Proceedings of The Institute of Acoustics

FFT ALGORITHM FOR DOPPLER DIFFERENTIAL CORRECTION

R. O. ROWLANDS

APPLIED RESEARCH LABORATORY
THE PENNSYLVANIA STATE UNIVERSITY

INTRODUCTION

The process of estimating the time delay between the same signal that may be present in two separate channels involves the generation of their cross-correlogram. The selection of the instant at which this peaks is the best estimate of the time delay. The processing technique generally employed involves taking Discrete Fourier Transforms (DFT) of the signals and multiplying one by the conjugate of the other. A fact that has not received much attention in the literature is that the conventional form of the fast algorithm for calculating the transform (FFT) does not accurately preserve the phase of the signal. This does not matter if there is no Doppler shift between the signals because the phase error is made negative when the conjugate of the spectrum of the second signal is formed and this cancels the phase error in the first signal. If there is a Doppler differential between the signals, however, the errors are different and they do not completely cancel. This paper describes a phase corrected FFT algorithm that may be used to overcome this problem.

THEORY

Let a continuous signal be sampled at intervals of 1/2W where W is the signal bandwidth. A batch of n samples will define a portion of this signal that has a duration T where n=2WT. This definition specifies that T encompasses n time intervals rather than the (n-1) intervals that lie between the first and last sample. A DFT enables the spectrum of the signal to be computed at discrete frequencies h/T using the formula

$$F(h/T) = \sum_{k=0}^{n-1} G(k/2W) \exp(-j2\pi hk/n) \qquad (1)$$

where h is the set of integers $0 \leq h \leq n/2$. Assume, for analysis purposes, that G(t) is a complex signal consisting of a single frequency whose location is offset from one of the DFT eigenfrequencies, h_1/T by an amount δ/T, where $|\delta| \leq 0.5$. The samples of G(t) may be defined as follows:

$$G(k/2W) = A \exp(j\theta) \exp j2\pi(h_1 + \delta)\, k/n. \qquad (2)$$

If θ is defined as the phase of the signal at the reference time k=0, it is independent of the value of δ.

The complex output of the transform at h_1/T is given by

$$F(h_1/T) = \sum_{k=0}^{n-1} A \exp(j\theta) \exp(j2\pi(h_1 + \delta)k/n) \exp(-j2\pi h_1 k/n)$$

$$= A \exp(j\theta) \sum_{k=0}^{n-1} \exp(j2\pi\delta k/n).$$

In order to perform the summation of the exponential terms the following substitution is made

$$k' = k-(n-1)/2. \tag{3}$$

If n is even, k' is not an integer, but increases in integer steps. With this substitution

$$F(h_1/T) = A \exp(j\theta) \sum_{k'=-(n-1)/2}^{(n-1)/2} \exp j2\pi\delta(k' + (n-1)/2)/n$$

$$= A \exp j(\theta + \pi\delta(n-1)/n) \sum_{k'=-(n-1)/2}^{(n-1)/2} \exp(j2\pi\delta k'/n) \quad . \tag{4}$$

The imaginary term inside the summation is a sine function that possesses odd symmetry and cancels out over the range of the summation. The real term can only affect the amplitude of the transform. The phase of the signal as determined by the FFT algorithm is, therefore, $\theta + \pi\delta(n-1)/n$. This can vary over a range of almost π as δ varies over -0.5 to +0.5, and, in practice, since the value of δ is unknown, the phase information is completely ambiguous.

To illustrate this point, consider the signal shown in Figure 1. The dashed line represents a signal of frequency $(h_1 + 0.5)/T$ that is being analyzed while the heavy lines in the upper and lower parts of the figure are the sine and cosine components of the eigenfrequency h_1/T. Although the input signal is a sine wave, it is evident that its correlation with the sine component of h_1/T is zero whereas it correlates strongly with the cosine component. A clue as to how this error in phase interpretation may be corrected becomes apparent on inspecting the figure. It will be noticed that if the zero reference time is shifted from the beginning of the interval T to the middle of this interval, the input signal correlates to a greater degree with the component of the eigenfrequency having the same phase at this reference time.

