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Vibration of Steel Turbo-Alternator Foundations

by

R.R. Wilson* and C.A. Brebbia**

Introduction

Until recently, large steel turbines have been mounted on massive concrete foundations. However, with the increasing size of turbines and auxiliary equipment, it is now thought preferable to use a steel-frame type of structure. The older type of foundation had all their natural frequencies well above running speed, but for steel foundations the lowest natural frequencies are below running speed. Hence the vibrational behaviour of such a foundation becomes of greater importance.

This paper is concerned with the analysis of steel foundations using the finite element displacement method. The structure is considered as an assemblage of beams, columns and plates. In-plane and transverse components of vibration are considered for each type of element together with the torsional vibration of the beams and columns.

The mass and stiffness matrices for the complete structure are formed, and the resulting eigenvalue problem solved to give the natural frequencies and corresponding mode shapes. A matrix is also derived to represent the structural damping of the foundation with additional terms to represent the damping of the oil films at the bearings. The structure is subjected to sinusoidal excitation, caused, for example, by shaft eccentricity. The steady-state response to such forces is estimated by solving a set of simultaneous complex equations to give the displacements in amplitude and phase at various points in the structure.

Idealization of the Structure

This type of structure is made up of a number of box beams, columns and plates. A typical foundation is shown in fig. 1, together with the idealization used.

For beams and columns, a cubic variation is assumed for the transverse displacements in two orthogonal directions and a linear variation for the longitudinal and torsional displacements.

* Central Electricity Research Laboratories

** Civil Engineering Department, Southampton University

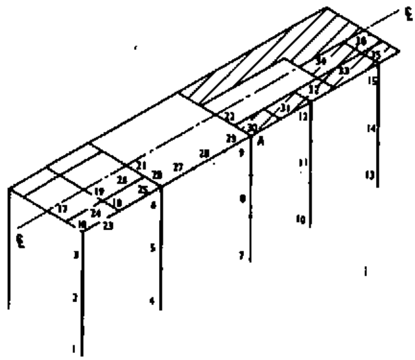

Figure 1: Idealization of the structure

The plates are of box construction. However, in this analysis, they are represented by uniform plates whose thickness is determined so as to give the correct moment of inertia about the neutral surface.

Because of the geometry of the structure, it was decided to use rectangular elements for the plates. For the vibration of plates with various boundary conditions, a good rate of convergence was obtained using a fully compatible element with the transverse displacement represented by Hermitian polynomials. [1], [2]. This is a sixteen degree of freedom element which requires the $\partial^2 w/\partial x \partial y$ term to be considered at each node. However, with the formulation used in the present analysis, it is not possible to combine this term with the degrees of freedom considered for the beams and columns. By using an approximate expression for the twist as a function of the slopes at neighbouring nodes, four constraint equations are formed, reducing the sixteen degrees of freedom to twelve. It can be shown [3] that this modified element can still represent the constant strain conditions. The displacement and tangential slope between adjacent elements are continuous but the normal slope is no longer continuous. However, very good rates of convergence have been obtained using this element.

Finally the mass of the casing and the machines has been considered by assigning different densities to those elements which support them.

Free Vibration

Since this type of foundation is to a very large extent symmetric about the vertical plane through the shaft, only half of the structure needs to be studied in the free vibration analysis. The symmetric and asymmetric cases are considered separately by applying appropriate boundary conditions.

The analysis has been carried out for the particular foundation shown in fig. 1 for which experimental results are available. Once the eigenvalues and eigenvectors have been found, the values of the displacements along the element sides can be calculated. These displacements were plotted in a convenient scale using a Calcomp plotter. A typical mode shape is shown in fig. 2.

