Scattering of sound by sound in the presence of a cylinder or a sphere

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A preliminary report on the scattering of sound by sound in the presence of cylindrical obstacles was given by the present authors at the Austin symposium in 1969[1]. In the experiments, primary beams of 7 and 5 Mhz frequency intersect perpendicularly in water. The transducers are circular quartz crystals of 1-in. diameter. Solid metal cylinders and, later, solid metal spheres, were placed in the interaction region, and the sum frequency was detected by a 12 MHz transducer, mounted on a rotatable arm pivoted at the center of the interaction region.

The signals from the sources were pulsed, and the gating and travel distances so arranged that the pulses had the same length as the width of the source transducers and arrived in the interaction region at the same time.

The presence of the cylinder in a beam of low intensity ultrasound produces a multi-lobe diffraction pattern. Therefore, the presence of two mutually perpendicular beams means that at virtually every detecting angle θ, there will be two beams of sound travelling collinearly, and hence capable of nonlinear interaction.

Theoretical Analysis

The following procedure was employed to derive an approximate theoretical expression for the pressure of the sum-frequency signal.

The pressure in a sound wave emitted radially by an oscillating cylinder is in the linear case given by

$$p(r) = p(a_c) \frac{H_o(kr)}{H_1(ka_c)} \qquad (1)$$

where H_0, H_1 are Hankel function of the first kind.

For two such initial beams, Dean[2] has developed an expression for the pressure at the combination frequency, based on the analysis of Westervelt. If we use Dean's equation under the approximation that $r \gg a_c$, and express the result in terms of the pressure of the fundamental frequency at the field point r instead of at the surface of the cylinder, we obtain the following result for the pressure at the sum frequency:

$$p_+(r) = [\mathrm{Re}\, p_1(r)][\mathrm{Re}\, p_2(r)] \left(2 + \frac{B}{A}\right) \frac{k_+ r}{\rho_o c_o} \qquad (2)$$

where $k_+ = \frac{\omega_1 + \omega_2}{c_o}$ and $R_e[p_i(r)]$ represents the real part of Eq.(1) for each of the primary frequencies.

We now make a physical assumption about the system. Thus far, we have been talking about omnidirectional primary beams, whereas our experimental setup produces a diffraction beam with many side lobes. We shall therefore assume that the scattered (diffracted) field produced by the plane wave incident on the rigid cylinder is equivalent to a cylindrical source modulated by the diffraction pattern of a plane wave incident on the cylinder.[3] This latter expression has the form[3]

$$p_{s_i}(r) = p_i(I) \left[\frac{a_c}{r} \sin \tfrac{1}{2}\Theta_i + \frac{1}{\pi k_i r} \cot^2 \tfrac{1}{2}\Theta \sin^2(k_i a_c \sin \Theta)\right]^{\frac{1}{2}} \cdot e^{-r/(r + \ell_i)} \tag{3}$$

where $p_i(I)$ is the pressure amplitude of the plane wave at the face of the cylinder. The subscript i = 1 refers to the 7-MHz signal, i = 2, to the 5 MHz. To take into account the attenuation of the fundamental component in each of the primary beams because of finite-amplitude wave distortion, we have multiplied the expression for the pressure at low intensities by $e^{-r/(r+\ell)}$ where ℓ_i, the discontinuity distance, is given by

$$\frac{1}{\ell_i} = \left(1 + \frac{B}{2A}\right) M_i k_i \tag{4}$$

A,B, being the well-known coefficients in the expansion of the pressure in terms of the density, M_i, k_i the acoustic Mach number and the wave number, respectively, for the appropriate primary beam.

