Proceedings of The Institute of Acoustics

THE APPLICATION OF JET NOISE SOURCE LOCATION TECHNIQUES TO THREE-DIMENSIONAL SOURCE DISTRIBUTIONS

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1. Introduction

Jet noise source location techniques, in particular Polar Correlation [1] and the Acoustic Telescope [2], evaluate an equivalent line source model of the actual source distribution. In practice the sources on a jet engine are contained within a source region which has significant displacements from the jet axis. For example on the RB 211 the inlet duct has a diameter of twelve acoustic wavelengths at 2000 Hz. This paper will discuss the significance of the line source approximation and demonstrate the degradation of resolution which occurs for a source which is moved off axis.

2. The application of polar correlation to three-dimensional source distributions

The source image which is obtained using Polar Correlation has previously been evaluated in terms of a one dimensional source distribution by Fisher et al. [1] However, the same analysis may be applied to the more general case of a three dimensional distribution and this highlights the effect which the one dimensional approximation has on the measured results. In order to evaluate this effect, the pressure signal measured from a three dimensional source distribution at $\bar{x}$ in the acoustic far field will be defined in the frequency domain as

$$p(\bar{x},\omega) = \int_V q(\bar{y},\omega)e^{ikr}\ \frac{d\bar{y}}{r} \qquad (2.1)$$

where $q(\bar{y},\omega)$ defines the source fluctuations at $\bar{y}$ and $r = |\bar{x} - \bar{y}|$

Polar Correlation evaluates the source distribution by considering the cross spectral density between the signals from measurement points at $\bar{x}$ and $\bar{x}_o$. This quantity may be defined as

$$G(\bar{x},\bar{x}_o) = Ex\left[p(\bar{x},\omega)p^*(\bar{x}_o,\omega)\right]. \qquad (2.2)$$

In this equation the asterisk denotes a complex conjugate and the frequency dependence of $G(\bar{x},\bar{x}_o)$ has been dropped for convenience. In all that follows the frequency will be kept fixed.

The cross spectral density may be related to the source fluctuations by

$$G(\bar{x},\bar{x}_o) = \int_V Q(\bar{y})e^{ik(r-r')}\ \frac{d\bar{y}}{rr'}, \qquad (2.3)$$

where $Q(\bar{y})$ is defined as an effective source strength as viewed from $\bar{x}_o$ and defined by

$$Q(\bar{y}) = \int_{V_o} Ex\left[q(\bar{y},\omega)q(y_o,\omega)\right] e^{ik(r'-r_o)}\ \frac{r'}{r_o}\ d\bar{y}_o \qquad (2.4)$$

where $r_o = |\bar{x}_o - \bar{y}_o|$ and since measurements are taken on a polar arc of radius R in the acoustic far field, the propagation distances r and r' may be approximated by

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$$r = |\bar{x} - \bar{y}| = R - y_1\sin\alpha - y_2\cos\alpha + \ldots\ O(|\bar{y}|^2/R)$$

$$r' = |\bar{x}_o - \bar{y}| = R - y_1\sin\alpha - y_2\cos\beta + \ldots\ O(|\bar{y}|^2/R)$$

so that

$$r - r' = -y_1(\sin\alpha - \sin\beta) - y_2(\cos\alpha - \cos\beta). \qquad (2.5)$$

Also in the amplitude term $(rr')^{-1} \approx R^{-2}$.

These far field approximations may be incorporated in (2.3) to give for the cross spectral density

$$G(\bar{x},\bar{x}_o) = \frac{1}{R^2}\int_v Q(\bar{y})e^{-iky_1(\sin\alpha-\sin\beta)-iky_2(\cos\alpha-\cos\beta)}\,d\bar{y}. \qquad (2.6)$$

