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NUMERICAL CODE FOR DESCRIBING ACOUSTIC WAVE PROPAGATION IN SOLIDS CONTAINING CRACKS OF KNOWN GEOMETRY

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Experimental techniques for nondestructive testing using nonlinear ultrasound stimulate the theoretical interest in wave propagation in materials containing crack-type defects (i.e. internal contacts). The presence of cracks invokes two major mechanisms of nonlinearity: an asymmetric reaction of the crack to normal compression/tension, and friction-induced hysteresis activated by shearing action. The generated nonlinear response of a sample highly depends on its geometry, for which a numerical description is most suitable. Our numerical tool consists of two components: a unit for solving the elasticity equations in the bulk volume and a unit that provides appropriate boundary conditions to be imposed at the internal boundaries in the material. The crack model has to provide load-displacement relationships for any value of the drive parameters. The traditional Coulomb friction law written for loads does not have this property, and therefore we use another concept that is, however, based on Coulomb’s friction law as well. The approach includes the account for roughness of the defect faces which results in the appearance of an additional contact regime of partial slip, when some parts of the contact zone slip and some do not. This situation is successfully dealt with by using the previously developed method of memory diagrams. In this method, the hysteretic load-displacement solution is constructed with the help of an internal system function (memory diagram) that contains all memory information. This displacement-driven solution can be easily extended to two other contact regimes (contact loss and total sliding) and is finally computed for any normal and tangential displacement histories. Memory diagrams have to be maintained at each discretization point on the crack surface and updated following the applied displacement fields. The load-displacement data provides input to the solid mechanics unit programmed in COMSOL®. We present an exemplar simulated configuration and discuss the results.

Keywords: contact acoustical nonlinearity, crack, friction, crack-wave interaction

1. Introduction

Planar defects in solids generate a strong nonlinear response once activated by the action of an elastic wave or vibrational stresses. Contact nonlinearity is usually much stronger than other types of acoustic nonlinearity such as material or geometric ones. This fact underlies at least two applications: nonlinear acoustic nondestructive testing (NDT) and creation of new nonlinear materials with unusual properties (metamaterials). In the former case, internal contacts act as secondary nonlinear sources in a material thus allowing one to detect their presence and than make a conclusion about the health or serviceability of the structure under study. In the former case, strong nonlinearity is combined with the effect of internal resonances in a sample eventually generating rich effects that can be potentially used for enhancing properties of existing linear metamaterials. These applications stimulate the interest to modelling for elastic and related phenomena in materials containing internal contacts. The development of a proper numerical tool offers a possibility to estimate sizes of defects by comparison measured and synthetic responses and therefore build up a complete NDT strategy. For the creation of metamaterials, numerical modelling represents a principal instrument that allows one to optimise the geometry of a prototype material before performing actual experiments.

Modelling for elastic behaviour of materials with internal contacts is possible via at least four existing approaches:

1. Homogenisation techniques [1] that aim to express the effective material properties through the parameters of inclusions or cracks. It is supposed that the material contains a large number of cracks with certain orientation distribution while the actual position of detects are not important.
2. Numerical methods that use phenomenological contact models such as frictionless contacts with different stiffness for normal compression and tension [2], Preisach model [3] for hysteresis, etc.
3. Methods of numerical contact mechanics [4] with a detailed meshing of the contact zone and arbitrary contact configurations in 3D, taking into account a variety of movement types, contact interaction laws including dynamic friction, etc.
4. Multi-scale numerical approach based on finite element or boundary element modelling in the presence of frictional contacts considered as mesoscopic elements whose reaction is obtained as a solution of an auxiliary contact problem.

The method proposed here belongs to the former group. The actual locations of defects are considered as known, in contrast to the homogenisation techniques that use statistical approaches. The advantage with respect to purely phenomenological methods is that our modelling is based on physics of normal and tangential contact interactions instead of artificial assumptions. Finally, complete contact mechanical modelling in which an arbitrary contact zone is mashed with finite elements is too detailed and cumbersome for acoustical applications when a signal can contain millions oscillations per second. To avoid that difficulty, our approach includes the consideration of a mesoscopic cell in which a problem of frictional shift between two bodies having certain contact topography is solved. The obtained relationship between vector contact load and displacement represents a boundary condition posed at the internal boundary corresponding to the contact.