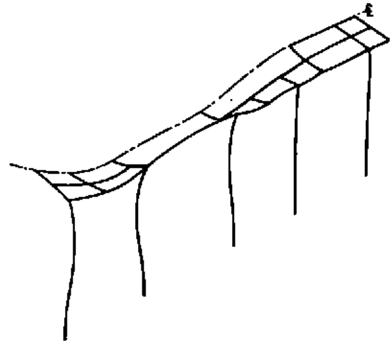

Figure 2: Typical mode shape

In general, the vertical vibrations of the plates and beams are coupled to the transverse vibration of the columns through the slope at the joints. Despite this, in all the modes calculated, the displacement throughout the structure was shown to be either predominately vertical or longitudinal and transverse, the latter corresponding to twisting of the columns. Indeed, there was very little change in the vertical frequencies when all the other degrees of freedom were constrained. Similarly, constraining the vertical degrees of freedom did not appreciably alter the frequencies of the twisting modes.

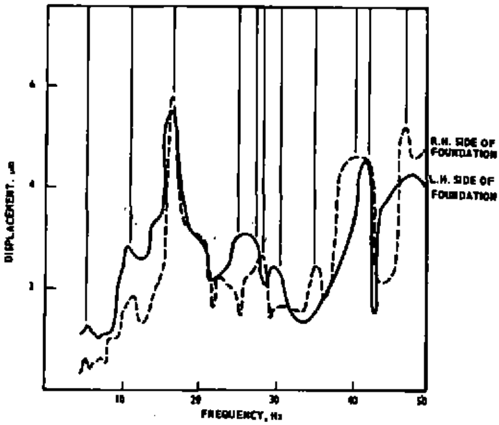

Figure 3: Vertical displacement and calculated natural frequencies

Fig. 3 shows a typical plot of the vertical displacement measured at point A in fig. 1, as the turbine speed is increased from 0 to 50 Hz. Also shown in the same figure are some of the calculated natural frequencies of the foundation. These frequencies correspond to these mode shapes which were found to have an appreciable displacement at this point in the structure. Although the distribution of the theoretical frequencies appears to be in reasonable agreement with the experiment, it is generally difficult to predict the relative importance of the different natural frequencies. A better estimate of this can be obtained by using a method of predicting the response of the structure.

Response Analysis

If the foundation is subjected to a set of sinusoidal forces, $\underline{p}$, varying at a frequency ω, then we have

$$\underline{M}\,\ddot{\underline{u}} + \underline{C}\,\dot{\underline{u}} + \underline{K}\,\underline{u} = \underline{p}\,e^{i\omega t}, \qquad (1)$$

where $\underline{M}$, $\underline{C}$ and $\underline{K}$ are the mass, damping and stiffness matrices, and $\underline{u}$ gives the displacements at the nodes.

The mass and stiffness matrices are as used in the free vibration case. Since the foundation is of steel, the structural damping may be expected to be low and, in the present analysis, it is represented by having the damping matrix proportional to the stiffness matrix. The damping of the oil films at the bearings can be estimated [4], and these additional terms added to the damping matrix.

For the steady state response of the structure,

$$\underline{u} = \underline{u}^*\,e^{i\omega t}. \qquad (2)$$

Substituting in equation (1), we have

$$(\omega^2\,\underline{M} + i\omega\,\underline{C} + \underline{K})\,\underline{u}^* = \underline{p} \qquad (3)$$

This set of simultaneous equations is solved to give the complex vector $\underline{u}^*$, the components of which represent in amplitude and phase the steady state displacement at the nodes. The forcing function chosen, $\underline{p}$, will be determined mainly by shaft eccentricity and thus it will vary as the square of the frequency.

In the idealization used for the response analysis, the complete foundation must be considered since, in general, the response will have components from both symmetric and asymmetric modes.

Conclusions

The work outlined in this paper is intended to give a practical method of checking designs of steel turbo-alternator foundations by means of free vibration and response analysis programs. If a particular mode of vibration gives unacceptable amplitudes, the free vibration analysis can be used to estimate the effect of a structural modification on the resonance. In this way, any proposed alteration can be checked. The response analysis provides, at the design stage, an estimate of the level of vibration that can be expected for a given shaft eccentricity.

References

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