The modified form of the DFT equation that will give the correct phase of the signal at the mid point of the time interval T is

$$F(h/T) = \sum_{k' = -(n-1)/2}^{(n-1)/2} G(k'/2W) \exp(-j2\pi hk'/n). \qquad (5)$$

If the use of the standard form of the FFT algorithm to perform this computation is desired, the zero time reference point must be transferred back to the point at which the first sample is taken, even though the reference point for the phase is still in the middle of the time interval. To do this, Equation (3) is again used to obtain

$$F(h/T) = \exp\{j\pi h(n-1)/n\} \sum_{k=0}^{n-1} G(k/2W) \exp(-j2\pi hk/n) . \qquad (6)$$

The phase corrected FFT algorithm (PCFFT), therefore, amounts to computing the FFT in the ordinary way and then multiplying each complex frequency term by the approximate phase correcting factor $\exp\{j\pi h(n-1)/n\}$.

APPLICATION

In order to illustrate the applicability of this algorithm, a signal consisting of two tones was used, those in one channel being Doppler shifted with respect to those in the other. After transforming both sets of signals, the output of the bins containing the unshifted frequencies in one channel were multiplied by conjugates of the complex outputs of the nearest bins in the Doppler channel containing the highest signal amplitude, and then inverse transformed to obtain a correlgoram. The same procedure was carried out using both the standard FFT algorithm and the PCFFT algorithm described above. Figure 2 shows a plot of the time resolution bin in which the peak of the correlogram occurs, versus the percentage Doppler shift for both cases. Also shown are the sinc x frequency responses of adjacent spectrum bins so that in the extreme case, there is a two bin difference between the reference and the Doppler channels. The true delay corresponds to bin number 40. It will be seen that with the standard FFT algorithm, large errors occur when the Doppler shifted frequencies are in the vicinity of cross-over points between two bins, whereas the error with the PCFFT consists of a gradual linear shift. This can be accounted for in the following way: Let the frequency of the reference signal be f_o and that of the Doppler shifted signal be f_1. If there is a delay of τ between the two signals, the phase of f_1 will be $\omega_1 \tau$. This is the quantity that is measured by the FFT. However, in multiplying the spectra, the shift in the frequency is ignored so that this phase shift at the frequency f_o would correspond to a time delay of $f_1 \tau / f_o$. Since the extent of the Doppler shift will be known to within one resolution element in the frequency domain, the error can be corrected in incremental steps corresponding to the spacings of the FFT eigenfrequencies.

ACKNOWLEDGMENTS

The author gratefully acknowledges the support of the Naval Sea Systems Command (SEA-06H1 and SEA-06H2) and the assistance of J. A. Macaluso, who was responsible for the computer simulation portion of the study.

