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AN APPROACH TO THE DERIVATION OF THE STATISTICAL ENERGY ANALYSIS EQUATIONS

R S Langley

College of Aeronautics, Cranfield Institute of Technology, Cranfield, Bedford MK43 OAL, England.

1. INTRODUCTION

The problem of predicting the response of a complex system to high frequency random loading may be approached using Statistical Energy Analysis (SEA) which seeks to relate the mean energy stored in different parts of the system to the distribution of input power [1]. To this end the system is modelled as a collection of subsystems whose mean energies and external power inputs are related via a set of linear equations, the coefficients of which are expressed in terms of quantities known as the loss factors and the coupling loss factors. Comprehensive reviews of SEA are given in references [2] and [3] where the historical background of the method and the physical principles involved are discussed in detail. For a complex dynamic system the theoretical justification of the SEA equations is normally based on either a diffuse wave field approach or a modal approach to the system dynamics. In the latter case the arguments are generally based on a heuristic extension of the exact results which may be derived for the power flow between two coupled oscillators or, more generally, the approximate results which may be derived for two coupled oscillator sets. A number of studies (for example, references [3-5]) have considered the theoretical background to SEA in detail, and considerable progress has been made in identifying the conditions under which the method is likely to yield reliable results. The aim of the present analysis is to complement existing approaches with a derivation of the SEA equations which is based on a continuum analysis of a general coupled dynamic system. It is shown that the general form of the SEA equations is widely applicable providing a suitable energy definition is adopted, and expressions for the coupling loss factors are derived in terms of the Green functions of the system. The conditions under which the continuum results reduce to the standard SEA theory are discussed, and particular attention is paid to the role of "non-direct" coupling loss factors.

2. DERIVATION OF THE SEA EQUATIONS

The present analysis is concerned with the vibration of a general system which is composed of N coupled subsystems. The j'th subsystem is taken to have a single scalar response variable $u_j(x,t)$ which for harmonic vibration of frequency ω is governed by the equation of motion

$$L_j(u_j) - \rho_j\omega^2(1-i\gamma_j)u_j = F_j(x,\omega) + F_j^c(x,\omega) \tag{1}$$

where $u_j(x,\omega)$ is the complex amplitude of the response, L_j is a differential

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operator, ρ_j is the volume density (or equivalent) and γ_j is a dissipation factor. The distributed force $F_i(x,\omega)$ represents an external load, while the force $F_j^c(x,\omega)$ represents a load arising from coupling to another subsystem. The latter force will generally appear only if subsystem j forms the boundary of another subsystem; for example a plate forming one boundary of an airspace. With the present approach a structural member consisting of a beam (with coincident mass centre and shear centre) would be modelled as four subsystems with response variables corresponding to twist, axial displacement, and lateral displacement in two perpendicular directions. Similarly the bending vibrations of a plate would be modelled as a single subsystem with $u_j(x,t)$ equal to the out of plane displacement, while an airspace would be modelled as a single subsystem with $u_j(x,t)$ equal to the dynamic pressure. For structural elements whose various displacement components are governed by coupled differential equations (as for example in the case of a shell) the following analysis must be modified slightly, as detailed in reference [6], although the final results are not significantly effected.

If equation (1) is multiplied by the complex conjugate of the response velocity $(-i\omega u_j)$ and integrated over the subsystem volume (or equivalent), then the real part of the resulting equation yields the following power flow relationship

$$Q_j = 2\omega\gamma_j T_j + R_j \tag{2}$$

where Q_j is the time average of the input power, T_j is the time average of the total kinetic energy stored in the subsystem, and R_j is the time average of the power which is transferred to the neighbouring subsystems. If the coupling between the subsystems is conservative then the sum of R_j over j will be zero. Equation (2) is equally valid for the case of random excitation providing the notion of a temporal average is replaced with that of a statistical average, and the frequency band of interest is relatively narrow so that the frequency which appears on the left side of the equation may reasonably be replaced by a centre frequency (or alternatively, the dissipation factor is inversely proportional to frequency). Statistical Energy Analysis postulates that R_j is proportional to the energy difference between neighbouring subsystems; the validity of this premise is investigated in what follows.

