GEOMETRY RELATED PERFORMANCE BOUNDS FOR PASSIVE SYNTHETIC APERTURE SONAR

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INTRODUCTION

The idea of using the motion of an active sonar array, to create an *active* synthetic aperture, was introduced by Cutrona [1] and Sato, Ueda and Fukada [2]. Passive Synthetic Aperture Sonar has received less attention (see, for example, Pusone and Lloyd [3], Fay and Wolcin [4] and, more recently Stergiopoulos and Sullivan [5]). In common with these papers I shall use the term passive synthetic aperture loosely, to mean any system that uses the motion of the array platform to obtain an improvement in performance.

One, often quoted veto, against passive synthetic aperture sonar (see Autrey [6], for example), is that the frequency, or spectral shape, of the target must be known *a priori*. This is certainly true, for a single hydrophone, or at the output of a narrow band phase-shift beamformer. However if the hydrophone signals are available separately, then it is easy to show that the bearing/frequency ambiguity can be resolved by solving a set of non-linear equations.

Given that there are no fundamental restrictions on passive synthetic aperture sonar, then it is reasonable to ask:"What are the lower bounds on the performance of a passive synthetic array?" An appropriate and mathematically appealing bound is the Cramér-Rao Lower Bound.

For a data vector x , and a model for the probability density function of the data, $f(x \mid \theta)$, dependent on some parameters θ , then the log likelihood function is defined as $L(\theta) = \log(f(x \mid \theta))$ and the Fisher Information matrix can be written as

$$J(\theta) = \left\{ J_{ij} \right\} = -E\left\{ \frac{\partial^2 L}{\partial\theta_i \, \partial\theta_j} \right\} \qquad (1)$$

The multi-parameter Cramér-Rao bound is given by the inverse of J.

FARFIELD GEOMETRY

The geometry shown in Figure 1 was chosen to be applicable to a linear towed array in a sound field produced by a stationary farfield target. The target is at a bearing θ and is emitting a tone of frequency ω. An M element array is used to observe the target, for N time samples, over a period $-T/2 \leq n\,\tau \leq T/2$.

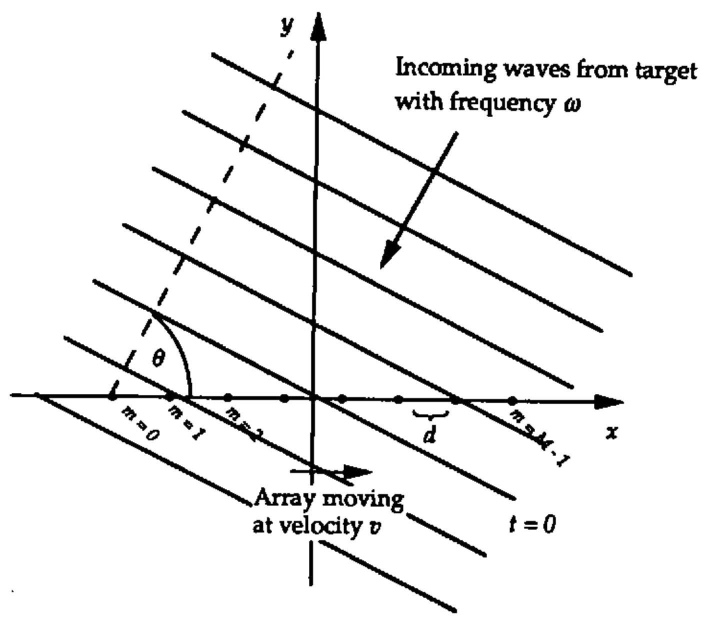

Figure 1: Farfield geometry for a line-array towed along the x-axis *at a speed* v

If the noise is Gaussian and white, with a standard deviation of σ, then the elements of the Fisher information matrix are given by

$$J_{ij} = \frac{1}{\sigma^2} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \frac{\partial r_{m,n}}{\partial \theta_i} \frac{\partial r_{m,n}}{\partial \theta_j} \tag{2}$$

where $r_{m,n}$ is value of the signal that would be measured at the m^{th} hydrophone after the n^{th} time sample in the absence of noise. Even though the target is characterised its frequency and bearing, an initial phase also has to estimated, giving a parameter vector $\theta = [\omega, \cos\theta, \phi]$, and a J matrix with nine elements:

