lines from interfering sources may be nulled out, and the processor can only produce approximations to broadband nulls in the presence of strong broadband noise. These properties can however be improved at the cost of increasing the array size and computational workload.

The conditions essential for successful operation include :

i) The CBF beam must have sufficiently low sidelobe levels. To obtain this the main array must have a fairly large number of sensors (>10) and a sufficient aperture.

ii) CBF beamforming must not be excessively degraded by sensor error levels. If this has occured then more processing is required before beamforming.

iii) Separate optimisation processes must be carried out for each octave when a field of several octaves bandwidth is involved. Processing in bandwidths of more than 1 octave would lead to a system that is very suboptimal, and bandwidths of less than 1/2 an octave make for a clumsy configuration.

4. THE ALGORITHM

4.1 CONVENTIONAL BEAMFORMER

In each processing band a main array sub-array of N elements is selected, giving a signal vector set $\{X_n\}$. For any array with elements at coordinates (x_n, y_n, z_n) a directivity pattern called the conventional beam can be obtained by placing the appropriate time delays after each of the sensor elements before summing. These time delays are given by:

$$\tau_n = [\, \cos\theta \,(x_n \cos\varphi + y_n \sin\varphi) + z_n \sin\theta \,] \,/\, C \qquad \ldots 1$$

Where θ and φ give the look direction, and C is the speed of sound through water. The CBF beam is thus defined as :

$$Y_o(t) = \frac{1}{N} \sum_{n=1}^{N} X_n \,(t - \tau_n) \qquad \ldots 2$$

4.2 PROCESSOR OUTPUT POWER

Past experimentation [1], has shown that for optimisation to take place over a bandwidth of 1 octave, the real adaptive array output must be passed down a tapped delay line with M taps, and a tap separation of $T_o=1/4F_m$. Where $M \simeq 4 \rightarrow 6$ and F_m is the median frequency of the processing band. This provides enough free variables in order to produce several broadband nulls of reasonable quality.

Now from figure 1, the optimised output beam Y(t) is formed from the main array CBF output $Y_o(t)$ plus the weighted sum of the MJ tap/sensor signals from the adaptive array :

$$Y(t) = Y_o(t) + \sum_{j=1}^{J} \sum_{m=1}^{M} U_{jm} X_j (t - (m - 1) T_o) \qquad \ldots 3$$

and in its matrix form :

$$Y(t) = Y_o(t) + \sum_{k=1}^{MJ} W_k \, . \, Z_k$$

$$= Y_o(t) + W^T \, . \, Z \qquad \ldots 4$$

Where Z(t) is the vector of real filtered signals on the MN taps of the adaptive array, and W is the adaptive array weight vector. The total power is thus given by :

$$P = \overline{Y}^2 = \overline{Y}_o^2 + 2\, W^T\, \overline{Z\, Y_o} + W^T\, \overline{Z\, Z^T}\, W$$

$$= P_o + 2\, W^T R + W^T Q\, W \qquad \ldots 5$$

Where P_o is the CBF power, Q is the correlation matrix of the MN tap/sensor signals $Z_k(t)$, and R is the correlation vector between the main beam output Y_o and Z.

4.3 OPTIMISATION OF W

4.3.1 APPLICATION OF A NORM CONSTRAINT

One method of obtaining a set of sub-optimal weight vectors would be to apply a norm limit on W in equation 5. A 'soft' norm limit on W can be applied by adding a term $W^T \Lambda W$ to the power P. Where Λ is a symmetric matrix, normally taken as diagonal. Hence :

$$P' = P_o + W^T Q W + 2 W^T R + W^T \Lambda W$$

and by minimising P' with respect to W :

$$dP' = 2dW^T \, (QW + R + \Lambda W)$$

$\Rightarrow \qquad (Q + \Lambda) W = -R$

$\Rightarrow \qquad W_{opt} = - (Q + \Lambda)^{-1} R$ ...6

Where Λ is put at : $\Lambda = \lambda \underline{1} Q_{11}$ such that it acts as a norm limiter. This approximately limits the norm of W_{opt} to $1/\lambda G$ where $G = P_o/Q_{11}$ when looking at a strong source. Now for the main beam to be preserved, the norm of W must be restricted to about 0.1 of the CBF norm. Hence the norm limit needs to be about 1/M/10 , or $\lambda \approx 10M/G$. Experimentation with some simple time stationary fields showed λ of typical values of 0.01 to give a satisfactory performance. The optimal value of λ is however dependant on the strength of the signal in the look direction.