The response of the coupled system to the harmonic loading $F_j(x,\omega)$ which appears in equation (1) may be written formally as

$$u_i(x,\omega) = \sum_j \int_{V_j} G_{ij}(x,y,\omega) F_j(y,\omega)\,dy \tag{3}$$

where V_j is the volume (or equivalent) occupied by subsystem j and $G_{ij}(x,y,\omega)$ is a Green function representing the response at location x on subsystem i to a harmonic point load situated at location y on subsystem j. Reciprocity implies that the Green functions which appear in equation (3) are symmetric.

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In SEA it is frequently assumed that the applied loading consists of uncorrelated ("rain on the roof") excitation such that the cross-spectrum between $F_i(y,t)$ and $F_k(z,t)$ has the form

$$S_{ik}(y,z,\omega) = \delta_{ik}\delta(y-z)a_i \tag{4}$$

where a_i represents the intensity of the excitation acting on subsystem i, which is assumed to be constant over a frequency band of interest Ω. It may be shown [6] that equations (3) and (4) lead to the results

$$T = [\rho]Ma \ ; \ Q = [q]a \qquad (5),(6)$$

where **T**, Q, and a are vectors containing T_i, Q_i, and a_i respectively, the diagonal matrix $[\rho]$ contains the densities ρ_i, and **M** and [q] are given by

$$M_{ij} = \frac{1}{2}\int_{V_i}\int_{V_j}\int_{\Omega} \omega^2 |G_{ij}(x,y,\omega)|^2 dx dy d\omega \tag{7}$$

$$q_i = \mathrm{Re}\left\{ \int_{V_i}\int_{\Omega} i\omega G_{ii}(x,x,\omega)\, dx d\omega \right\} \tag{8}$$

Equations (5) and (6) may be used to derive a relationship between the input power and the subsystem energies in the form

$$Q = CE \ ; \ C = (1/\pi)[q]M^{-1}[q] \ ; \ E = \pi[\rho^{-1}q^{-1}]T \qquad (9)-(11)$$

where the new subsystem energy measure E_i has been introduced. This measure is similar to that introduced by Smith [7] in the study of strongly coupled systems. Equation (2) and the summation property of the power flows R_j may be used to show that equation (9) can be rewritten in the form

$$Q_i = \omega_c \eta_i n_i E_i + \sum_{j\neq i} \omega_c \eta_{ij} n_i (E_i - E_j) \tag{12}$$

$$\eta_i n_i = (2/\pi)\gamma_i \rho_i q_i \qquad\qquad -\omega_c \eta_{ij} n_i = (1/\pi) M^{-1}_{ij} q_i q_j \qquad (13),(14)$$

where ω_c is the centre frequency of the excitation and the coefficients $\eta_i n_i$ and $\eta_{ij} n_i$ are defined by equations (13) and (14). Equation (12) is precisely the standard form of the SEA equations although the relationship between the coefficients defined by equations (13) and (14) and the familiar loss factors and coupling loss factors of SEA is not immediately apparent. Similarly the energy definition of equation (11) generally differs from the modal energy definition which is normally used in SEA. It is shown in the following section that under certain conditions the standard SEA parameters may be recovered from equations (11), (13), and (14).

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Although the present analysis has focussed on "rain on the roof" excitation, it is shown in reference [6] that equations (11)-(14) are equally valid for the case of random point loading providing the power inputs and subsystem energies are averaged over the range of possible point load locations. Also, random system properties may be incorporated into the above analysis by averaging the various matrices (and therefore effectively the Green functions) over the statistical distribution of the system properties.

