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SPECTRAL CONDITIONS FOR THE ACTIVE CANCELLATION OF BROADBAND NOISE

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INTRODUCTION

Considerable interest is now being shown in the techniques of active noise control (ANC). In part this is due to recent developments in theory and technology. The application of modern control algorithms and the availability of fast digital signal processing devices presents the possibility of overcoming many of the practical problems that have so far inhibited the successful implementation of ANC systems. However, these developments require an analysis from a systems approach; this paper presents a basic analysis of the spectral conditions required for the active cancellation of broadband noise.

STRUCTURE OF ANC

Consider a primary (unwanted) source of noise P emitting a signal with frequency content given by the *auto power spectral density* function $G_{pp}(f)$. Introducing a secondary (active) source S with a spectrum given by $G_{ss}(f)$ results in a combined spectral density $\overline{G}_{cc}(f)$ as measured by an observer at O.

Assuming non dispersive propagation, the primary signal will be attenuated by H_1 and delayed by $t_1 = r_1/c$ to produce an observed spectrum $\overline{G}_{pp}(f)$. Similarly, the secondary source will be attenuated by H_2 and delayed by $t_2 = r_2/c$. In this theoretical development the secondary source is assumed to be obtained directly from the primary and inverted.

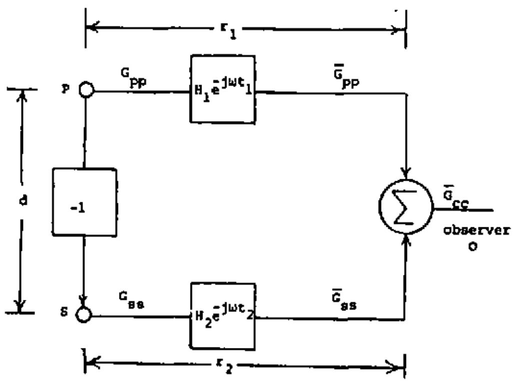

Fig.1. Theoretical structure of ANC

Using this description a measure of the degree of cancellation is given by the *cancellation factor* as the ratio of the cancelled spectrum to the primary spectrum for a given observation point.

$$k \triangleq \frac{\bar{G}_{pp}(f) - \bar{G}_{cc}(f)}{\bar{G}_{pp}(f)} = 1 - \frac{\bar{G}_{cc}(f)}{\bar{G}_{pp}(f)} \tag{1}$$

Assuming $\bar{G}_{ss}(f) \leqslant \bar{G}_{pp}(f)$ cancellation is obtained for $\bar{G}_{cc}(f) < \bar{G}_{pp}(f)$ giving $0 < k \leqslant 1$. However, the relative delay between the two signals due to the difference in distance $\Delta r = r_1 - r_2$ can result in reinforcement of the original signal, i.e. $\bar{G}_{cc}(f) > \bar{G}_{pp}(f)$. The worst case is given by the signal amplitude being twice that of the primary giving $\bar{G}_{cc}(f) = 4\,\bar{G}_{pp}(f)$ with $-3 < k < 0$. The cancellation factor is, therefore, bounded by +1 for cancellation and -3 for reinforcement.

$$-3 \leqslant k \leqslant 1 \tag{2}$$

In general, the cancellation will be dependent upon: the relative time delay, and therefore Δr, the frequency and the relative amplitudes of the signals. To obtain an expression for the cancellation factor the input-output auto spectrum and cross spectrum relationships can be used:

$$\bar{G}_{pp}(f) = |H_1 e^{-j\omega t_1}|^2\, G_{pp}(f) = H_1^2\, G_{pp}(f) \tag{3}$$

and

$$\overline{G}_{cc}(f) = |H_1 e^{-j\omega t_1} - H_2 e^{-j\omega t_2}|^2 \; G_{pp}(f) \qquad (4)$$

to obtain

$$k = -(H_2/H_1)^2 + 2(H_2/H_1)\cos\omega\,\Delta r/c \qquad (5)$$

where $(H_2/H_1)^2 = \overline{G}_{pp}(f)/\overline{G}_{ss}(f)$ is interpreted as the *spectral density ratio* at O. Fig. 2 shows a plot of cancellation factor $(0 < k \leq 1)$ against spectral density ratio for given frequency and Δr.