To obtain the source image Polar Correlation evaluates the inverse Fourier transform of (2.6) with respect to $k(\sin\alpha-\sin\beta)$. However in practice this inverse Fourier transform cannot be explicitly evaluated since $G(\bar{x},\bar{x}_o)$ cannot be measured for values of $k(\sin\alpha-\sin\beta)>k_m$ where $k_m< k$. This limitation is most conveniently described by considering $G(\bar{x},\bar{x}_o)$ to be multiplied by a weighting function $w(k(\sin\alpha-\sin\beta))$ which is zero outside the range of measurement where $k(\sin\alpha-\sin\beta)>k_m$. Thus the source image is given by the inverse transform:

$$Q_I(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\bar{x},\bar{x}_o)w(k(\sin\alpha-\sin\beta))e^{ikz(\sin\alpha-\sin\beta)}d(k(\sin\alpha-\sin\beta) \qquad (2.7)$$

Then from (2.6) the source image is obtained in terms of the three dimensional source distribution as

$$Q_I(z) = \frac{1}{R^2}\int_v Q(\bar{y})W(y_2,y_1- z)d\bar{y} \qquad (2.8)$$

where

$$W(y_2,y_1- z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} w(k(\sin\alpha-\sin\beta))e^{-ik(y_1-z)(\sin\alpha-\sin\beta)-iky_2(\cos\alpha-\cos\beta)}$$
$$\times\ d(k(\sin\alpha-\sin\beta)). \qquad (2.9)$$

This result describes the effect of the three dimensional source distribution in terms of a three dimensional window function $W(y_2,y_1-z)$ which operates on the actual distribution to give the source image. In assuming that the jet noise source distribution is one dimensional it is implied that the sources are accurately defined as a function of y_1 by

$$Q'(y_1) = \int_{y_2}\int_{y_3} Q(\bar{y})\ dy_2dy_3. \qquad (2.10)$$

However in practice this approximation is only valid when the window function $W(y_2,y_1-z)$ is independent of y_2. Clearly for values of $ky_2(\cos\alpha-\cos\beta)\ll 1$ this appoximation will hold, but in practice this may not always be the case. The critical parameters are the maximum displacement $(\cos\alpha_{max}-\cos\beta)$ and the non-dimensional off-axis displacement y_2/λ (where λ is the acoustic wavelength and $k=2\pi/\lambda$). A numerical study has been undertaken to define the window function (2.9) in terms of these parameters using $\beta=0$, α_{max} = 10°, 30°, and y_2/λ= 0, 5, 10, 15. The results of this study are shown in terms of a distorted Barlett window function in Figs. 1 and 2, and these demonstrate that for small aperture angles of α_{max} = 10° there is no significant error for off-axis displacements of up to 15 wavelengths. However for the larger apertures there is a significant error for all displacements. From these results it would appear that a limit of ky_2 $(\cos\alpha_{max}-\cos\beta)<\pi/2$ ensures no significant off-axis displacement error.

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3. Application of the acoustic telescope to three-dimensional source distributions

The analysis used above may be extended to the Acoustic Telescope described by Billingsley and Kinns [2]. If it is assumed that the sources within the distribution are all incoherent and the distance from the jth microphone in the telescope to the focal point is L_j, then the evaluated source image is given by

$$Q_I(z) = \int_v Q(\bar{y}) \left|\sum_{j=1}^{N} e^{-ik(r_j-L_j)}\right|^2 d\bar{y} \qquad (3.1)$$

By using the far field approximation for r_j given above the summation in the integral (3.1) may be represented as a window function given by

$$W(y_2-z,y_1) = \left|\sum_{j=1}^{N} e^{-ik(y_1-z)\sin\alpha_j - iky_2\cos\alpha_j}\right|^2 \qquad (3.2)$$

This summation has been evaluated numerically and the images of an off-axis point source are illustrated for a typical case in Fig.3. These results show similar characteristics as those for Polar Correlation, the distortion increasing with off-axis displacement. Comparing equation (2.9) and equation (3.2) shows that in both cases the error is a function of $ky_2\ (\cos\alpha_{max}-1)$.

Since the telescope uses a smaller aperture to obtain the equivalent resolution, the distortion is correspondingly less for a given off-axis displacement.