3. WEAK COUPLING APPROXIMATIONS

The concept of weak coupling plays a central role in SEA, although there is no single agreed definition of weak coupling [2]. In the present analysis the coupling will be said to be weak if

$$|G_{ij}(x,y,\omega)|^2 = O(\varepsilon^n) \tag{15}$$

where ε is a small parameter and n represents the least number of couplings which lie between subsystem i and subsystem j. Thus n=1 if the subsystems are directly coupled, and n=2 if the shortest route between the two subsystems is across one intervening subsystem. There are two main classes of system which can be expected to meet the requirements of equation (15), being (i) those systems which have a high wave decay rate such that the reduction in the energy of a wave which crosses a subsystem is $O(1-\varepsilon)$, and (ii) those systems whose couplings have a wave transmission coefficient which is $O(\varepsilon)$. It can be noted that various plate junctions tend to have a low wave transmission coefficient [1] even though the mechanical coupling at such junctions is strong. From a modal, rather than a wave, point of view it can be shown that the present weak coupling condition will be met if the generalized coupling coefficients between the modal coordinates of the connected subsystems are of order $\varepsilon^{1/2}$. The present definition of weak coupling therefore encompasses most previous definitions [2] without limiting the analysis to either a wave or a modal approach.

Equation (14) indicates that the coupling loss factors η_{ij} are dependent on the inverse of the matrix M which is defined by equation (7). This matrix may generally be written in the form

$$M = [\lambda] + A_1 + A_2 + \ldots.+A_m \tag{16}$$

where $[\lambda]$ represents the diagonals of M and each off-diagonal entry of M is contained in one of the matrices A_i. If the minimum number of couplings which separate subsystem i from subsystem j is n, then M_{ij} is assigned to the matrix A_n. Thus the ij'th entry of A_1 will be zero unless subsystems i and j are directly coupled. The term m which appears in equation (16) represents the maximum "width" of the system. If the weak coupling condition of equation (15) is met then $[\lambda]$ will be $O(1)$ while A_n will be $O(\varepsilon^n)$. The inverse of M may then be written approximately in the form

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$$M^{-1} = [\lambda^{-1}] - [\lambda^{-1}]A_1[\lambda^{-1}] - [\lambda^{-1}]A_2[\lambda^{-1}] + [\lambda^{-1}]A_1[\lambda^{-1}]A_1[\lambda^{-1}] + \ldots \quad (17)$$

which may then be used together with equation (14) to calculate the coupling loss factors. It can be seen from equations (9), (10), (12) and (17) that the present definition of weak coupling implies that the coupling loss factors will be of a lower order of magnitude than the loss factors, a condition which has formed the basis of a number of previous definitions of weak coupling [2].

In SEA it is normally assumed that the coupling loss factor η_{ij} is zero unless subsystems i and j are directly coupled. In the present approach this is equivalent to retaining only the first two terms on the right of equation (17), which may initially seem to be justified on the grounds that the remaining terms are of second and higher order. However, a Statistical Energy Analysis normally consists of (i) calculating the loss factors and coupling loss factors, and (ii) inverting the coupling loss factor matrix to yield the subsystem energies for a given distribution of input power. Step (ii) is equivalent to using the loss factors and coupling loss factors to estimate the matrix M, since equations (9) and (10) imply that

$$E = \pi[q^{-1}]M[q^{-1}]Q \quad (18)$$

In practical situations only one of the subsystems may be subjected to excitation, in which case the second and higher order terms which appear in M are vitally important as in their absence equation (18) would predict a non-zero response only in those subsystems which are directly coupled to the excited subsystem. The second and higher order terms which appear in equation (17) may not therefore be lightly discarded if the calculated coupling loss factors are to yield an estimate of M which is accurate beyond first order. To assess the conditions under which "non-direct" coupling loss factors may reasonably be neglected it is convenient to write the inverse of M in the form

$$M^{-1} = [\delta] + B_1 + B_2 + \ldots\ldots + B_m \quad (19)$$

where, as in equation (16), the ij'th entry of B_n is zero unless the minimum distance between subsystems i and j is across n couplings. The matrix B_1 therefore accounts for the direct coupling loss factors, while the non-direct coupling loss factors are contained in the subsequent matrices. Equation (17) implies that $[\delta]$ is $O(1)$ while B_1 is $O(\epsilon)$. By inverting equation (19) and making use of the structure of the B_n matrices it may be shown that the leading order terms of each of the A_n matrices which appear in equation (16) may be expressed solely in terms of B_1 providing that $B_n < O(\epsilon^n)$ for $n>1$. Under this condition the inclusion of the first two terms in either equation (17) or equation (19) is sufficient to yield a reliable estimate of the system response.