In accord with our assumption, therefore, we explicitly identify the $p_s(r)$ with the Re $[p_i(r)]$ of Eq. (2), so that the pressure $p_+(r)$ will be given by

$$p_+(r) = p_{s_1}(r)\, p_{s_2}(r) \left(\frac{B}{A} + 2\right) \frac{k_+ r}{\rho_0 c_0^2} \tag{5}$$

(cylinder)

Entirely analogous arguments can be used for the case of a solid metal sphere, beginning again with an interaction formula developed by Dean. As a final result, we obtain

$$p_+(r) = p_{s_1}(r)\, p_{s_2}(r) \left(\frac{B}{A} + 2\right) \frac{k_+ r}{\rho_0 c_0^2} \ell n\, k_+ r \tag{6}$$

(sphere)

with

$$p_{s_i}(r) = p_i(T) \frac{a_s}{2r} \left[1 + \cot^2 \tfrac{1}{2}\Theta\, J_1(k_i a_s \sin \Theta_i)\right]^{\frac{1}{2}} e^{-r/(r+\ell_i)} \tag{7}$$

for the diffraction field of a plane wave by a sphere[4]. Here, of course, a_s represents the radius of the sphere.

Experimental Results

The experimental arrangements have been described in detail previously[1]. The formulas of diffraction theory (3) and (7) were verified for the high intensity sources by using each source separately and measuring the fundamental component as a function of the angle θ. The results are shown for the cylinder in Fig. 1 (where both of the sources were tested) and for the sphere in Fig. 2.

The corresponding interaction patterns are plotted in Figs. 3-5. Figure 3 gives the interaction at the sum frequency of 12 MHz for the 1/8" radius rod used for Fig.1, while Fig. 4 shows the corresponding interaction for a rod of 1/32" radius. Fig. 5 gives the sum interaction for the 1/8" radius sphere of Fig. 2.

In all these instances, there is substantial agreement between the experimental data and the calculated values. Another check of the theoretical analysis is the measurement of the peak in the sum frequency ($\theta = 0^\circ$) as a function of the distance of the receiver from the cylinder. For the intensities used in Figs. 3,4, this peak falls off with increase in the distance over the entire range available for measurement, because the wave quickly reaches the sawtooth shape and the conditions of Eqs.3,5 applied.

A different picture could be obtained by greatly reducing the initial sound pressures. These results are shown for the 1/8" rod in Fig. 6. For $r < 20$ cm, the sum frequency contribution is seen to increase with distance. This corresponds to the region of the parametric array and eliminates the possibility that the nonlinear interaction occurred at the surface of the cylinder. The range $20 \text{ cm} < r < 35$ cm corresponds to a transition zone. For $r \gtrsim 35$ cm, the sum frequency contribution decreases with increasing distance in accord with Eq. 5. This latter equation is indicated by the dashed line in the figure.

The results of the measurements therefore substantially confirm the appearance of interaction of sound with sound when two beams are incident on a cylindrical or spherical obstacle, and the fact that this interaction takes place in the manner of the parametric array proposed by Westervelt.

