THEORETICAL AND EXPERIMENTAL INVESTIGATION OF DYNAMIC CHARACTERISTICS OF SAND COLUMN

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INTRODUCTION

It is well known that increasing damping of machinery structures will reduce noise radiation due to ringing vibration. However, increasing damping of industrial machinery structures is not often a simple procedure. Damping achieved by attached viscoelastic layers has been used successfully for many years on industrial machines, but it is only effective for light beam and plate-like structures. Furthermore, it cannot be used in hostile environment.

Squeeze-film damping can also be used to increase the loss factor of structural components [1]. This method uses a sandwich construction consisting of two parallel plates with a layer gap filling with air or a heavy fluid such as oil. However, in order to obtain high loss factor of structures, it is necessary to make the gap very thin; the applied plate very heavy and flexible.

An alternative damping treatment, granular infill treatment, can be used to increase the structural loss factor. Using granular materials such as sand to fill the existing cavities of the structural components, the damping will increase. Chow and Pinnington studied the damping of plates with a sand infill treatment using the impedance approach [2]. However, some of the explanations of the results are uncertain. This current work is a step towards a better understanding of these mysteries.

The following sections will discuss the dynamic response of a sand column with a base excitation. Experimental and theoretical results are also compared.

SAND COLUMN MODELLING AND ITS DYNAMIC CHARACTERISTICS

Consider a column of sand which is free at one end and excited at the other end as shown in figure 1. It is assumed that only longitudinal motion is allowed in the column, therefore it can be treated as a rod. Both the elastic and viscous stresses are assumed to exist in the sand column. The wave equation of this case can be written as

$$E^* \frac{\partial^2 u}{\partial y^2} + C \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) = P \frac{\partial^2 u}{\partial t^2} \qquad (1)$$

where u is the displacement along the rod, E*, C and P are the complex Young's modulus, the viscous coefficient and the density of sand respectively.
The boundary conditions are as follows: the strain is equal to zero at free surface and the product of stress and the cross sectional area of the column is

equal to exciting force. The solution of the wave equation (1) is

$$u(y.t) = -\frac{F_o e^{j\omega t}}{SK\,(E^* + j\omega c)}\,(\mathrm{ctg}\;kl\;.\;\cos ky + \sin ky) \qquad (2)$$

The driving point and transfer accelerance can then be evaluated respectively as

$$I(o) = \frac{\ddot{u}\,(o,t)}{F\,(o,t)} = \frac{\omega^2}{SK\,(E^* + j\omega c)}\,\mathrm{ctg}\;kl \qquad (3)$$

$$I(y) = \frac{\ddot{u}\,(y,t)}{F\,(o,t)} = \frac{\omega^2}{SK\,(E^* + j\omega c)}\,(\mathrm{ctg}\;kl\;.\;\cos ky + \sin ky) \qquad (4)$$

Where S is the cross-sectional area of the column and K is the wavenumber in the sand.

A piston-like circular disc was placed in between the sand column and the excitation in the experiments (see figure 2). This effect has to be taken into account in the theory. The actual driving point and transfer accelerance which are measured in the experiments will then be equal to

$$I(o) = \frac{\mathrm{ctg}\;kl}{m_p\;.\;\mathrm{ctg}\;kl + \dfrac{SK\,(E^* + j\omega c)}{\omega^2}} \qquad (5)$$

$$I(y) = \frac{\mathrm{ctg}\;kl\;.\;\cos ky + \sin ky}{m_p\;.\;\mathrm{ctg}\;kl + \dfrac{SK\,(E^* + j\omega c)}{\omega^2}} \qquad (6)$$

where m_p is piston mass.

Figure 3 displays the estimated driving point accelerance of a 1 m height sand column using equation (3). The response is directly proportional to frequency at high frequencies. Figure 4 gives the estimated transfer accelerance at 0.2 m and 0.6 m level of the column using equation (4). The attenuation appears at high frequencies and the rate of attenuation increases rapidly as the sand level increases.

Figures 5 and 6 show the estimated driving point accelerance (including the effect of the circular disc) of 1 m sand column using equation (5) and the estimated transfer accelerance at 0.2 m and 0.6 m level of the sand column using equation (6) respectively. As can be seen in figure 5, the driving point accelerance at low frequencies is sand column mass (3.2 kg) controlled whereas at high frequencies the circular disc mass (0.06 kg) dominates. In figure 6, the attenuation is increased by including the effect of the circular disc.

EXPERIMENTAL RESULTS

To verify the theory, the initial experiments were carried out to measure the driving point and transfer accelerance of a sand column. Figures 7, 8 and 9 compare the measured and predicted driving point and transfer accelerance. In figure 7, the measured and predicted point accelerance agrees very well at high frequencies except that the peak predicted occurs at lower values. This is probably due to the fact that the friction between the wall of the steel tube and the sand raises the elastic modulus of the system. Therefore, the resonante frequencies measured will accordingly increase. It is suggested that a tube with material having the elastic modulus the same order of magnitude as the sand should be used and this will probably give better agreement.