The condition $B_n < O(\epsilon^n)$ for $n>1$ may be re-expressed in terms of the coupling loss factors as $\eta_{ij} < O(\varepsilon^n)$ for $n>1$, where n is the minimum number of couplings which lie between subsystems i and j. The physical significance of this requirement may be inferred by considering the situation in which only subsystem j is subjected to excitation. This will produce a response in subsystem j of O(1) and a response in subsystem i of $O(\varepsilon^n)$. The response of the subsystem immediately preceding i on the shortest path between i and j will be $O(\varepsilon^{n-1})$. Given that the coupling loss factor is $O(\varepsilon)$ for directly coupled subsystems, equation (6) predicts that the power flow into subsystem i which is directly attributable to its nearest neighbour is $O(\varepsilon)\times O(\varepsilon^{n-1})=O(\varepsilon^n)$. Further, the indirect contribution to the power flow into subsystem i due to the energy content of subsystem j will be $O(\eta_{ij}) \times O(1)$. The requirement that $\eta_{ij} < O(\varepsilon^n)$ is therefore equivalent to requiring that the "direct" power flow is at least one order of magnitude greater than the "indirect" power flow for this situation.

A comparison between equations (17) and (19) demonstrates that the order of magnitude condition on B_n requires that there be a relationship between the Green functions which appear in A_n $(n>1)$ and those which appear in A_1. For example, the condition on B_2 requires that to leading order $A_2 = A_1[\lambda^{-1}]A_1$, considering only the non-zero terms in A_2. Although this condition, or alternatively the foregoing condition on the power flows, would seem to be intuitively reasonable (particularly for reverberant systems) it is difficult to verify this analytically. A study of three subsystems which are connected in a chain may be used to demonstrate fairly readily that the condition that the modal coupling coefficients are $O(\varepsilon^{1/2})$, in line with the present definition of weak coupling, does not guarantee that $A_2 = A_1[\lambda^{-1}]A_1$. Similarly, Hodges and Woodhouse [3] have considered the power flow through a chain of oscillators and have shown that in general $\eta_{ij} = O(\varepsilon^n)$, which does not meet the present requirements. These aspects have been considered further in reference [8], where it is shown that the requirements on the Green functions are in fact likely to be met for reverberant systems.

Under the assumption of weak coupling a first approximation to the diagonal Green function G_{ii} which is needed for the calculation of q_i and λ_i is

$$G_{ii}(x,y,\omega) = (1/\rho_i) \sum_{n=1}^{\infty} \phi_n(x)\phi_n(y)/[(\omega_n^2-\omega^2) + i\gamma_i\omega^2] \qquad (20)$$

where ω_n is the n'th natural frequency and ϕ_n the n'th normal mode shape of the uncoupled subsystem. Given equation (20), q_i and λ_i may be evaluated using equations (7), (8) and (16), and the results substituted into equations (11), (13) and (14) to yield

$$E_i = 2T_i/n_i \qquad \eta_i = \gamma_i \qquad (21),(22)$$

$$\eta_{ij}n_i = (2/\pi)\omega_c\gamma_i\gamma_j\rho_i\rho_j \int_{V_i}\int_{V_j}\int_{\Omega} \omega^2|G_{ij}(x,y,\omega)|^2 dx\,dy\,d\omega \qquad (23)$$

where n_i is the number of resonant modes in the frequency band Ω and equation (23) relates to the direct coupling loss factors only. Equations (21) and (22) are in agreement with the standard approach to SEA, while it is shown in reference [8] that a number of well known results for the coupling loss factors (including those yielded by both the wave and modal approaches to SEA) may be recovered from equation (23). Under the appropriate conditions the continuum analysis of section 2 therefore results in the standard SEA equations.