For a given spectral density ratio, the maximum cancellation k_{max} occurs for:

$$H_2/H_1 = \cos\omega\,\Delta r/c \qquad (6)$$

with a value $k_{max} = (H_2/H_1)^2 = \overline{G}_{pp}(f)/\overline{G}_{ss}(f)$ verifying the intuitive conditions that the maximum possible cancellation occurs for $H_2/H_1 = \cos\omega\,\Delta r/c = 1$. For a given frequency, the maximum cancellation will occur at observation points where the difference in distance from the sources is exactly an odd multiple of half wavelengths. The cancellation around these points will decrease with the cosine of the ratio $2\Delta r/\lambda$. The limit of cancellation (k=0) occurs for:

$$\Delta r = \frac{c}{\omega}\cos^{-1}\frac{H_2}{2H_1} \qquad (7)$$

from which it is clear that the area of cancellation varies inversely with frequency, for a given amplitude spectral density.

EXPERIMENTAL RESULTS

Using two similar loudspeakers the curves in Fig. 2 were verified. Using a fixed frequency of 100 Hz, the distance between the speakers was varied to give various Δr values. These results agreed with equation (5), to within experimental accuracy. The cancellation for broadband noise, up to 5 kHz, was investigated by driving the loudspeakers with a prbs signal and displaying the cancellation factor in dBs. The results, shown in Fig. 3 show clearly the cosinusoidal variation of cancellation with frequency. Cancellation and reinforcement diminishes at higher frequencies due to the mismatch in the loudspeaker characteristics. The limit of cancellation was verified by setting $H_1 = H_2$ and with $\Delta r = 0.3$m (d = 0.57m). The maximum frequency for cancellation is given by $f = 56/\Delta r = 187$ Hz, which corresponds to the results of Fig. 4.

CONCLUSIONS

The basic theoretical relationships for ANC have been developed and verified by experimentation. In practice, the secondary source will be obtained by detecting the primary signal, processing and then generating the cancelling waveform. Thus, the inversion in Fig. 1 will be replaced by a transfer function which will include the acoustic feedback path and the loudspeaker characteristic.

REFERENCES

[1] M. VOGT "General conditions of phase cancellation in an acoustic field", ARCHIVES OF ACOUSTICS, Vol. 1, No. 2, 109 - 125 (1976).

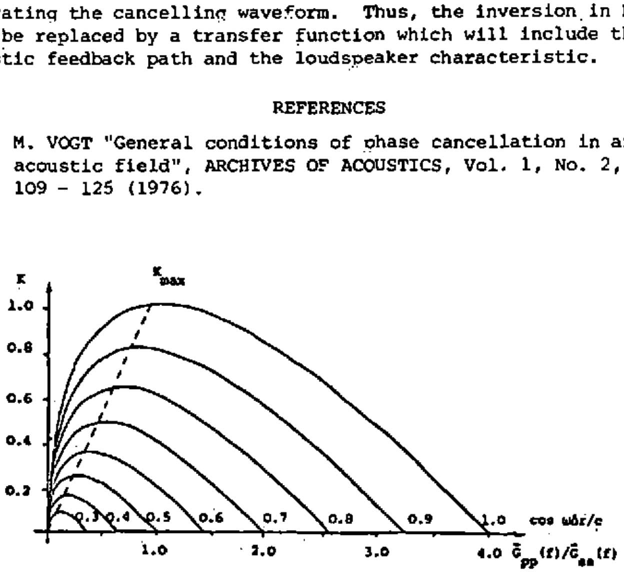

Fig. 2 Cancellation factor as a function of Δr

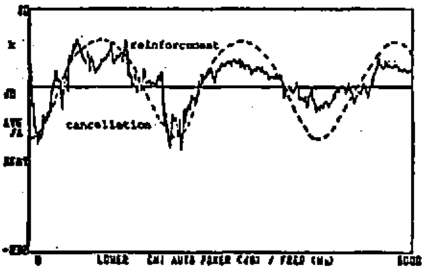

Fig. 3 Cancellation as a function of frequency

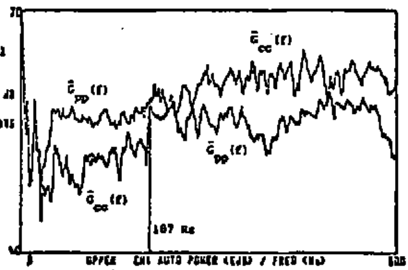

Fig. 4 Cancellation for Δr = 0.3m