$$J_{ij} = \frac{1}{\sigma^2} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{m,n}^2 \begin{bmatrix} b_{m,n}^2 & b_{m,n}\,c_{m,n} & b_{m,n} \\ b_{m,n}\,c_{m,n} & c_{m,n}^2 & c_{m,n} \\ b_{m,n} & c_{m,n} & 1 \end{bmatrix} \tag{3}$$

$$a_{m,n} = \sin\left(\omega(n\tau - T/2)\left[1 + \frac{v\cos\theta}{c}\right] + \frac{\omega(md - L/2)\cos\theta}{c} + \phi \right) \tag{4}$$

$$b_{m,n} = (n\tau + T/2)\left[1 + \frac{v\cos\theta}{c}\right] + \frac{(m\,d - L/2)\cos\theta}{c} \tag{5}$$

$$c_{m,n} = \frac{\omega}{c}\left[v(n\tau - T/2) + m\,d - L/2 \right] \tag{6}$$

Figure 2 shows the variation of the bounds, as a function of bearing, for a target of frequency 100 Hz. The array has an inter-element spacing of 2.5 m and is sampled at 400 Hz. The sound speed is assumed to be constant at 1500 ms^{-1}.

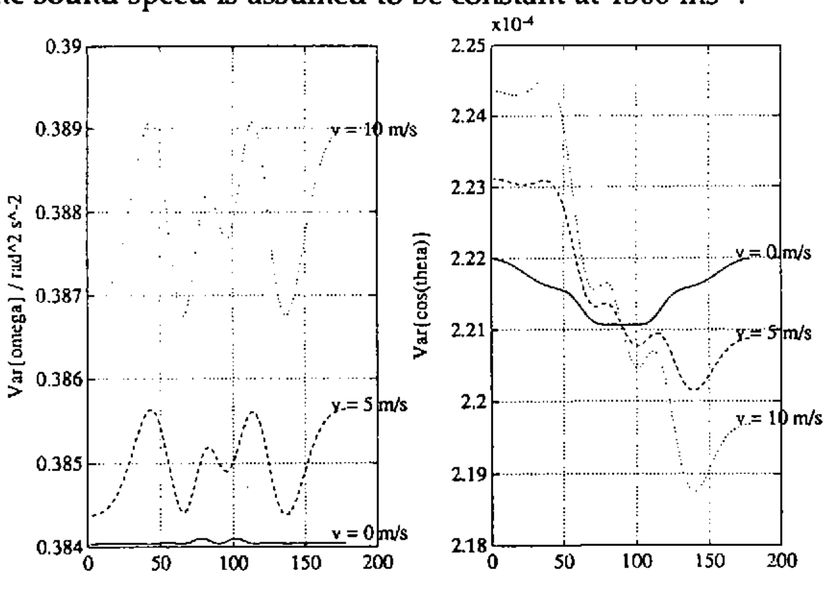

Figure 2: Cramér-Rao bounds for parameters of farfield targets, (a) frequency, (b) bearing

PERFORMANCE BOUNDS FOR PASSIVE SYNTHETIC APERTURE SONAR

The curves shown are for tow speeds of 0, 5 and 10 ms^{-1} for 100 time samples from a 10 element array. The signal to noise ratio at a single hydrophone is 0 dB. The bounds for the moving array are not symmetric about the broadside bearing and whilst there is an increase in the frequency bound, the bearing bound decreases for tows away from the target. Furthermore it would appear that there is an optimum operating point at any given velocity. At 5 ms^{-1} for instance, the best results are achieved at a bearing of about 140°. Nevertheless the moving array does not seem to offer any major improvements over the static array. Initially, this may seem a surprising result but with further thought it can be seen that the conventional geometry, used here, is constraining the observed frequency to be constant. In comparison, synthetic aperture radar relies a frequency variation as the array platform sweeps past the target. A more realistic bound can be obtained by modelling the target as being within the nearfield of the synthetic aperture.

NEARFIELD GEOMETRY

The bounds for the nearfield case are more intractable than those for the farfield case - especially as there is now an increase in the number of parameters required to describe the target (frequency, bearing, range and phase).