2. Assumptions and simplifications

The developed code allows one to calculate all elastic fields including the nonlinear terms in a sample of known geometry with cracks of known configuration. The numerical tool consists of two components: a solid mechanics module programmed in 2D using an available finite element software (COMSOL) and an external contact model integrated into the solid mechanics unit. Our contact mechanical approach has the following essential features:

- the considered contact interactions model includes friction and is based on the Coulomb friction law;
- the internal contacts/cracks surfaces have a nontrivial topography (e.g. roughness);
- the normal load-displacement dependency for rough surfaces requires some information on roughness statistics; otherwise it can be measured directly for an engineered contact;
- the tangential interactions appear during shift; rolling and torsion as movement types are not considered;
- plasticity and adhesion are neglected;
- the model is quasi-static i.e. frictional dynamics effects are not ignored.

- the contact load-displacement solution is obtained via the Method of Memory Diagrams (MMD) [5] that uses the assumptions of the reduced elastic friction principle [6] also called Ciavarella-Jäger theorem [7].

The former two features are importantly linked to each other. The matter is that the Coulomb friction law for plane surfaces does not provide a load-displacement relationship in an explicit form. Indeed, Coulomb friction law just defines in which state - contact loss, stick, or sliding - the system evolves at some given stress values. The calculation of displacement is only possible via an implicit procedure that redistributes strains and stresses in the whole sample trying to match the sliding condition (shear stress equals normal stress times friction coefficient). In the next section we explain how the introduction of surface relief such as random roughness helps avoid this difficulty. Some comments on the MMD are also given.

3. Contact model

3.1 Mesoscopic scale

Our approach to calculation of the desired physics-based load-displacement relationships requires the introduction of an intermediate scale or, in other words, of a mesoscopic cell. On one hand, the size of a cell is considerably less than both the crack size and the wavelength so that the macroscopic elastic fields calculated by the solid mechanics unit are approximately uniform within each cell. On the other hand, the cell size is much greater than the scale of roughness or other microscopic features (e.g. in the case when the internal contacts are not defects but structure elements having certain profile). The concept of the mesoscopic cell is illustrated in Fig. 1. Its lower part shows the geometry the auxiliary problem to be solved for determining the load-displacement relationship. With the notations introduced in Fig. 1, the link between loads (contact forces per unit nominal contact surface) N and T and displacements a and b should be determined.

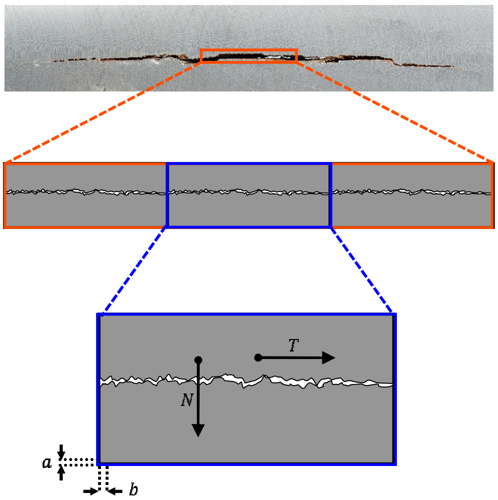

Figure 1: Auxiliary contact problem at the mesoscopic scale: a link between loads (contact forces per unit nominal contact surface) N and T, and displacements a and b should be determined.

3.2 Partial slip regime

The presence of surface relief (e.g. roughness) means that the contact system can evolve in one of three regimes: contact loss (N=T=0), partial slip ($|T|<\mu N$), and total sliding ($|T|=\mu N$). In the regime of partial slip, some contact points at the opposite faces of the crack slide and some do not. This regime does not exist in the case of perfectly smooth surfaces in which the condition $|T|<\mu N$ actually corresponds to the state of stick in accordance to the Coulomb friction law. The partial slip regime was firstly discovered for contact of two spheres [8] (Hertz-Minldin or Cattaneo-Mindlin system) and is also observed for other non-plane contact geometries. In this regime, according to the reduced elastic friction principle [6] (called also Ciavarella-Jäger theorem [7]), the tangential reaction of a contact system loaded by constant forces or displacements can be expressed through its normal reaction curve $N(a)$ in the following way:

$$\begin{cases} b = \theta\mu\left(a-\alpha\right) \\ T = \mu\left(N\left(a\right)-N\left(a=\alpha\right)\right) \end{cases}. \tag{1}$$

This solution is valid [6] for axisymmetric profiles with

$$\theta = \frac{2-\nu}{2\left(1-\nu\right)}, \tag{2}$$

as well as for a variety of other geometries [7] including rough surfaces [9]. For non-axisymmetric geometries, values of θ can differ from Eq. (2). It is straightforward to rewrite Eq. (1) as

$$\begin{cases} b = \theta\mu\int_0^a D\left(\eta\right)d\eta \\ T = \mu\int_0^a D\left(\eta\right)\left.\frac{dN}{da}\right|_{a=\eta} d\eta \end{cases}, \tag{3}$$

where function $D(\eta)$ is introduced as shown in Fig. 2(b).