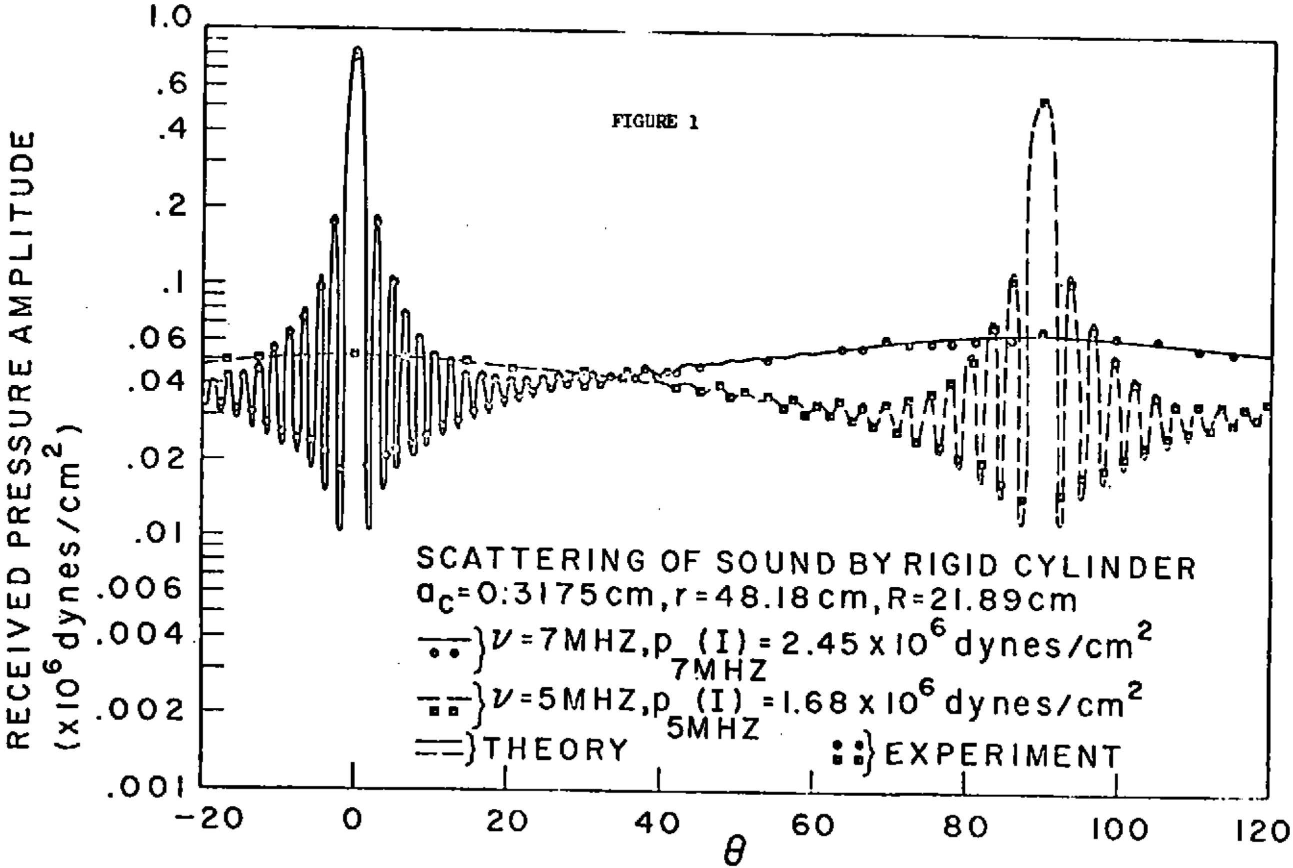

RECEIVED PRESSURE AMPLITUDE
(x10^6 dynes/cm^2)
1.0
.6
.4
.2
.1
.06
.04
.02
.01
.006
.004
.002
.001
-20
0
20
40
60
80
100
120
θ
SCATTERING OF SOUND BY RIGID CYLINDER
a_c = 0.3175 cm, r = 48.18 cm, R = 21.89 cm
ν = 7MHZ, p_7MHZ (I) = 2.45 x 10^6 dynes/cm^2
ν = 5MHZ, p_5MHZ (I) = 1.68 x 10^6 dynes/cm^2
THEORY
EXPERIMENT

