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# EXPERIMENTAL STUDY OF ROTATIONAL MOBILITIES OF CONCRETE FLOORS

R.S. Ming and B.M. Gibbs

Acoustics Research Unit, School of Architecture and Building Engineering, Liverpool University, P.O. Box 147, LIVERPOOL, L69 3BX

## INTRODUCTION

The majority of building services machinery, such as centrifugal fans, chillers etc., is resiliently mounted on plantroom floors. Although vibrational power is reduced by such mountings, some is still transmitted through them into the floor, the magnitude of which is directly related to the floor mobilities at the mounting points. In general, floors can be excited in six degrees of freedom (three translations along and three rotations about the main axes) and six corresponding point mobilities may be required to describe the power into floors. However, it may be that two or three mobilities are enough to predict input power accurately although it is common practice to assume one degree of freedom only, vertical translation, contributes to power flow.

Vertical translational mobility, the quotient of translational velocity and component point force, is relatively easy to measure[1,2,3], but relatively little has been published on rotational mobility measurement methods. Sattinger [4] proposes a method where the rotational mobilities of a structure are measured by finite spatial derivatives of the measured translational mobilities. Verheij [5] proposes a statistical estimate of the frequency average of real parts of point (translational or rotational) mobilities. This method involves measurement of translational mobility and two acceleration auto-spectra.

There remains a need to consider power into a concrete floor by moments particularly at high frequencies but there is litter information on floor rotational mobilities. In this paper, three measurement methods have been considered and the results compared with simple prediction.

## PREDICTION OF ROTATIONAL MOBILITY

Floors in plantrooms vary in constructions and boundary conditions but in general they can be considered to be isotropic and the effects of boundary conditions can be neglected if the excitation point is not close to the edges. If the floor is treated as an infinite thin plate the point rotational mobility is given by [6]

$$Y_m = \left[ 1 - \frac{4}{\pi} j \ln(0.9ka) \right] \frac{4.8 f}{C_l^2 \rho h^3} \qquad (1)$$

where h is the thickness of the floor, $\rho$ is density, $C_l$ is longtudinal wave speed, a is the half distance of the exciting force couple and k is wave number. In Figure 1 is shown a comparison of the magnitude, real and imaginary parts of the rotational mobility of a concrete floor (h=125 mm, a=1mm) obtained from equation (1). At low frequencies, the imaginary part dominates but the difference between the real and imaginary parts decreases with increased frequency. This means that the most input power in lower frequencies is reactive with a small part dissipated by the floor.

Figure 2 shows the comparison of the predicted point translational mobility which is pure real and the real part of the point rotational mobility of a concrete floor (h=125 mm). The translational mobility is frequency invariant whereas the real part of the rotational mobility increases with frequency. This means the power into a floor by a moment may become more important at high frequencies.

## MEASUREMENT METHODS

Three methods, which were used to measure rotational mobilities of structures, have been investigated. The methods of Sattinger [4] and Verheij [5] involve indirect measurements, in which a point rotational mobility is derived from corresponding translational mobilities. The third method is a direct method, in which a point rotational mobility is given by the measurement of the transfer function of the acceleration differences of a floor and a moment driver blocking mass.

Sattinger's method has the advantage of using conventional measurement techniques without a requirement for the use of special fixtures. Rotational mobilities are obtained as spatial derivatives of translational mobilities, which are approximated by finite difference sums of sets of those translational mobilities. A point rotational mobility is determined from four translational mobilities. When the points on which force and translational velocity are measured are identical, then reciprocity allows three translational mobilities only to be determined: one transfer ($Y_{12}$ or $Y_{21}$) and two point translational mobilities ($Y_{11}$ and $Y_{22}$) measured on two reference positions (1 and 2), both at a distance, d, from the point of interest (point 0). A point rotational mobility is given by:

$$Y_m = v_\theta / M = (Y_{11} + Y_{22} - 2Y_{12}) / (4d^2) \quad (2)$$

Where $v_\theta$ is rotational velocity and M is moment. Using a Taylor series expansion for the double spatial derivative, it can be proven that a systematic error proportional to the inverse of the third power of the distance is introduced. This bias can be estimated by repeating the measurements with different distances, d. Enlarging the distance results in a reduction of the bias. However, the distance must keep small with respect to the governing wavelength in order to preserve a good correlation between the various translational mobilities and rotational mobility of interest. Thus the distance, 2d, must be smaller than half the (governing) bending wavelength.