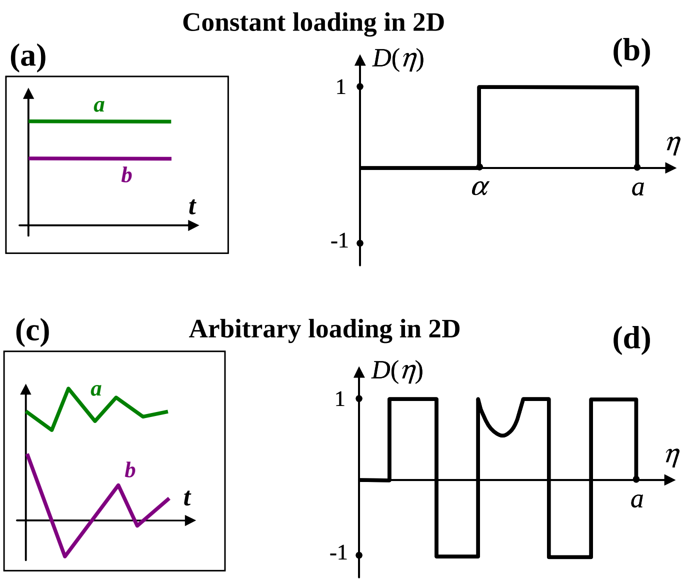

Figure 2: Illustration for loading histories ((a) and (c)) and memory diagrams ((b) and (d)).

The MMD [5] is a method that allows one to calculate the solution in the form of Eq. (3) not only for constant loading (Fig. 2.(a)) but also for an arbitrary loading history (Fig. 2 (c)). In the general case of arbitrary loading, function $D(\eta)$ called memory function or memory diagram has a more complex form (Fig. 2(d)) in comparison to the simple rectangular one depicted in Fig. 2(b). The memory diagram evolves in accordance to a number of prescribed rules following from the Coulomb friction law written for microscopic contact stress and displacement fields and can consist of straight horizontal and curvilinear sections [5]. In particular, due to the Coulomb friction law the absolute value $\left|D(\eta)\right|$ can not exceed 1, etc.

The MMD requires the knowledge of the normal contact reaction N=$N(a)$ for the system under study. There exist theoretical [5] and experimental [10] arguments supporting the quadratic approximation for $N(a)$,

$$N(a) = \frac{1}{4} C^2 a^2, \tag{4}$$

with $C \approx 6 \cdot 10^{10}\, Pa^{1/2} m^{-1}$ obtained by an indirect ultrasound-based estimation [10] for two aluminium blocks with rough surfaces. In our modelling, we use this value as an example. The approximation Eq. (4) only works for small a which corresponds to weak acoustic strains.

The MMD offers a possibility of calculating the tangential reaction curve $T = T\left[a(t), b(t)\right] \equiv MMD(b)$ as a function of input parameters histories (in our case, displacement histories $a(t)$ and $b(t)$). This solution is only valid in the partial slip regime defined by the condition $\left|T\right| < \mu N$ or $\left|b\right| < \theta\mu a$. Indeed, the second Eq. (3) reflects the force balance principle which means that the external tangential force is equilibrated by the shear stress distribution in the contact zone. When the external tangential force exceeds the limit μN, the force balance considerations do not apply anymore. For the traditional representation of a crack as a slit with perfectly plane surfaces this becomes a serious difficulty since the tangential displacement b remains undetermined in the framework of the Coulomb friction law in the case of total sliding. The difficulty can be overcame by accepting an additional assumption [11] linking b with known parameters or by applying an iterative procedure that recalculates all stress and strain fields for a guessed b in order to satisfy the condition $\left|T\right| = \mu N$. The latter can be problematic as such a situation can occur in many mesoscopic cells at the same time. Matching tangential displacements at n points to simultaneously satisfy n conditions represent a cumbersome task. Modern generic finite element modelling software such as COMSOL does not provide this possibility [12]. However, following our approach, this difficulty can be avoided.