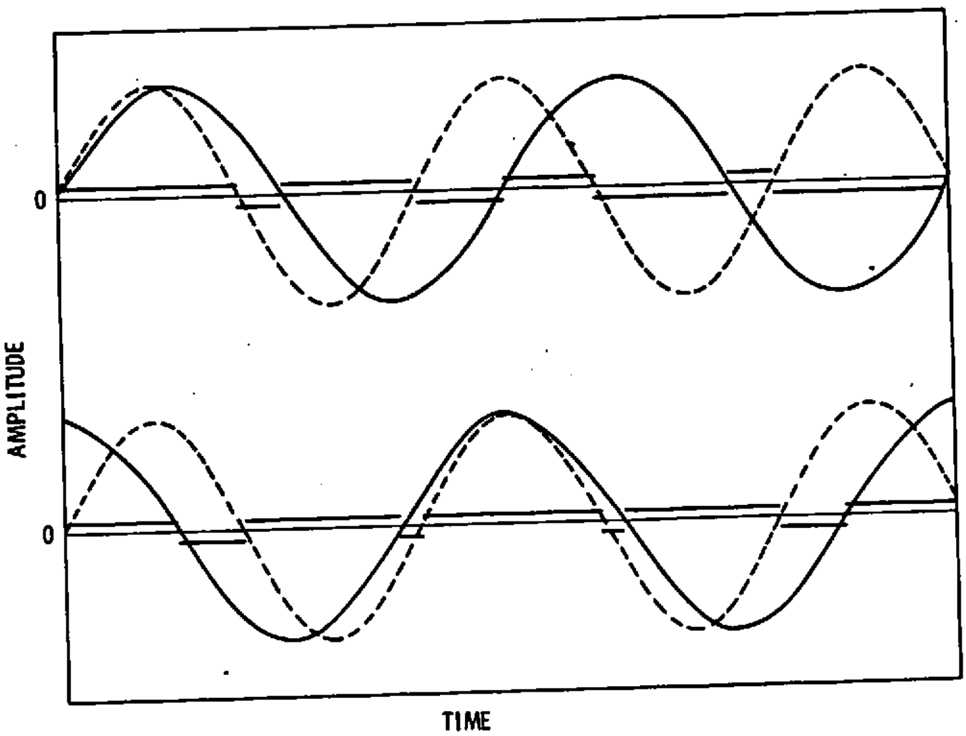

Figure 1. Phase Relationship between Two Frequencies.

--- Unknown Frequency

____ Nearest Eigenfrequency

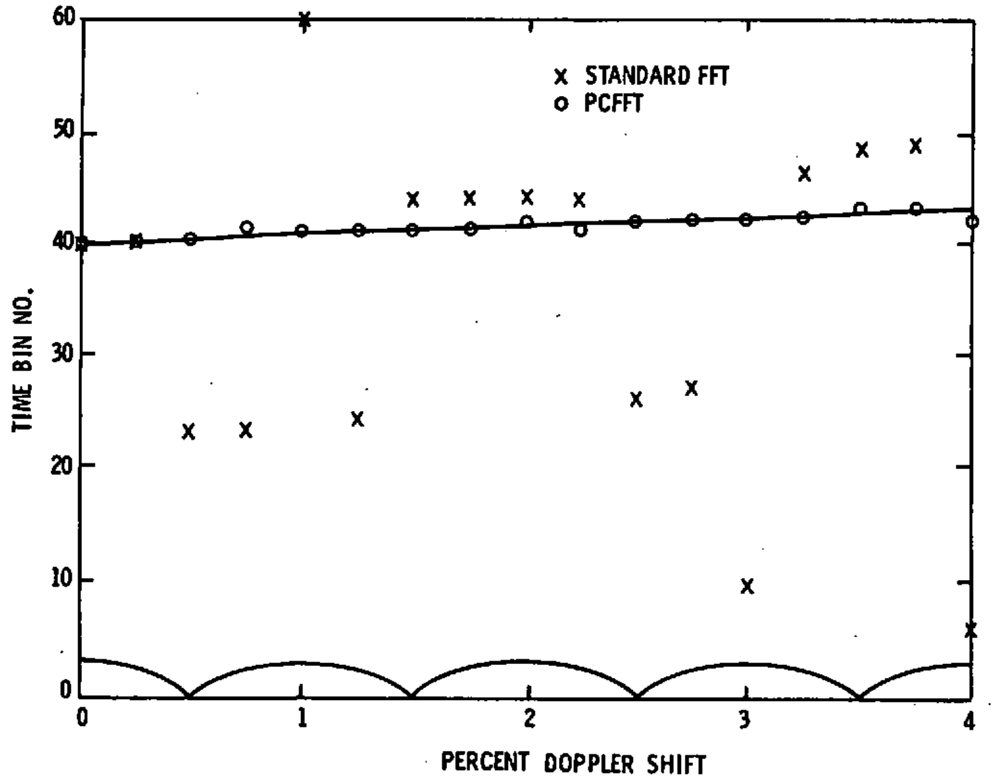

Figure 2. Location of Correlation Peak as a Function of Doppler Shift.