FIGURE 1

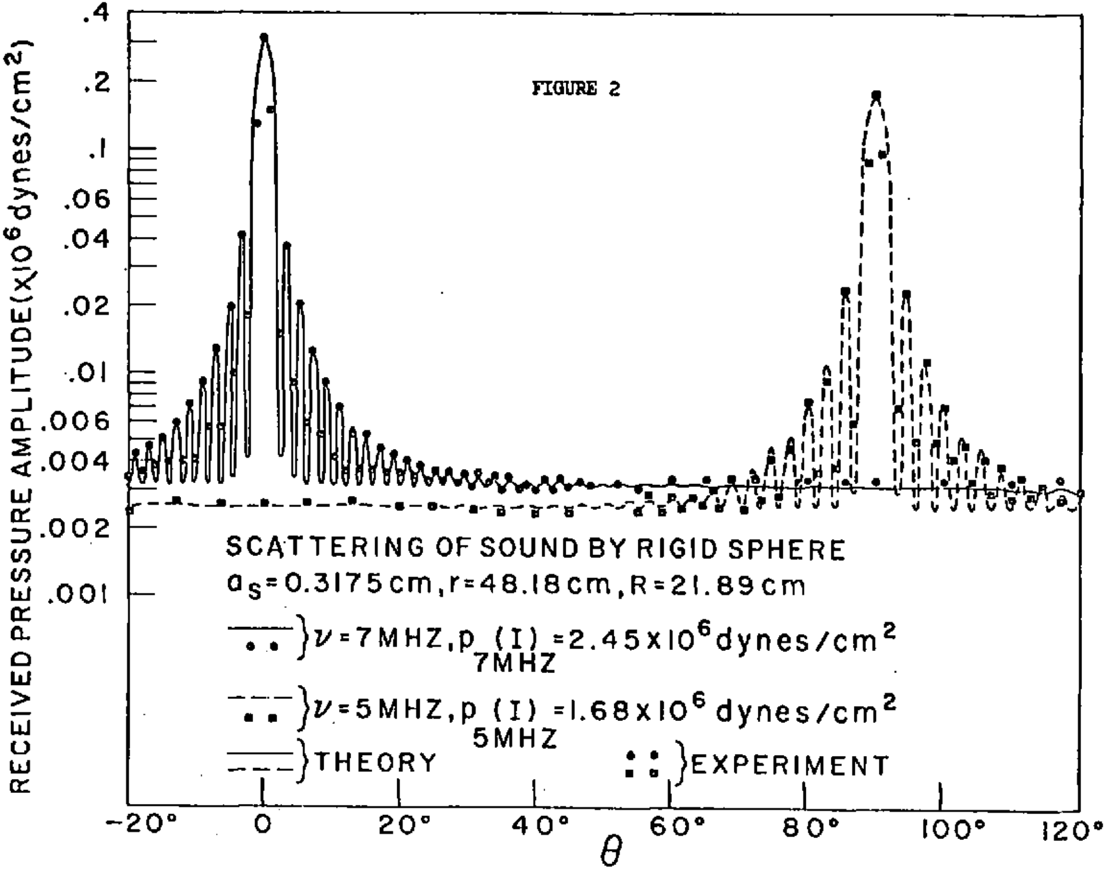


FIGURE 2
RECEIVED PRESSURE AMPLITUDE(x10^6 dynes/cm^2)
.4
.2
.1
.06
.04
.02
.01
.006
.004
.002
.001
SCATTERING OF SOUND BY RIGID SPHERE
a_s=0.3175 cm, r=48.18 cm, R=21.89 cm
ν=7MHZ, $p_{7MHZ}(I)$=2.45x10^6 dynes/cm^2
ν=5MHZ, $p_{5MHZ}(I)$=1.68x10^6 dynes/cm^2
THEORY
EXPERIMENT
−20°
0
20°
40°
60°
80°
100°
120°
θ