4. CONCLUSIONS

It has been shown that under certain conditions the standard form of the SEA equations may be deduced using a continuum approach. Particular attention has been paid to the concept of weak coupling, and it has been shown that this condition alone does not guarantee the validity of the assumption that non-direct coupling loss factors can be neglected. This condition is, however, likely to be valid for weakly coupled reverberant systems [8]. The equations of section 2 are not restricted to the case of weak coupling and this analysis may be used as a basis for the development of improved models of the system dynamics for those cases in which the standard SEA approach is not valid.

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EXPLORING STATISTICAL ENERGY ANALYSIS ON A CYLINDRICAL STRUCTURE

M Blakemore, B Jeffries, R J M Myers and J Woodhouse

Topexpress Ltd, Poseidon House, Castle Park, Cambridge CB3 0RD

1. INTRODUCTION

A test structure of sections of cylindrical shell connected by bolted, flanged joints has been constructed, generically typical of aerospace structures. This is being used to explore various aspects of SEA modelling of vibration levels within the structure for given external excitation. The aim is to study coupling loss factors between the subsystems by a variety of approaches. These can include (a) inverse estimates based on measuring mean modal energy in the subsystems under conditions of known power injection, either with a system containing many subsystems, or with one assembled with just two sections; (b) measurements of individual coupled modes of the system, and then calculation of appropriate averages of these; (c) calculation of modal coupling strengths or wave transmission coefficients through the flange separating the systems. In each case one seeks information about the statistical properties of the reverberant field in the entire structure for given driving, not just a calculation of the average vibrational energy.

2. THE TEST STRUCTURE

The test structure has been designed to be as versatile as possible while remaining easy to handle in the laboratory. It embodies features relevant to certain aerospace applications, and also, on a more abstract level, some features likely to raise difficulties with SEA modelling. Idealised SEA studies in the past have often used rather simple structures in which these difficulties do not arise, and this might give a misleading impression about the methodology and expected accuracy of SEA modelling of more general systems.

The system consists of five sections made from aluminium plate 1.6mm thick, bent to cylindrical form with radius 0.25m and welded with a single axial seam. The lengths of all the sections are different, ranging from 0.545m to 0.845m. There is also one section made of plate 2.5mm thick, with length matching that of one of the thinner sections. All are drilled with 12 equally spaced holes near the ends, so that bolted connections to simple L-section flanges can be made. These in turn can be used to bolt sections together, with or without an intervening plane circular baffle made of 2.5mm aluminium plate. These components can be assembled in a variety of configurations, with two or more sections, and with or without heavy wooden end-plates.

It is expected that experiments based around this test structure will continue for some time, as different possibilities are explored. In this paper, we report some preliminary results from early stages of this investigation. These include both deterministic and statistical results. The deterministic measurements allow a distinction to be drawn between two different views of the vibrational behaviour of this structure. If it were exactly axisymmetric (i.e. if the welded seams and the discrete bolted joins are not too significant), then all vibration transmission must occur in one or other of the various waveguide modes having variation with azimuthal angle θ like $\cos n_c\theta$ where the angular order n_c can take the values 0, 1, 2, 3, etc. If the behaviour is separated into these components, we see a set of one-dimensional propagation problems associated with the different values of n_c. On the other hand, if we treat each cylindrical section as a single subsystem, then we obviously have two-dimensional subsystems.

It is not *a priori* obvious how well SEA will work in such a system. If we use the former view, then we have many subsystems per section of cylinder, and the modal density in each will be very low. If we take the second route we have more modes, but we are "sweeping under the carpet" a significant aspect of the physics of the problem, namely that the vibration transmission is actually occurring in a set of weakly-coupled one-dimensional systems over which we are averaging in some way. This is one of the senses in which this test structure embodies features likely to expose difficulties in SEA modelling.

Similar difficulties can arise from the fact that the structure has the connectivity of a simple chain: each section is only connected mechanically with its nearest neighbours, and if the air-borne pathway for vibration transmission is not significant then the coupling loss factors will be zero except between neighbouring susbsystems. Also, we expect the nature of the mechanical coupling to be rather similar across each boundary, since all the joints are nominally identical. Any SEA model obeying these two conditions predicts a very simple pattern of decay along the system from excitation at one end. However, very simple observations confirm what one might expect from other such problems, that the actual decay behaviour is not of this form.