The measured and predicted transfer accelerance shows similar trend at both the low and high frequencies. The agreement is reasonably good at high frequencies.

CONCLUSION

Experimental and theoretical investigation of the sand column dynamic responses have been studied and the agreement of these initial results is encouraging, but indicate that a more ideal experiment should be conducted. The sand modulus as a function of the sand column height requires further study.

REFERENCES

1. L.C. Chow and R.J. Pinnington, 1986, "Predicted Industrial Methods of Increasing Structural Damping in Machinery", Proceedings of the Institute of Acoustics, Vol. 8, 339-346, Salford.

2. L.C. Chow and R.J. Pinnington, 1986, "On the prediction of the loss factors of plates using sand granular material", ISVR Technical Report No. 141, University of Southampton.

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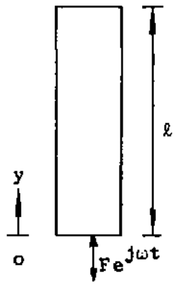

Figure 1

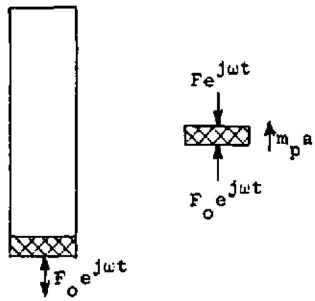

Figure 2

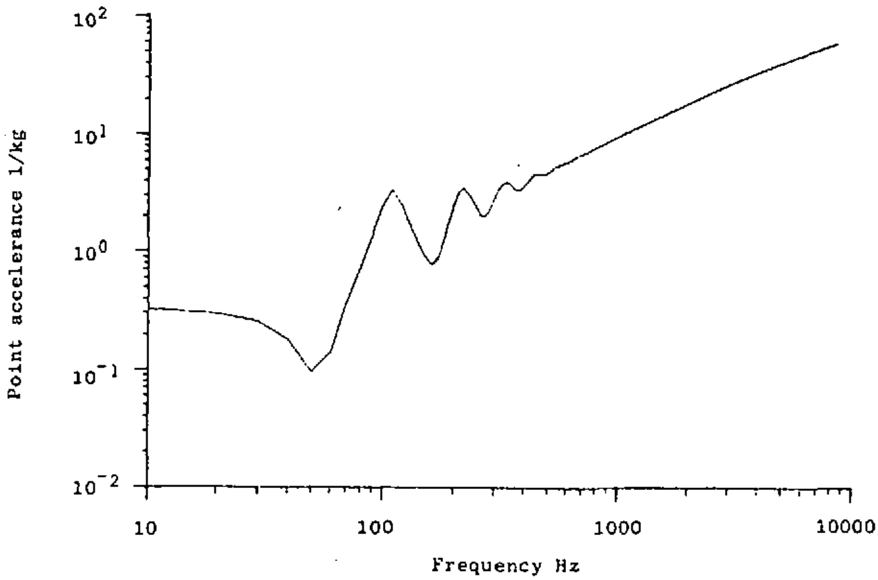

Figure 3. The estimated point accelerance of 1 m height sand column.

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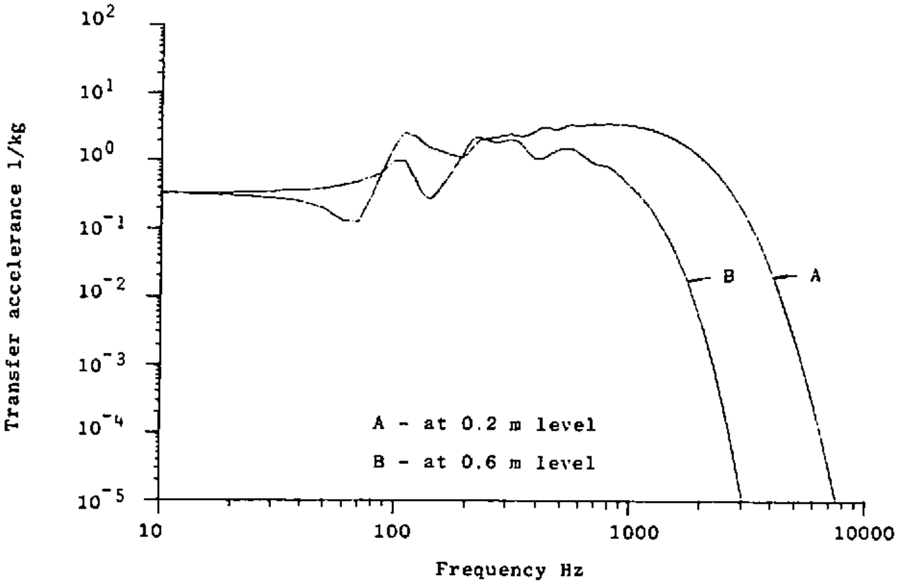

Figure 4. The estimated transfer accelerance at 0.2 m and 0.6 m level of 1 m height sand column.