Verheij's method involves estimating frequency bandwidth averages of real parts of point mobilities at adjacent positions or at the same position but for different degrees of freedom. With this method all 6 point mobilities can be estimated. A point rotational mobility can be determined from one corresponding point translational mobility and the ratio of the corresponding mean square band-filtered accelerations measured when the structure is excited by a broadband distant sound source.

$$Re(Y_{2,m})_{\Delta f} = Re(Y_{11}) * a_{2,\Delta f}^2 / z_{1,\Delta f}^2 \quad (3)$$

where $Re(\ )_{\Delta f}$ is the $\Delta f$-bandwidth mean of the real part. $Y_{2,m}$ and $Y_{11}$ are the point rotational and translational mobilities at points 2 and 1 respectively, $a_{2,\Delta f}^2$ and $z_{1,\Delta f}^2$ are mean square rotational and translational accelerations measured at points 2 and 1, respectively. Point 1 is the reference point. The method is statistical in nature and it is essential to obtain a high modal density in the frequency bands of interest. This is not likely to occur at low frequencies, even in third octave bands.

The direct method involved the use of a moment driver, shown in Figure 3, constructed to a design by Petersson[7]. Two pairs of accelerometers are used to measure the transfer function of the rotational velocities of interest points on the floor and the seismic mass of the moment driver respectively. Point rotational mobility, the quotient of a rotational velocity and a moment, was then obtained from the transfer function.

## RESULTS

Two concrete floors, of dimensions 4820*3000*125 mm and 3150*2080*125 mm, were investigated. Five randomly chosen positions were used on each floor to obtain spatial and time averages. Measurements were carried out using a dual channel FFT analyser giving 400 point spectra over the frequency range of interest. Post-processing using a BBC Master microcomputer and IEEE488 interface transfer allowed division by frequency and a computation of mobilities in a calibrated logarithmic format of dB($20\log_{10}$) re a $ms^{-1}N^{-1}$.

The set-up for translational mobility measurements is shown schematically in Figure 4. Different hammers and tips were employed for different frequency ranges: From 1 KHz to 5 KHz a small hammer and steel tip were used and below 1 KHz a large hammer with a head of mild steel of mass 0.8 Kg was used. A rubber tip was used in the frequency range of 0 to 500 Hz and plastic tip in 500 Hz to 1 KHz. The hammers, tips and accelerometers were calibrated before the measurements by using a freely suspended concrete cube of mass of 20 Kg.

For the Verheij method, a 200 watts speaker was located in the chambers under the floors and driven by a periodic stationary random noise. Above the floors, the rotational and translational accelerations were obtained by means of two B&K 4378 accelerometers and sum/difference amplifers.

Results obtained by the Sattinger method are presented in Figures 5 and 6, and those by the Verheij method in Figures 7 and 8, and it can be seen that the agreement between measured and predicted values is reasonable good. The Verheij method yields results which independent of the distance, d, but are not reliable for frequencies below 100 Hz.

Results for the floor (4820*3000*125 mm) excited by the moment driver are shown in Figure 9. The measured value is higher than prediction in the frequency range of 300 Hz to 2500 Hz and this may be due to the fact that the floor response is controlled by local stiffnesses. The discrepancy is large below 300 Hz where the floor cannot be assumed to be an infinite thin plate.

## CONCLUSIONS

Three methods appear to provide the practical means for measuring rotational mobilities of floors. From the limited survey undertaken, the following conclusions can be drawn:

1: Verheij's method is not suitable for frequencies where modal density is low.

2: Sattinger's method seems to not give a reliable results in very low frequencies. Also care must be taken for choosing hammers, tips and the distance between measurement accelerators.

3: The direct method requires spatial averages over at least four or five positions.

4: The theory is not suitable for low frequencies because floors can not be considered as infinite thin plates.