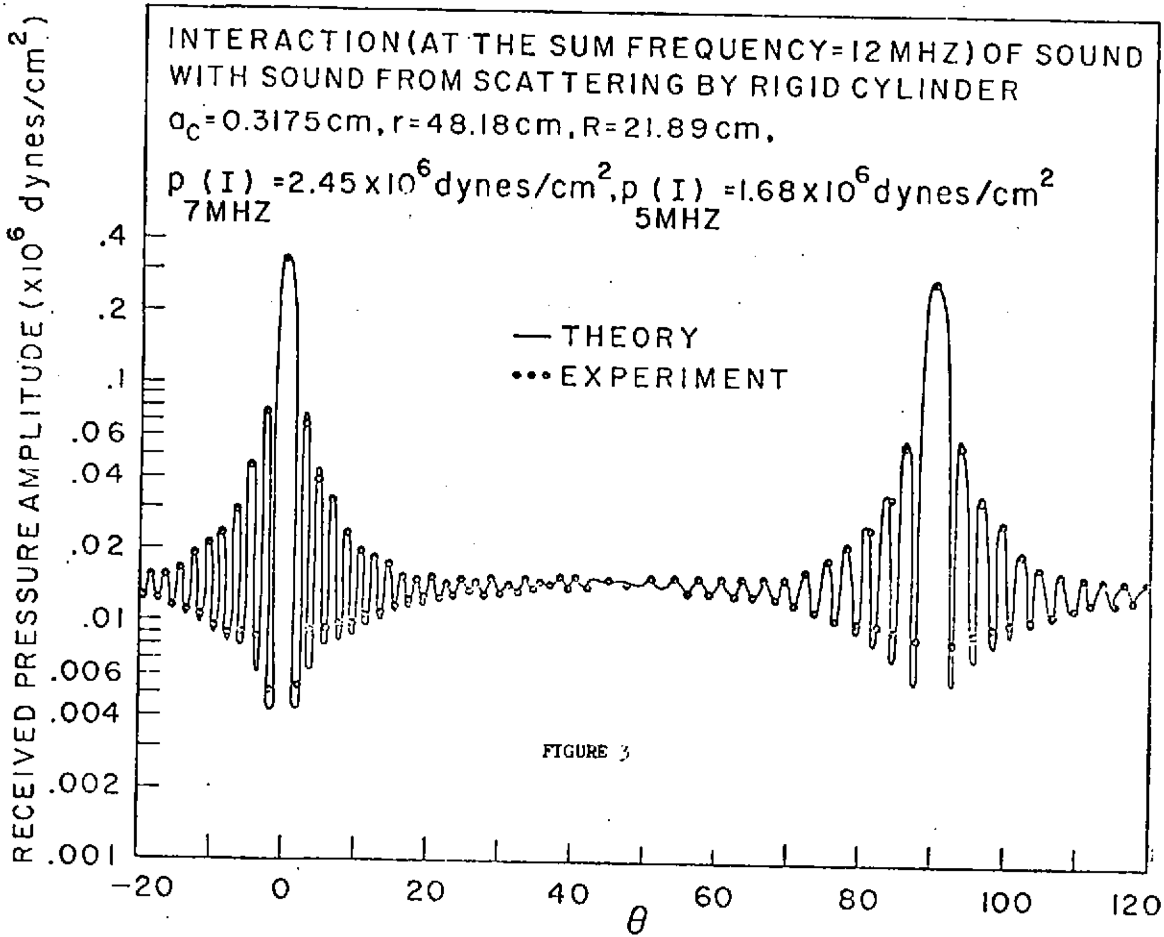
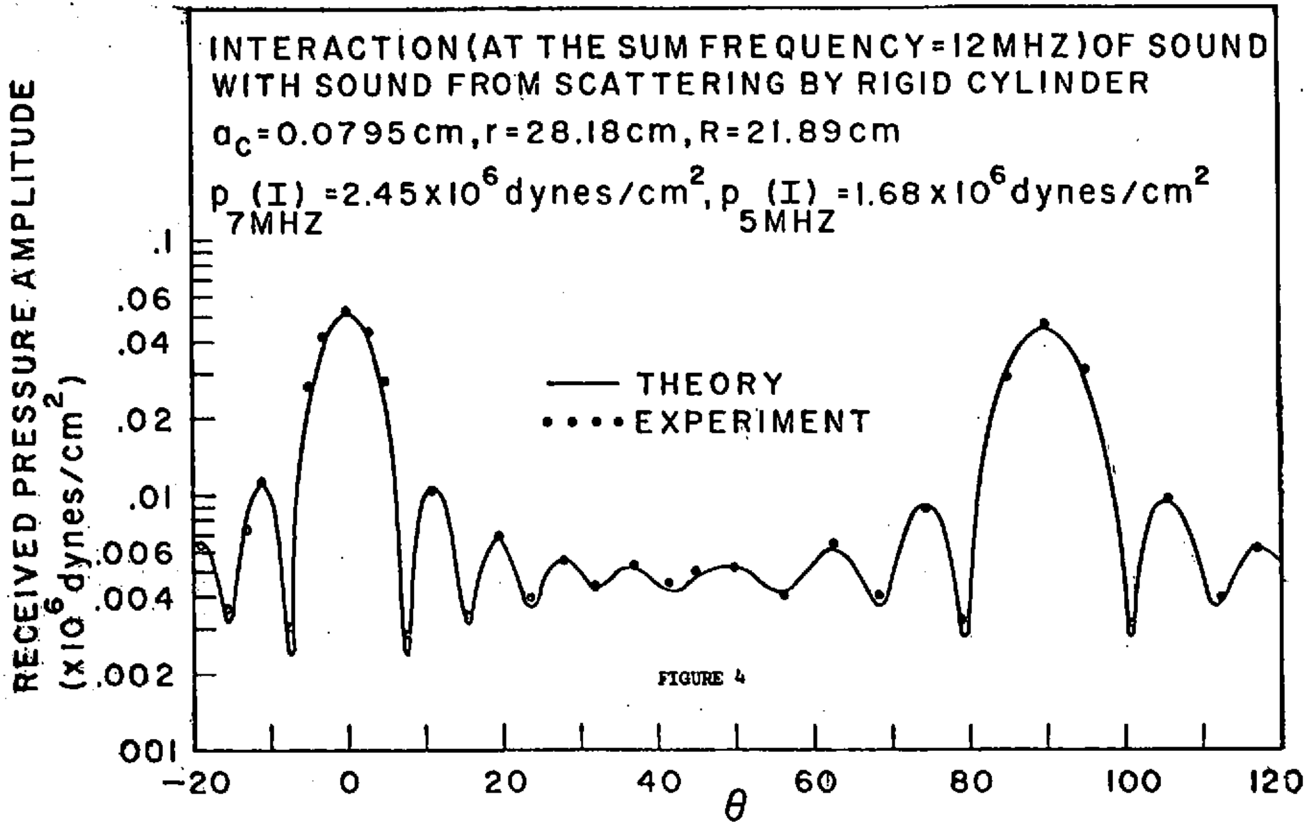