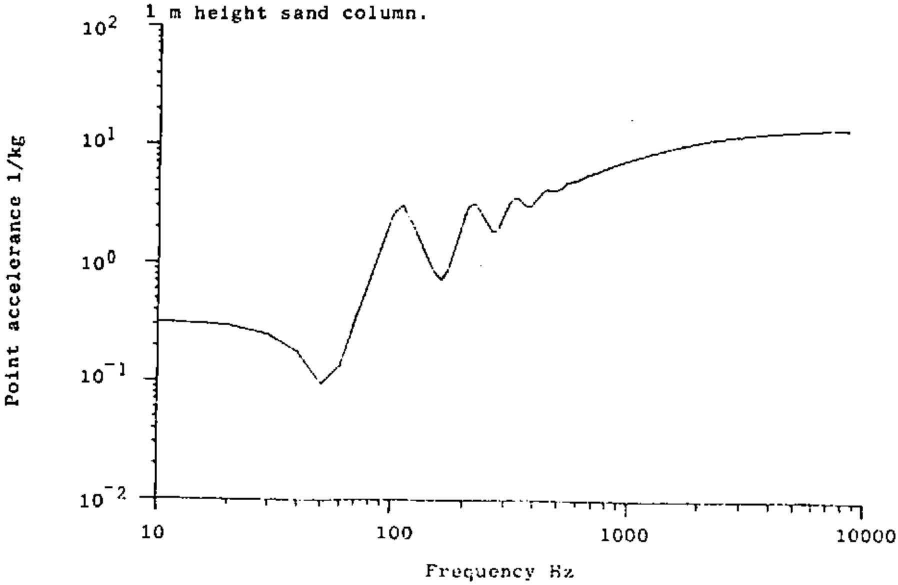

Figure 5. The estimated point accelerance of 1 m height sand column (including the effect of the circular disc).

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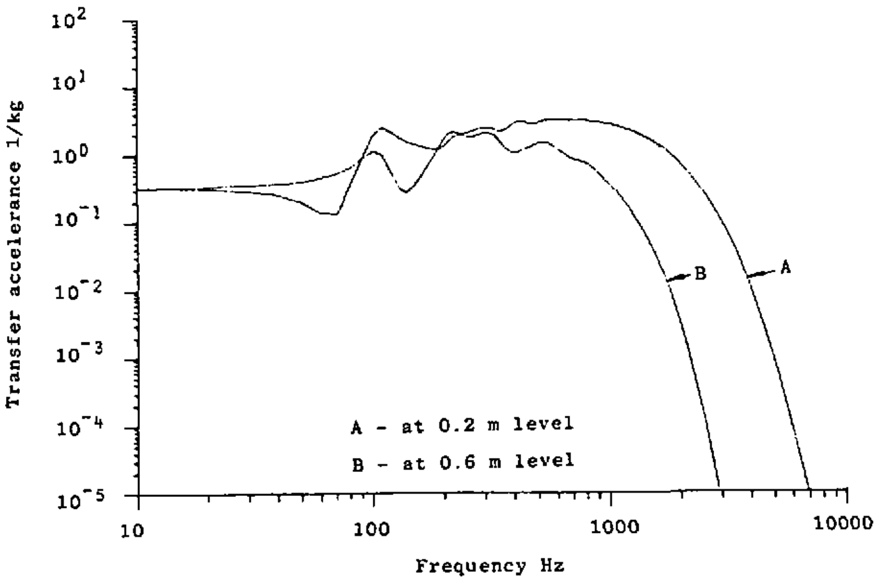

Figure 6. The estimated transfer accelerance at 0.2 m and 0.6 m level of 1 m height sand column (including the effect of the circular disc).