4. The source image from an incoherent ring source

To model broadband noise emitted from a by-pass duct, consider a ring of equal amplitude, incoherent sources. The cross spectral density measured between points in the acoustic far field will be

$$G(\bar{x},\bar{x}_o) = \frac{Q}{2\pi R^2}\int_{-\pi}^{\pi} e^{ika\cos\theta(\cos\alpha-\cos\beta)}d\theta = \frac{Q}{R^2} J_o(ka(\cos\alpha-\cos\beta))$$

for sources at $y_1 = 0$ and strength $Q/2\pi a$ per unit length. The amplitude data is therefore weighted by the zero order Bessels function $J_0(\ )$ with argument $ka(\cos\alpha-\cos\beta)$. This will start to have a significant effect on the data when $ka(\cos\alpha-\cos\beta) > \pi/5$, and will result in a broadening of the source image. However the apparent distortion on the source image will not be important until $J_0(\ ) < 0.5$, which corresponds to $ka(\cos\alpha_{max}-\cos\beta) > \pi/2$, as before. This places a finite limit on the resolution which may be obtained of a ring source, since α_{max} is restricted by this criterion. For a given radius and a given value of α_{max}, an upper limiting frequency and maximum resolution may be defined which satisfy this criterion. These are given in Table 1 for ring source of 4' radius.

TABLE 1

α_{max}	Max.Freq.	max. resolution (ft)
10^o	4.6 kHz	1.4'
20^o	1165 Hz	2.82'
30^o	524 Hz	4.2'

When modelling a ring source for the curve fitting method given by Fisher [3], the effective mean source strength Q will be reduced by more than 5% when $ka(\cos\alpha-\cos\beta) > \pi/5$. For instance with a 4' radius ring source, this criterion is invalidated for a 30^o aperture at frequencies above 210 Hz, unless the appropriate corrections are made.

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References

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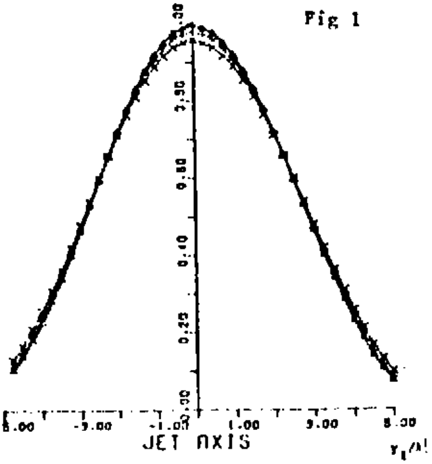
2. J. BILLINGSLEY and R. KINNS 1976 J. Sound Vib 48(4), 485-510. The Acoustic Telescope.
3. M.J.FISHER. Proc. I.O.A. Meeting on Noise Source Location, May 1981, "Jet Noise Source Breakdowns".

Fig 1



FIGURES: Source Images for Sources displaced by y_2/λ from axis.

Fig 1: Polar Correlation $\alpha_{max}=10^0$

Fig 2: Polar Correlation $\alpha_{max}=30^0$

Fig 3: Acoustic Telescope $\alpha_{max}=13.4^0$

——— $y_2/\lambda = 0$

—△— $y_2/\lambda = 5$

--+— $y_2/\lambda = 10$

--×— $y_2/\lambda = 15$

Fig 2

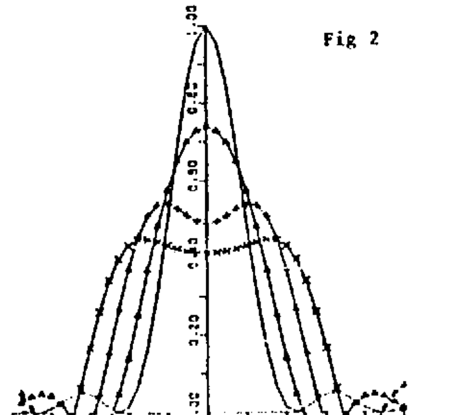


Fig 3