3.3 Load-displacement relationship

In order to obtain the solution to the mesoscopic contact problem and derive the load-displacement relationship in all three regimes i.e. for an arbitrary combination of displacements together with their histories, the following technique is suitable. The tangential displacement should be presented as a sum of two components,

$$b = b_0 + \tilde{b}, \tag{5}$$

where b_0 corresponds to the displacement achieved in the total sliding regime, and $\tilde{b}$ is a component that reflects partial slip and the ability of asperities to recede under tangential load. Equation (5) allows one to write down the solutions for each contact regime:

- Contact loss that occurs when $a \le 0$. Since no contact interaction is present, N=T=0, and asperities remain unstrained i.e. $\tilde{b} = 0$. Correspondingly, $b_0 = b$ in this case. The memory function equals 0 everywhere for $0 \le \eta \le N$ (Fig. 3(a)).
- Partial slip that takes place when $a > 0$, $\left|\tilde{b}\right| < \theta\mu a$. In accordance to the MMD applicable in this situation, $T = MMD(\tilde{b})$ while the total sliding contribution b_0 remains unchanged. The memory diagram has a certain form depending on loading history (Fig. 3(b)).

- Total sliding that happens when a>0 and the memory function equals +1 or -1 on the whole interval $0 \le \eta \le N$ (Fig. 3(c)). According to Eq. (3), $\tilde{b} = \pm\theta\mu a$ where the sign depends on the direction of sliding. The asperities recede under tangential loading, so that the actual full sliding contribution should account for this effect: $b_0 = b - \tilde{b}$.

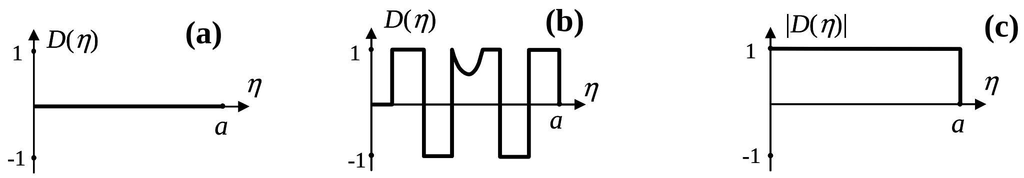

Figure 3: Memory diagrams corresponding to contact loss (a), partial slip (b), and total sliding (c).

Figure 4 illustrates an example of the tangential load-displacement curve (b) calculated for displacements histories (a) containing a dozen of oscillations for imitating a fragment of a typical acoustic signal.

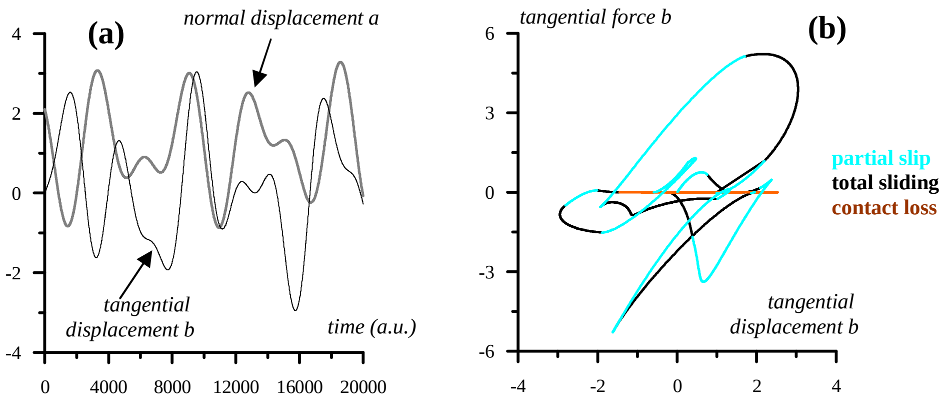

Figure 4: Tangential load-displacement relationship (b) calculated using our contact model for an exemplar displacements history (a).

4. Finite element modelling results

The presented contact model has been used as an external boundary condition that is to be set at the boundary corresponding to the internal contact. Such a possibility is offered by the "thin elastic layer" feature available in the solid mechanics module of COMSOL Multiphysics. Figure 5 illustrates a simulation example for a test sample (aluminium block) with an inclined crack of known geometry. The geometry has been automatically meshed with a variable mesh size which drastically decreases in the vicinity of the crack. The normal reaction curve was taken from literature [10] on ultrasonic assessment of properties of contact between two aluminium samples. Finally, the two pictures at the right of Fig. 5 present snapshots of the simulated wave propagation pattern at two different moments of time.