This method of optimisation was found to give a satisfactory performance in the common case of the system looking at a weak wanted signal. Problems were however encountered when the signal in the look direction was of significant strength. In this case by trying to minimise the power P' the algorithm partly cancels the signal in the main beam with the signal in the adaptive channel $W^T Z$. Although the extent of this cancellation is limited by the norm constraint, the resolution of the system is greatly reduced due to the "flatter" responses.

4.3.2 LINEARLY CONSTRAINED OPTIMISATION OF W

By introducing directional null constraints into the adaptive channel, the presence of the wanted signal in this channel can be avoided. Minimising the processor output power has no effect on the wanted signal; thus leaving its power equal to that contained in the main beam, and the sidelobes greatly reduced.

<u>The constraints :</u> Consider a monochromatic signal of the form EXP(jωt) at the centre of the array, coming from the look direction bearing φ_1, θ_1. This would appear at the output of the adaptive channel as :

$$W^T Z = \sum_{k=1}^{MN} W_k \text{ EXP } \{ j \omega (t + \tau_n(\varphi,\theta) - (m-1) T_o) \}$$

Where $\omega = 2\pi f$, $j = \sqrt{(-1)}$, m = tap number , and n = sensor number. For this signal not to apear at the output of adaptive channel $W^T Z$, W must satisfy the linear constraint :

$$C^T W = 0 \quad ; \quad C_R^T W = 0 \quad ; \quad C_I^T W = 0$$

Where :

$$C_k^T = \text{EXP } [j \omega \{ \tau_n(\varphi,\theta) - (m-1) T_o \}],$$

$$C_R = \text{REAL}(C) \quad ; \quad C_I = \text{IM} (C)$$

Now, since the above constraint only applies to a single frequency, several (N_c) of these point constraints are required to be located at frequencies spaced across the frequency band. Typically, $N_c = 5$ should be sufficient for a one octave processing band. Hence a collection of linear point constraints must be satisfied :

$$\left.\begin{array}{l} C_R^T\ (\ f_p\ ,\ \varphi_p\ ,\ \theta_p\ )\ W = 0 \\ C_I^T\ (\ f_p\ ,\ \varphi_p\ ,\ \theta_p\ )\ W = 0 \end{array}\right\}\ p = 1\ ,\ N_c$$

So far it has been assumed that the main array CBF beamforming does not shift the phase of the wanted signals unless assymetric errors are present. This is used to arrive at the further assumption that the out of phase constraints $C_I^T.W = 0$, are probably not needed where asymmetric errors have not occured. Neglection of these constraints does not introduce any approximations that have not already been made, but care must be taken as the system is now being made slightly less robust. The total matrix constraint on W is thus reached by retaining the in-phase constraints :

$$C^T\ .\ W = 0 \qquad \ldots 7$$

$$C = [\ C_R(f_1,\varphi_1,\theta_1)\ \mid\ C_R(\ldots\ \ \ldots\ \mid\ C_R(f_{Nc},\varphi_{Nc},\theta_{Nc})\ ] \qquad \ldots 8$$

So the system is now to minimise the broadband output power P of equation 5 with respect to W, and subject to the constraints of equation 7. A similar quantity P' as in the previous system (4.3.1) can be defined here to apply a soft norm constraint :

$$P' = P_o + W^T\ (Q + \Lambda)\ W + 2\ W^T\ R + \eta\ C^T\ W$$

Where η is a matrix of Lagrange multipliers and Λ is a similar matrix to that of the previous system, but its value given by λ is different here. Experimentation with some time stationary noise fields showed $\lambda \approx 0.01$ to give good performance, but again more research is needed to find a system that would automatically calculate the optimal λ for a time varying noise field. Now, differentiating the above equation to obtain the minimum power P' gives :