## ACKNOWLEDGEMENT

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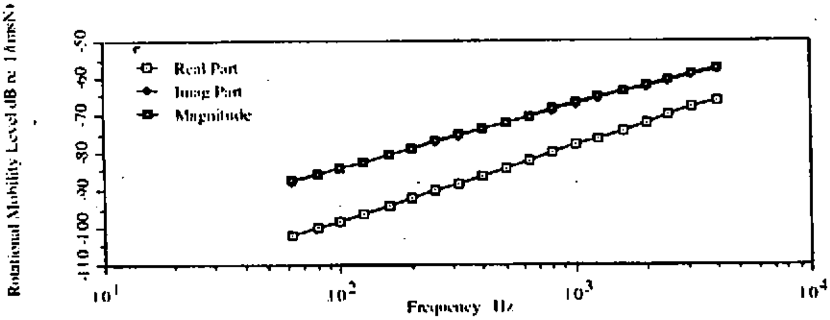


Figure 1. Comparison of the magnitude, real and imaginary parts of the rotational mobility of a concrete floor, h = 125 mm, d = 10 mm.

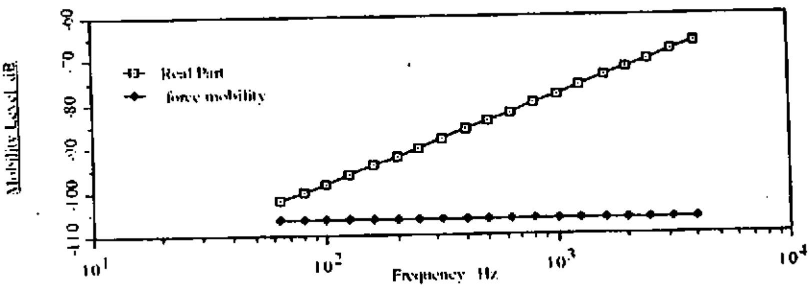


Figure 2. Comparison of the point translational mobility and the real part of the rotational mobility of a concrete floor, h = 125 mm.

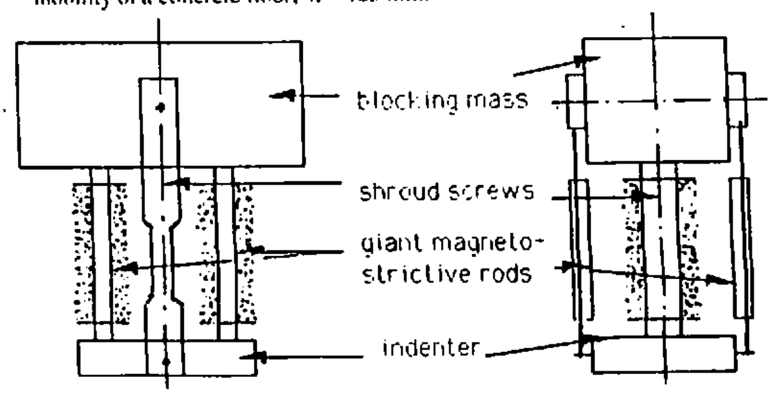


Figure 3. Sketch of a moment driver

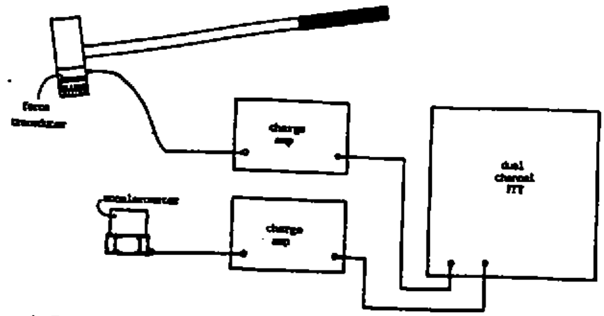

Figure 4. Schematic of equipment for translational mobility measurement.

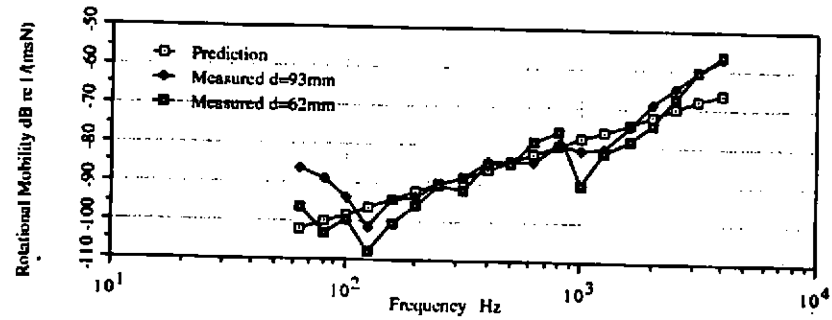


Figure 5. The real part of the rotational mobility, predicted and measured by Sattinger Method, of a concrete floor: 4820*3000*125 mm.