Nonlinear analysis [13] of the wave propagation simulation results provides nonlinear signatures of damage and finally an opportunity of using the method in nonlinear nondestructive testing.

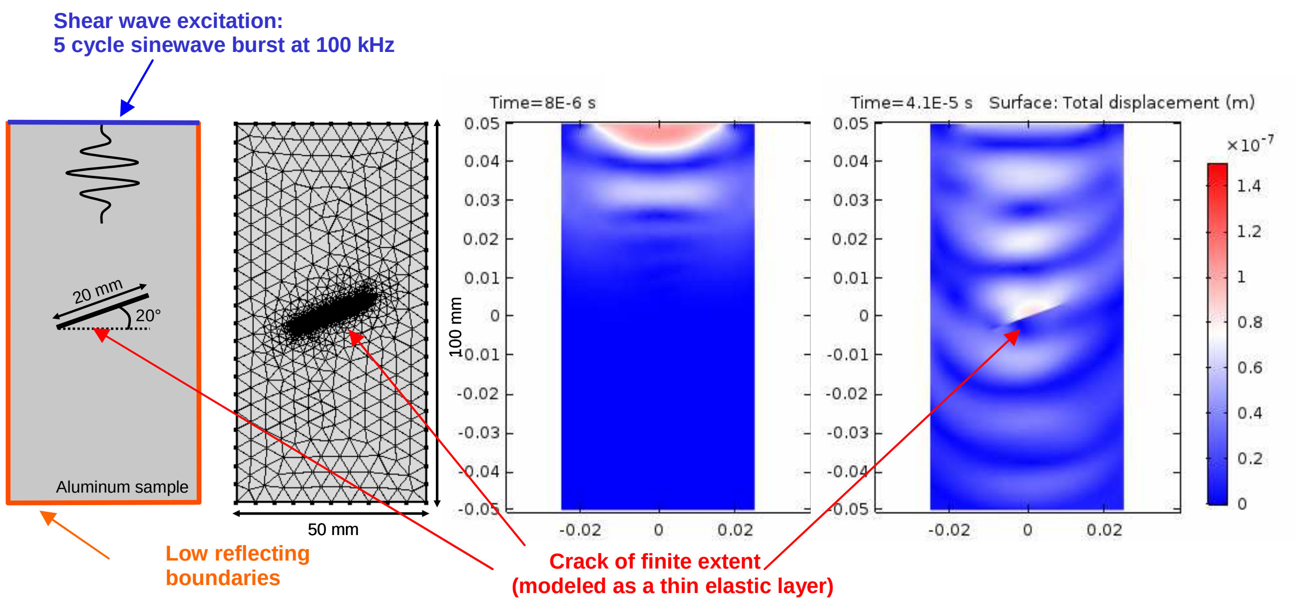

Figure 5: Simulation for a wave propagation in an aluminium sample containing an inclined crack. The considered geometry (original and meshed) is shown as well as two snapshots of the simulated wave propagation pattern.

5. Conclusions

This paper contains a description of a numerical tool created for modelling wave propagation and vibrations in materials containing internal frictional contacts (cracks, delaminations, etc). The theoretical analysis includes elaboration of a contact model that takes into account non-trivial (non-plane) geometry of internal contacts, in particular, rough surfaces. For non-plane contact topographies, there exists a contact regime called partial slip when some points of the contact area slide and some do not. The partial slip regime is the case when our previously developed method of memory diagrams applies. The method allows one to get a tangential load-displacement relationship for any loading history and represents a modern extension of the Hertz-Mindlin solution [8] applicable to a wide range of contact geometries. The hysteretic tangential reaction is obtained in an automated way using and internal memory function (memory diagram) that evolves according to the prescribed rules. In two other regimes - contact loss and total sliding - the exact solution can also be obtained.

The numerical tool can be applied as modelling support for modern nonlinear acoustic NDT methods. Upon the experimental validation, the developed tool is to be used for comparison of data and modelling results and for estimation of geometrical parameters of damage. Its application actually completes an NDT algorithm that starts with nonlinear acoustic measurements and finally results in the estimation of the damage "degree of gravity" and in possible predictions for the lifetime of the sample. Generally, numerical modelling considerably increases the visibility and "transparency" of all physical processes used for damage detection.

The presented method is capable of accounting not only for rough surfaces but other (regular) contact topographies. Combining nonlinear wave propagation with the use of internal resonances in periodic structures offers an opportunity to use the method for designing nonlinear acoustic metamaterials based on contact nonlinearity.

6. Acknowledgments

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