$$dP' = 2\ dW^T\ [\ (Q + \Lambda)\ W + R + 0.5\ C\ \eta^T\ ]\ = 0$$

$$\Rightarrow \quad W_{opt} = -\ (Q + \Lambda)^{-1}\ .\ (R + 0.5\ C\ \eta^T) \qquad \ldots 9$$

and on substituting equation 9 into the constraint of equation 7 :

$$C^T\ [\ (Q + \Lambda)^{-1}\ .\ (R + 0.5\ C\ \eta^T) = 0$$

$$\Rightarrow \quad \eta^T = -2\ (\ C^T\ (\ Q + \Lambda)^{-1}\ C\ )^{-1}\ .\ C^T\ (Q + \Lambda)^{-1}\ .\ R \qquad \ldots 10$$

thus by substituting equation 10 back into equation 9 , the optimum weight vector is obtained :

$$W_{opt} = -\ (Q + \Lambda)^{-1}\ [\ C^T\ (\ C^T\ (Q + \Lambda)^{-1}\ C\ )^{-1}\ C^T\ (Q + \Lambda)^{-1} - \underline{1}]\ .\ R$$

This is written in a more simplified form as :

$$S = (\ Q + \Lambda\ )^{-1} \qquad \ldots 11$$

$$V = C^T\ .\ S \qquad \ldots 12$$

$$W_{opt} = S\ [\ C\ (\ V\ .\ C\ )^{-1}\ V - \underline{1}\ ]\ .\ R \qquad \ldots 13$$

5. SUMMARY OF RESULTS AND CONCLUSIONS

The following will present a general summary of the simulation runs carried out to assess the resolution performance of the processor. Full graphical details shall be made available at the conference.

The noise fields simulated were generally of one octave bandwidth containing combinations of narrow band and wideband signals, isotropic noise, sensor selfnoise and sensor errors. Linear arrays used were of sizes 10, 20, and 30 elements with interelement spacings of $\lambda/3$.

To measure the resolution performance of the processor a factor called 'dip' was defined. This was simply the depth of the trough occuring between signal peaks. Successful runs produced dips of more than 2.5 dbs. The cases studied gave the following results.

No errors : In the absence of all types of errors; resolutions of less than 0.2 degress were observed for two broadside wideband signals for the largest of the arrays.

Base case : The baseline case chosen contained two broadband signals arriving from directions 'normal' to the array, with added isotropic noise level of +3db, sensor self noise of −15db, and sensor errors of $E = \pm 0.5$dbs , $\delta\varphi = \pm 5$ degrees. A maximum resolution of 2 degrees was obtained using the largest of the arrays, with the smallest array giving a decreased resolution of 5 degrees.

Selfnoise : Using the base case , self noise was increased such as might result from increased array motion thorough water. The resolution was seen to be unaffected for P_{sn} of up to −7dbs. Increased selfnoise levels then started to reduce the resolution.

Isotropic noise : Isotropic noise causes deterioation in the resolution bacause (a) it fills the 'gap' between the 2 sources and (b) it supresses the weight vector norm. Taking the base case and increasing P_{iso} to +6dbs, deteriorated the resolution to about 3 degrees for the largest of the arrays. Increased levels of P_{iso} caused a rapid deterioration in the resolution.

Sensor errors : Taking the base case with different levels of amplitude and phase errors it was seen that the processor was more sensitive to phase errors.

Multiple sources : In the absence of noise and errors, resolutions of less than 0.5 degrees were achieved. Simulations showed dips of 15dbs resulting with 3 wideband sources placed at separations of 1 degrees.

To summarize : a robust adaptive array processor of one octave bandwidth has been presented. From simulations carried out it was discovered that the processor's resolution suffered little when subjected to low levels of noise and errors. Only small changes in the resolution were observed with high levels of sensor selfnoise and errors. However, the system showed signs of degradation in the presence of stronger isotropic noise. Larger arrays which provided the system with more degrees of freedom gave better resolutions at the cost of increased computational load. However due to the simplicity and sub-optimal nature of the algorithm, computing costs were much much less than that of a fully optimised system or that of the frequency domain equivalent. The time domain processor can thus be recomended as an economical and robust choice for real time implimentation.

6. REFERENCES

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