# THE FREQUENCY DEPENDENCE OF THE TEMPERATURE MICROSTRUCTURE AND ITS EFFECT ON FORWARD SCATTERING OF UNDERWATER SOUND

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## ABSTRACT

In this paper, the effect of the transmission frequency on the product of the spatial correlation distance and the mean-square fluctuation of the refractive index of the medium is examined. The investigation is limited to the interference region of propagation in which the RMS amplitude fluctuation of the forward scattered sound increases with the square root of the transmission range. It is shown from both a theoretical basis and from some experimental results, that the spatial correlation distance decreases with increasing frequency at a constant range. However, a limiting value of approximately 1 cm is reached and this particular value can be related to the minimum patch size of a thermal inhomogeneity in the ocean.

## 1. Interference Scattering

Acoustic wave propagation in an inhomogeneous medium such as the ocean can be divided into two broad areas - the focussing region as described by Bergmann [1], and the interference region as described by Mintzer [2]. The interference region, in which the mean-square amplitude fluctuation of the forward-scattered sound increases in proportion to an increase in the transmission path length, has received a considerable amount of attention in the literature and many experimental investigations have been reported. The work reported on in this paper also relates to the interference region.

The interference region is characterised by constructive and destructive interactions between the scattered and unscattered acoustic waves and in the limiting case (i.e. far-far field), the propagating acoustic wave becomes highly scattered and an unscattered pressure wave vanishes resulting in a Rayleigh-type distribution of the acoustic pressure wave. The mean-square amplitude fluctuation, $\overline{\Delta V^2}$ , of a propagating acoustic wave in the interference region can be written in the form [3],

$$\overline{\Delta V^2} = \bar{\mu}^2 k^2 R \int_0^\infty N(\xi)\, d\xi \qquad (1)$$

where $\overline{\mu^2}$ is the mean-square fluctuation of the refractive index; $k = 2\pi/\lambda$ is the acoustic wave number; R is the transmission range; and N ( $\xi$ ) is the correlation function of the medium. The mean-square fluctuation can also be written in the form [4].

$$\overline{\Delta V^2} = \rho \sigma_s \frac{R}{2} \qquad (2)$$

where $\rho$ is the density of the scatterers; and $\sigma_s$ is the scattering cross-section of the individual scatterers.

Mintzer [2] has suggested that interference scattering depends only on the radius of the thermal patches (inhomogeneities) and therefore, for simplicity, it will be assumed that the patches or scatterers will be spherical with a diameter, d.

The scattering cross-section, $\sigma_s$ , of a thermal patch may be obtained from a consideration of bistatic radar theory [5] and may be written in the form,

$$\sigma_s = \frac{4\pi}{\lambda^2} \left( \frac{\pi d^2}{4} \right)^2 \qquad (3)$$

By substitution, the density of the scatterers is

$$\rho = \frac{32 \overline{\mu^2}}{\pi d^4} \int_0^\infty N(\xi) \, d\xi \qquad (4)$$

For a correlation function of the form,

$$N(\xi) = \exp\left( -\frac{\xi^2}{\alpha^2} \right)$$

where $\alpha$ is the correlation distance, equation (4) can be re-written as

$$\rho = \frac{16 \overline{\mu^2} \alpha}{\sqrt{\pi}\, d^4} \qquad (6)$$

In the case in which the patch diameter, d, is much smaller than the wavelength, $\lambda$ , the correlation distance, $\alpha$, is approximately equal to $\lambda$ , while in the case in which d is much larger than $\lambda$ , the correlation distance is approximately equal to the patch size, d.[4]. Thus, the density of the scatterers will be approximately,

$$\rho_1 = \frac{16 \overline{\mu^2} \lambda}{\sqrt{\pi}\, d^4} \quad ; \quad d \ll \lambda$$

$$\rho_2 = \frac{16 \overline{\mu^2}}{\sqrt{\pi}\, d^3} \quad ; \quad d \gg \lambda \qquad (7)$$

For $\overline{\mu^2}$ = constant, $\rho_1 > \rho_2$ and therefore the density of scatterers will be higher in the case in which the correlation distance is equal to $\lambda$ . This would result in an increased probability of scattering and therefore a larger mean-square amplitude fluctuation, $\overline{\Delta V^2}$ . At a constant range, this effect would appear in the form of the correlation distance decreasing with increasing frequency as the scattering becomes more pronounced (i.e. further into the interference region).

## 2. Some Experimental Results [6] - [11]

The above postulates can be examined by considering some experimental investigations of scattering by thermal inhomogeneities. In the data analysed, the transmission ranges varied between 300 and 800 metres (approximately constant) and the values for $\alpha$ were determined using the appropriate values for k with $\overline{\Delta V^2}$ taken from the experimental data and $\overline{\mu^2} = 5 \times 10^{-9}$. The data shown in FIG 1 illustrates that the correlation distance, $\alpha$ , does decrease with increasing frequency. At low frequencies, 14.5 kHz, the correlation distances are much larger than the acoustic wavelength, while at the higher frequencies, the correlation distances are of the order of the wavelength, as expected. The data shown by dotted lines in FIG 1 were obtained from results presented by two authors [10], [11], from measurements in water tanks. Although the ranges tested in these two experiments were orders of magnitude smaller than those shown in solid lines, the values of the correlation distance lead to an interesting conclusion.

Whitmarsh et al [6] have stated that the minimum patch size of a thermal inhomogeneity in the ocean would be of the order of 1 cm, and the results shown in FIG 1 appear to agree with this statement. Thus, as the frequency increases, the patch size would again become larger than the wavelength and the density of scatterers would decrease. This effect is being studied further.

## 3. Conclusions

This paper has reported on an investigation into the frequency dependence of the spatial correlation distance of the ocean. It has been shown both theoretically and experimentally that such a dependence exists. However, further work is being carried out to more fully assess this phenomenon. (A more detailed report of the current investigation is listed as reference 12).

## 4. References


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FIG 1.