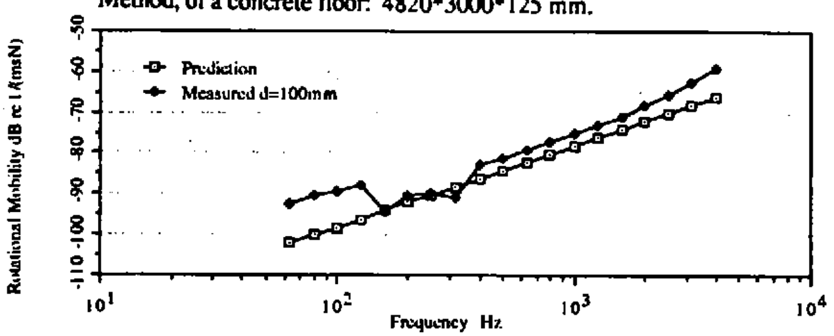


Figure 6. The real part of the rotational mobility, predicted and measured by Sattinger Method, of a concrete floor: 3150*2080*125 mm.

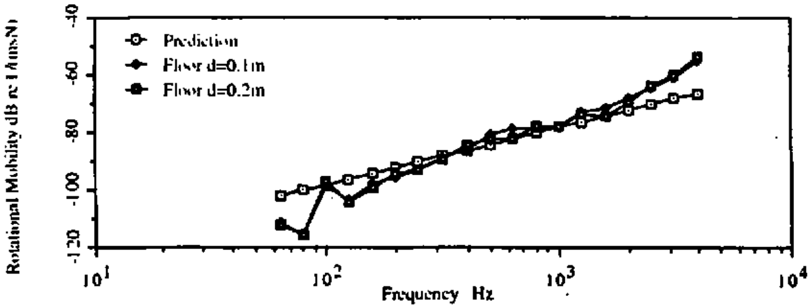


Figure 7.The real part of the rotational mobility, predicted and measured by Verheij Method, of a concrete floor: 4820*3000*125 mm.

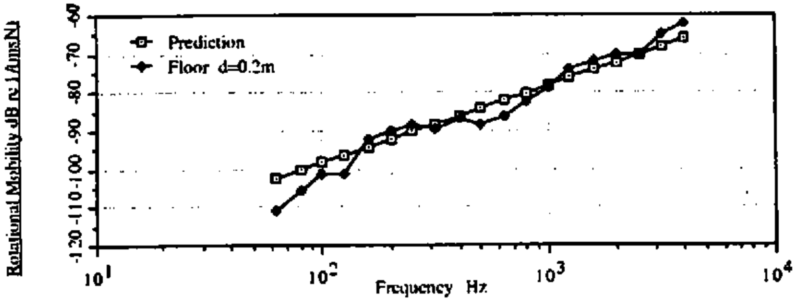


Figure 8.The real part of the rotational mobility, predicted and measured by Verheij Method, of a concrete floor: 3150*2080*125 mm.

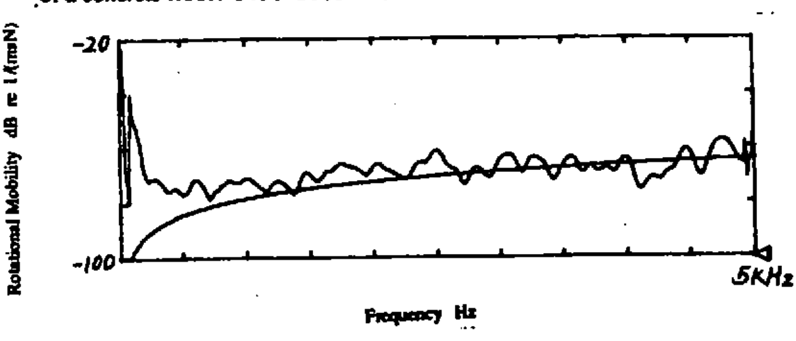


Figure 9. The real part of the rotational mobility, predicted and measured by a moment driver, of a concrete floor: 4820*3000*125 mm.