What we see is a large attenutation across the first junction (typically 20dB with the baffles in place), and then progressively less across the later ones (of the order of 10dB per junction). Behaviour of this kind is to be expected if there is a significant difference between different modes or travelling-wave directions in the reflection and transmission coefficients at the boundary. In the driven section we perhaps excite a broad range of modes to roughly similar energies, as assumed by standard SEA. Of these, some will be more strongly reflected than others at the first boundary, so that those which reach the adjacent section will have a much higher proportion of the ones with high transmission coefficients. Since the next boundary has similar characteristics to the first, the new mixture of modes is better suited to being transmitted through, and a smaller attenuation is thus seen. This process repeats, the weaker-transmitting components being filtered out of the mixture more and more effectively.

Since chain-like systems are not uncommon, it is of value to investigate ways in which SEA might be used to study them. This can be approached from more than one point of view. We might start with detailed modelling of the transmission characteristics of the boundary, to seek, for example, coincidence angles as a physical basis for the filtering phenomenon. We might then be able to incorporate this knowledge in a new SEA model, but we would perhaps have defeated the object of SEA, which is to obtain some kind of predictions of vibrational behaviour without detailed modelling. Another possibility might be to allow, say, two subsystems per section, made up of modes/waves with respectively higher and lower transmission coefficients, but *without* trying to decide in advance what these should be. We could then, provided enough experimental data were available, perform a best-fitting exercise of a model of this kind to the observed pattern of overall response in the sections. If by those means it proved possible to match the results well, one might have a model capable of predicting the effects of structural modification without having to know very much about the detailed physics underlying the split between the pairs of subsystems.

3. CHECKING THE DISPERSION RELATION

Measurements have been performed to examine the extent to which the modes of the system do indeed subdivide into different values of n_c, and also to check the behaviour of our cylindrical sections against the theoretical dispersion characteristics for a thin cylindrical shell. The structure was assembled with the dividing plates in place, to minimise coupling between sections for this first stage of investigation. The analysis method has been described before when investigating ribbed cylinders [1]. A set of transfer functions is determined to a fixed accelerometer from a ring of driving points equally spaced around the circumference. By Fourier analysing with respect to azimuthal angle, we then decompose the signal into the separate contributions from each value of n_c.

This was first done with both observing point and drive points in the same section of the structure. Since the coupling between sections is relatively weak, we then find a well-defined series of peaks corresponding to modes which "live" predominantly in that section. By looking at a single peak frequency, we can easily perform a modal analysis of that mode as a Fourier series in n_c. Almost always, this series is strongly dominated by a single term, so that a value of n_c can be associated unambiguously with each mode. The only exceptions are, not surprisingly, when two peaks are close enough together to have significant modal overlap. Then the two peaks both show a significant amount of two different n_c values, corresponding to what each mode "would have had" in the absence of the close proximity of the other.

This process has been carried out for the first 200 or so modes (mostly occurring in near-degenerate pairs), for frequencies up to about 2kHz. The results can be immediately compared with theoretical predictions of the dispersion behaviour for an infinitely-extended version of our cylinder. This behaviour is plotted as a series of curves in Fig. 1, for n_c ranging from zero to 16. Superimposed on this are plotted symbols corresponding to the modal measurements, different symbols being used for different deduced values of n_c. It is not difficult to assign an approximate axial wavenumber to each mode, simply by assuming that the ends of the section correspond to simple hinges, so that the axial variation of the modes consists of one half-wavelength between the ends, or two, or three, and so on. (This assumption has been checked by some measurements using an axial array of driving points.) The result of this process is shown by Fig. 1 to give very close agreement between theory and observation. This has been carried out for two different sections of cylinder with lengths differing by 30%, and worked well in both cases — since there is an end correction for the effect of the flanges which is not easy to calculate, the longer section worked slightly more accurately than the shorter one. We conclude that a first approximation to the modal behaviour of the system is just as one would guess. Each mode is largely confined to one bay, and we see an orderly sequence of modes associated with each value of n_c which agree well with the predictions of thin-shell theory.