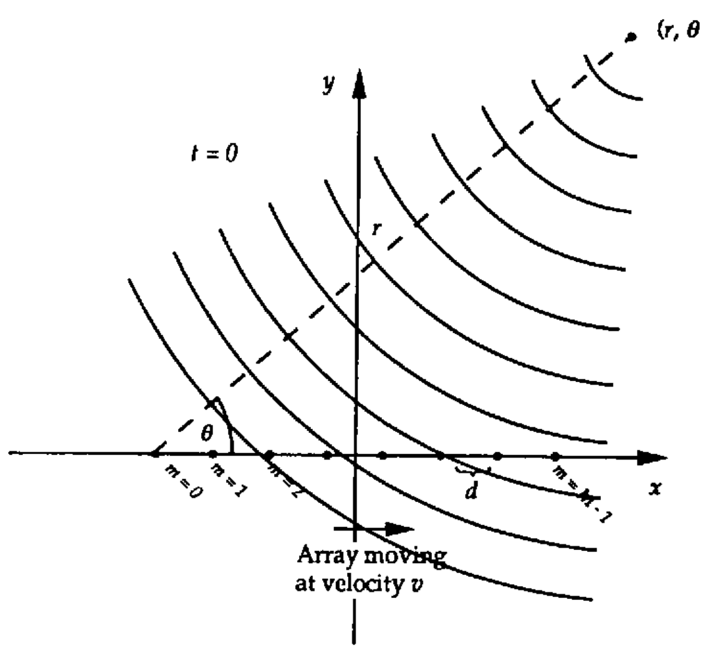

Figure 3: Nearfield geometry for a line-array towed along the x-axis at a speed v

In Figure 3, the target is at a range *r* and at a bearing of *θ*, and, as with the farfield case, the tow is symmetric about the *y*-axis. The Fisher Information matrix for a parameter vector $\theta = [\omega, \cos\theta, r, \phi]$, can be written as

$$J = \frac{a_0^2}{\sigma^2} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{m,n}^2 \begin{bmatrix} b_{m,n}^2 & b_{m,n} c_{m,n} & b_{m,n} d_{m,n} & b_{m,n} \\ b_{m,n} c_{m,n} & c_{m,n}^2 & c_{m,n} d_{m,n} & c_{m,n} \\ b_{m,n} d_{m,n} & c_{m,n} d_{m,n} & d_{m,n}^2 & d_{m,n} \\ b_{m,n} & c_{m,n} & d_{m,n} & 1 \end{bmatrix} \quad (7)$$

$$a_{m,n} = \sin\left(\omega n\tau - \omega(r^2 + (n\tau v + md)^2 - 2r(n\tau v + md)\cos\theta)^{1/2}/c + \phi\right) \quad (8)$$

$$b_{m,n} = n\tau - \left(r^2 + (n\tau v + md)^2 - 2\, r\, (n\tau v + md)\cos\theta\right) \quad (9)$$

$$c_{m,n} = \frac{(n\,\tau\, v\ +\ m\, d)\sin\theta}{c\, b_{m,n}} \quad (10)$$

$$d_{m,n} = \frac{r\ -\ (n\,\tau\, v\ +\ m\, d)\cos\theta}{c\, b_{m,n}} \quad (11)$$

Figure 4 shows the nearfield bounds for a target at a range of 100m and for tow speeds of 0, 50 and 100 $m.s^{-1}$.

The unrealistically high tow speeds are used here to avoid computing the bound with a large number of time samples. With the new geometry, the moving array gives a significant improvement in the lower bound for range accuracy. There is also a modest improvement in variance of the bearing estimate, as in the farfield case.

It would appear that the target can be more accurately localised when there is a significant geometry change during the tow. Further illustration of this can be seen in Figure 5. The bounds are computed for a broadside target, at a range of 1000 m, emitting a single tone at 75 Hz. As the array becomes longer the tow introduces less and less of a geometry change