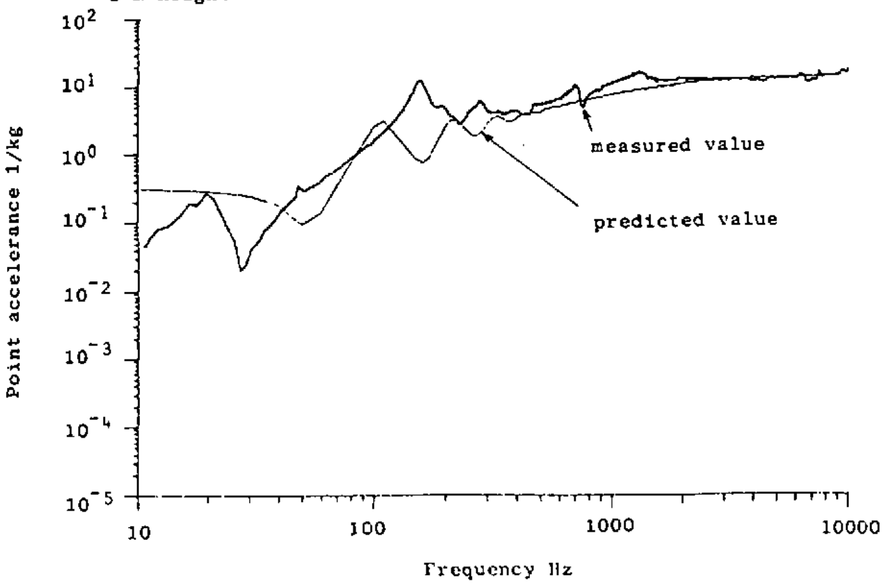

Figure 7. Comparison of measured and predicted point accelerance of 1 m height sand column.

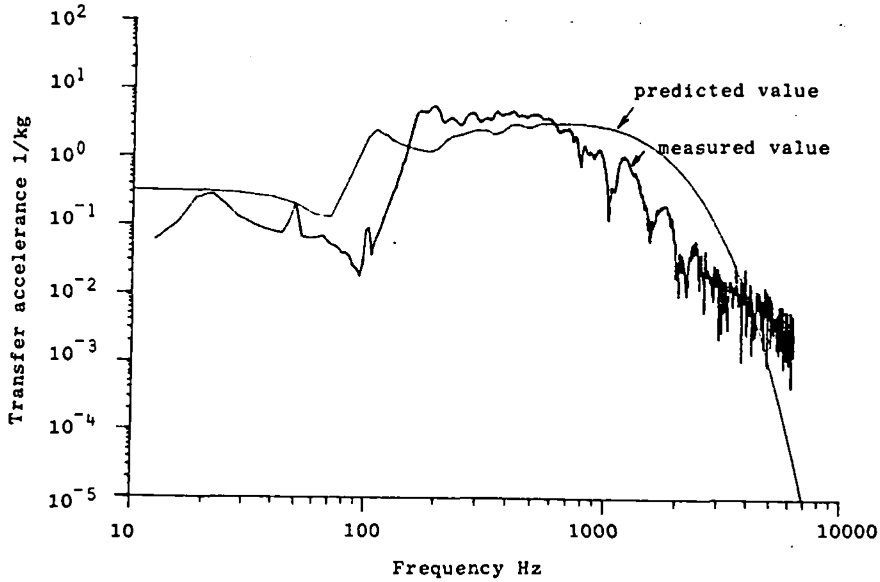

Figure 8. Comparison of measured and predicted transfer accelerance at 0.2 m level.

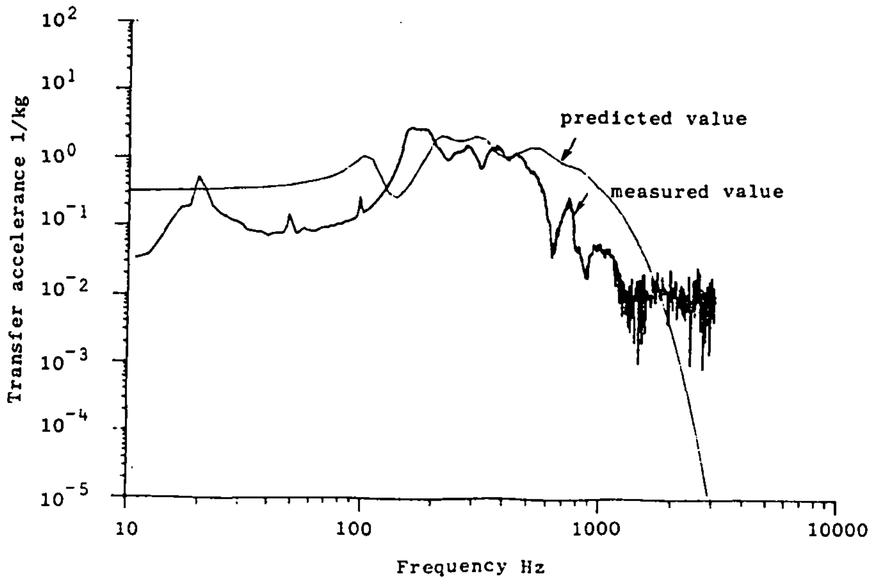

Figure 9. Comparison of measured and predicted transfer accelerance at 0.6 m level.