Naturally we did find some admixture of other values of n_c in modes nominally labelled with a given value. For measurements in the driving bay as just described, the typical level of these was about a factor 4 below the dominant value. This gives a measure of the influence of the departures from ideal cylindrical symmetry, due to the weld, non-circularity of cross-section, non-uniformity of thickness or material properties, etc.

When similar measurements are performed with driving and observing points in two adjacent sections, a rather different result is found. Since each mode is quite strongly confined to a single section, we find a high attenuation of general level in the adjacent bay. Numbers of the order of 20dB are typical of this attenuation. Performing the decomposition into n_c values again, we find that the level of predominance of the nominal value of n_c is so much reduced that it is often hard to tell which value this is without data from the driven bay. This phenomenon can be explained, at least qualitatively, in terms of modes available in the second section which might couple to a given mode confined mainly to the first section. Since the lengths of the sections are different, there is usually not a mode with the same value of n_c available for coupling. Instead, there may be modes with other n_c values much closer in frequency, and the combined effect of coupling to these produces a mixture of n_c values much less strongly dominated by a single value. Notice that this explanation does not involve any further sources of non-cylindricality — it arises simply from the solution to a mode-coupling problem.

4. COUPLING LOSS FACTORS

Attempts have also been made to measure *in situ* coupling loss factors by statistical methods. First, a system was assembled consisting of just two sections, separated by the circular plate. Each section was then subdivided by eight equally-spaced circumferential rings, and a random position chosen on each ring. This gave a set of sixteen points covering both sections, regularly in one direction and randomly in the other. A full matrix of transfer function between these points was then measured, using impulsive excitation and small accelerometers to receive. A hammer tip was selected which gave reliable data up to at least 6kHz (although with plating this thin, the hammer rebound time and hence the frequency spectrum of forcing was more dependent on the cylinder dynamics than on the hammer tip). All data were collected by digital datalogger, so that a wide range of subsequent processing could then be carried out.

From the driving point measurements (i.e. the diagonal terms of matrix of measurements), we can deduce power input, and also check modal density. We already have a good idea of the modal density, of course, from the results of the previous section. Each row of the matrix is then normalised and averages performed over the two sections separately to deduce the SEA matrix of mean energy per mode in the two subsystems when one or the other is subject to driving with a known rate of power injection. The averaging can be done in such a way that what is simulated is independent random drive at the eight excitation points in a section. This should be enough to give a reasonable average over the mode shape factors, achieving something approaching ideal "rain on the roof" driving. The extent to which this hope was realised in practice can also be assessed from the data, by looking at variances as well as average values.

By inverting the SEA matrix thus obtained, estimates of damping loss factors and coupling loss factors are obtained. This can either be done over a wide range of frequency, or in separate frequency bands. The damping loss factors can be compared with direct measurements of damping. Since we have weakly coupled subsystems here, the matrix is strongly diagonal-dominant and the inversion process is numerically well-conditioned. The coupling loss factor we expect to determine this way is, of course, the one corresponding to the rapid initial attenuation, as discussed earlier. When these two sections are later incorporated into a larger structure, we will expect to obtain different results for the coupling of these two bays depending on exactly how the data are interpreted.

As an initial calculation, the average behaviour over a wide frequency range was obtained, using the directly logged forces and accelerations (integrated to give velocities) with minimal processing. The equivalent power inputs and subsystem energies were then calculated, in response to uncorrelated random forcing at the set of points in one subsystem or the other with a frequency power spectrum matching that of the hammer blows. Values averaged over the whole matrix of transfer functions (with the expected error in the estimates) are given below, for forcing having an rms value of 1N:

Power in to subsystem 1	2.7×10^{-3} watts (±9%)
Power in to subsystem 2	2.7×10^{-3} watts (±9%)
$<v^2>_{11}$	1.3×10^{-4} m^2/s^2 (±9%)
$<v^2>_{12}$	2.3×10^{-6} m^2/s^2 (±7.5%)
$<v^2>_{21}$	2.5×10^{-4} m^2/s^2 (±7.5%)
$<v^2>_{22}$	1.1×10^{-4} m^2/s^2 (±14%)