INTERACTION (AT THE SUM FREQUENCY = 12 MHZ) OF SOUND WITH SOUND FROM SCATTERING BY RIGID CYLINDER
a_c = 0.3175 cm, r = 48.18 cm, R = 21.89 cm,
$p_{7MHZ}(I) = 2.45 \times 10^6$ dynes/cm^2, $p_{5MHZ}(I) = 1.68 \times 10^6$ dynes/cm^2
— THEORY
••• EXPERIMENT
RECEIVED PRESSURE AMPLITUDE ($\times 10^6$ dynes/cm^2)
.4
.2
.1
.06
.04
.02
.01
.006
.004
.002
.001
−20
0
20
40
60
80
100
120
θ

FIGURE 3



INTERACTION (AT THE SUM FREQUENCY = 12 MHZ) OF SOUND WITH SOUND FROM SCATTERING BY RIGID CYLINDER
a_c = 0.0795 cm, r = 28.18 cm, R = 21.89 cm
$p_{7MHZ}(I) = 2.45 \times 10^6$ dynes/cm^2, $p_{5MHZ}(I) = 1.68 \times 10^6$ dynes/cm^2
THEORY
EXPERIMENT
RECEIVED PRESSURE AMPLITUDE
($\times 10^6$ dynes/cm^2)
.1
.06
.04
.02
.01
.006
.004
.002
.001
−20
0
20
40
60
80
100
120
θ