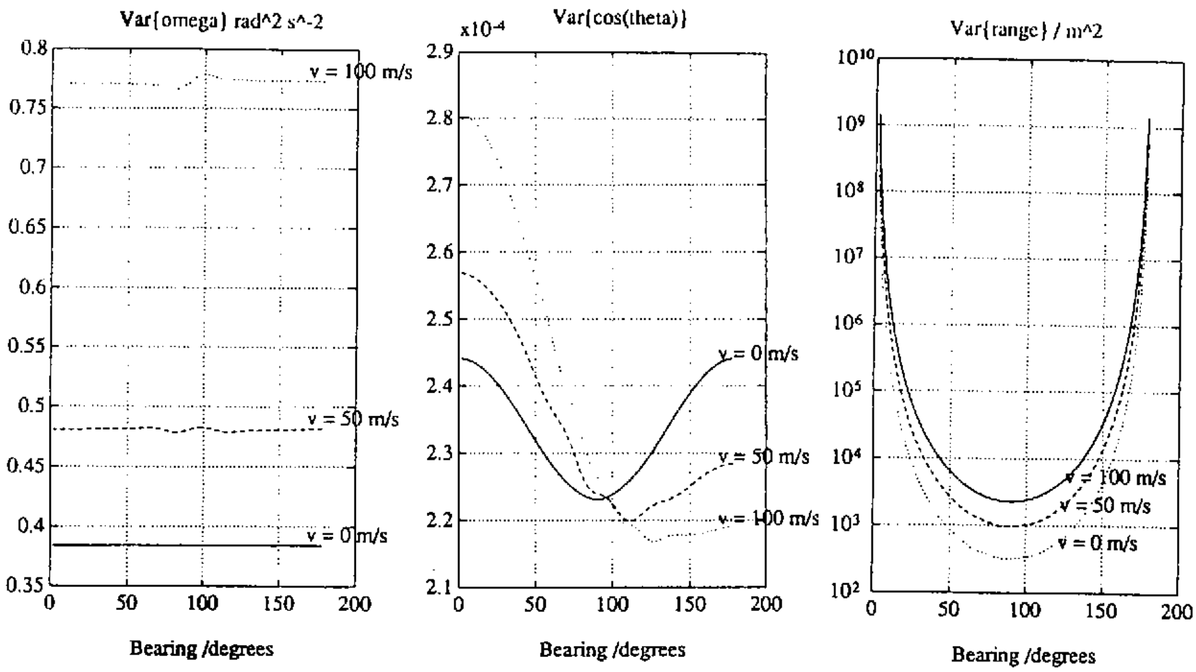

Figure 4: Cramér-Rao Bounds for Nearfield geometry, (a) variance of ω, (b) variance of cos θ, (c) variance of range, r.

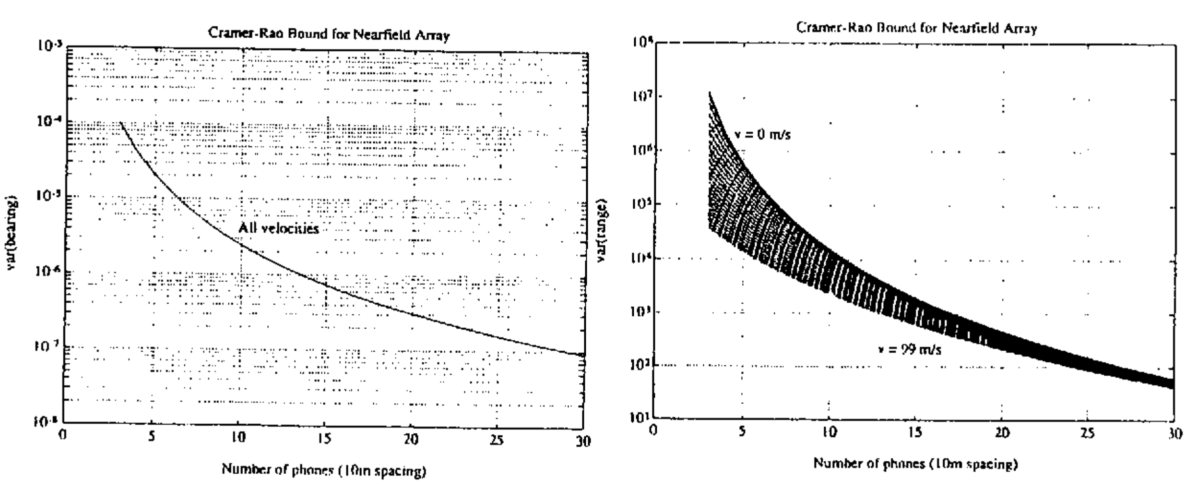

Figure 5: Variation in bounds for various tow speeds,(a) variance of r, (b) variance of θ

CONCLUSION AND DISCUSSION

Clearly, there are many other factors that may affect the performance of a passive synthetic aperture system, such as wavefront coherence and uncertaincy in hydrophone position. However many of these problems are applicable to other, more conventional, systems such as very long towed arrays.

The Cramér-Rao Lower Bounds for a restricted set of possible targets have been examined. Passive synthetic aperture sonar has the potential to produce improved target parameter estimates. An important factor when assessing the performance of passive synthetic aperture sonar systems is the geometry of the target/array encounter.

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