The corresponding damping loss factors η_1, η_2 and the coupling loss factors η_{12} and η_{21} for the two sections are given by the formal relations

$$\omega\eta_1 = 3.5\ (\pm20\%) \qquad \omega\eta_{12} = 0.074\ (\pm40\%)$$

$$\omega\eta_{21} = 0.11\ (\pm40\%) \qquad \omega\eta_2 = 5.9\ (\pm20\%)$$

where we might take the "centre frequency" to be 500Hz, giving $\omega = 3140$rads/s.

The usual SEA reciprocal relation $n_1\eta_{12} = n_2\eta_{21}$ (n_i being the number of modes in each section over the bandwidth) is satisfied to within 5%. This follows almost automatically from the way the experiment was performed and the data were analysed: we had a full matrix of measurements, each individually paired with a reciprocal one, and for impulsive forcing with such a short autocorrelation function (and consequently wide spectrum) the boundaries of the subsystems are not "felt" by the forcing so that power input is governed by the infinite cylinder admittance.

5. REFERENCE

[1] C H HODGES, J POWER AND J WOODHOUSE, 'The low frequency vibration of a ribbed cylinder, part 2: observations and interpretation', *J Sound Vib* 101 p237 (1985).

6. ACKNOWLEDGEMENTS

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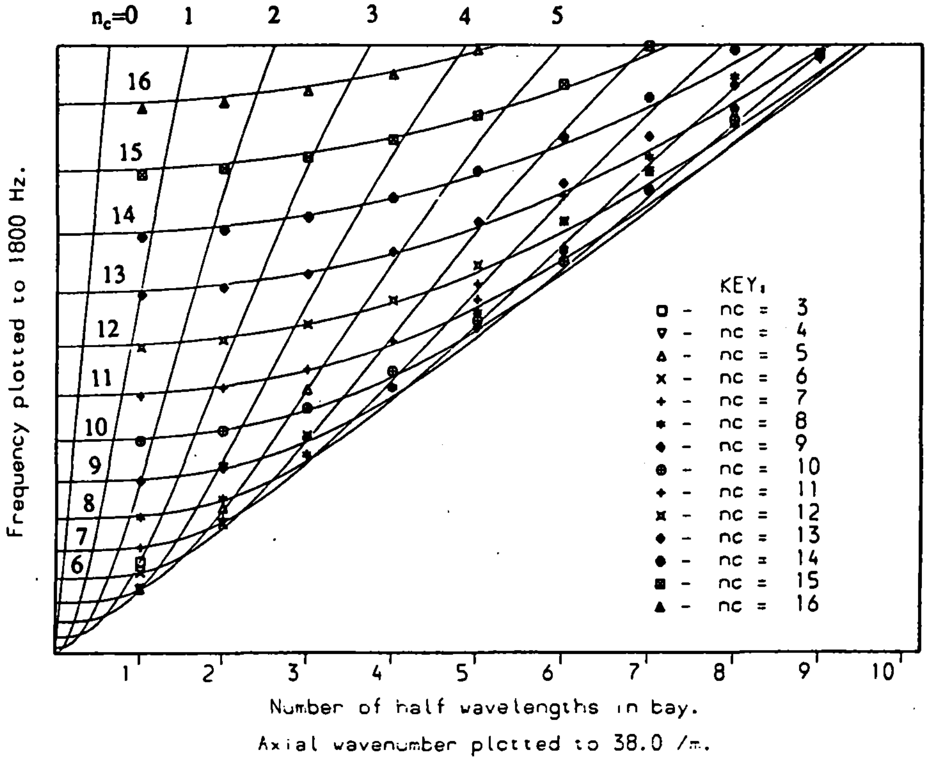

Figure 1. Computed dispersion curves for an infinite thin cylinder with the same parameters as the cylindrical sections of the apparatus used here (plotted as solid lines) compared with measurements of individual modes (plotted as symbols). Both theory and measurements discriminate between different angular orders, indicated by values of the variable n_c