FIGURE 4

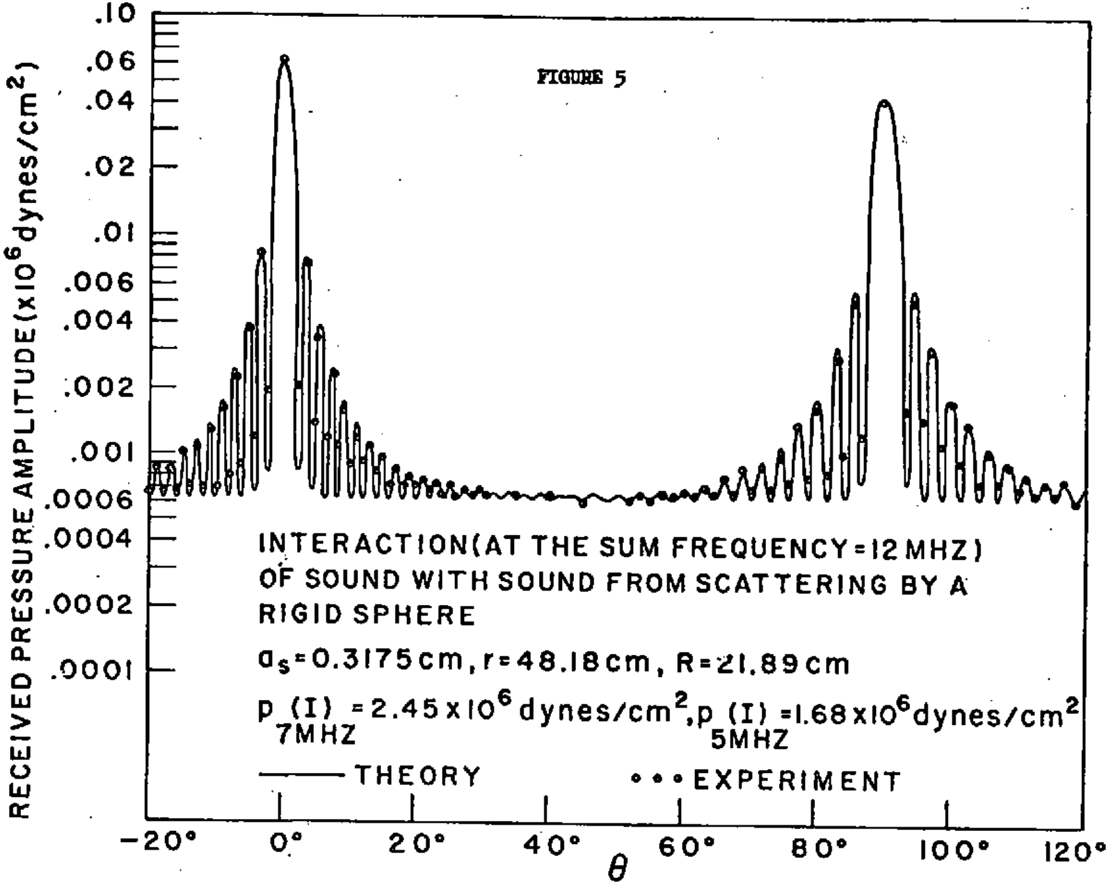


FIGURE 5
RECEIVED PRESSURE AMPLITUDE (x10^6 dynes/cm^2)
.10
.06
.04
.02
.01
.006
.004
.002
.001
.0006
.0004
.0002
.0001
INTERACTION (AT THE SUM FREQUENCY = 12 MHZ) OF SOUND WITH SOUND FROM SCATTERING BY A RIGID SPHERE
a_s = 0.3175 cm, r = 48.18 cm, R = 21.89 cm
p_7MHZ(I) = 2.45 x 10^6 dynes/cm^2, p_5MHZ(I) = 1.68 x 10^6 dynes/cm^2
THEORY
EXPERIMENT
-20°
0°
20°
40°
60°
80°
100°
120°
θ

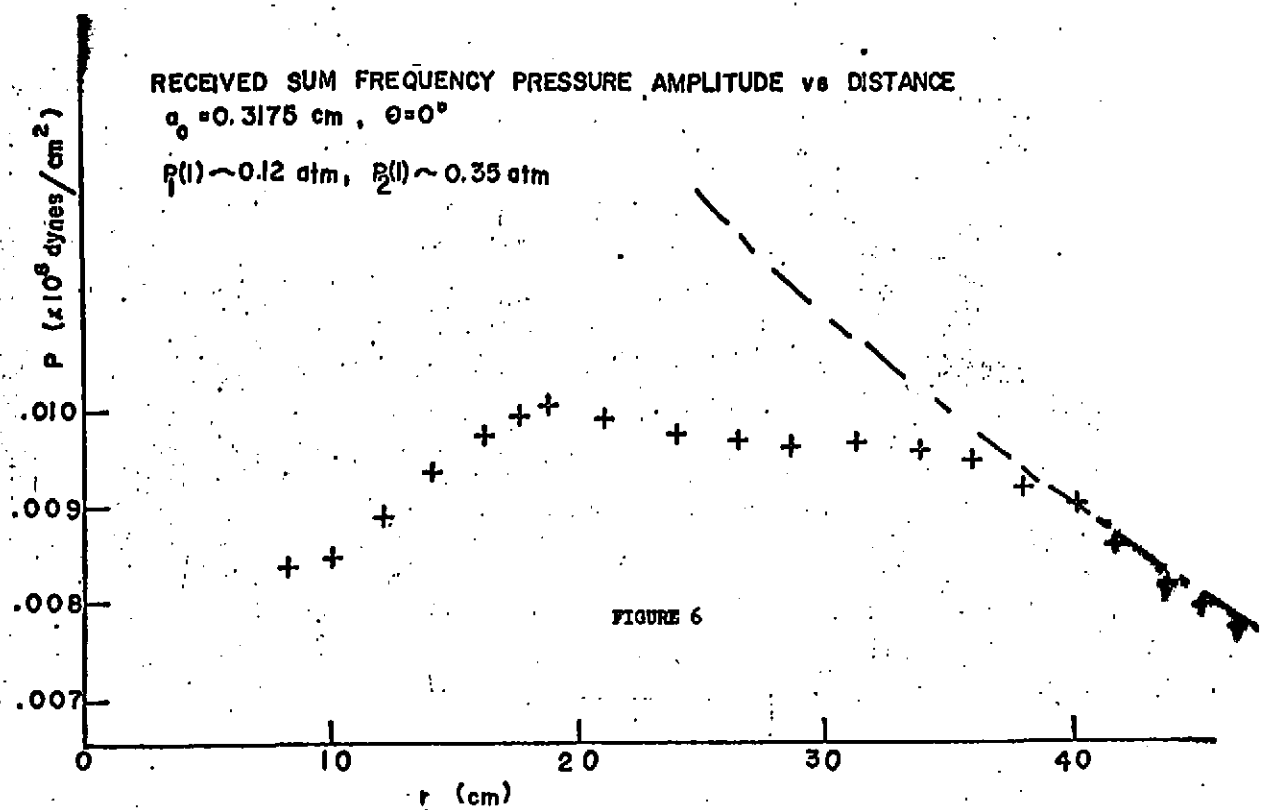

RECEIVED SUM FREQUENCY PRESSURE AMPLITUDE vs DISTANCE
a_0 =0.3175 cm , θ=0°
$P_1(1)$ ~ 0.12 atm, $P_2(1)$ ~ 0.35 atm
P (x10⁸ dynes/cm²)
.010
.009
.008
.007
0
10
20
30
40
r (cm)

FIGURE 6