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ACTIVE CONTROL OF STRUCTURE-BORNE NOISE

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1. INTRODUCTION

The successful application of active control requires an understanding of its fundamental physical limitations, and an understanding of the control strategies which allow its implementation. In this paper I will briefly review the acoustic limitations of global and local strategies for the active control of sound in enclosures using loudspeakers, and discuss various methods of control, particularly one developed at the ISVR over the past 8 years using adaptive digital filters. The use of this technique in controlling engine noise and road noise in cars will then be described. Much of this work has been carried out in collaboration with Dr. P.A. Nelson, and is the subject of a recent book (Nelson and Elliott, 1992). The physical basis for active vibration control, and the control strategies used in this field, are covered in another text under preparation, in collaboration with Professor C.R. Fuller at VPI (Fuller, Elliott and Nelson, 1993). In general, we can identify three important elements in the design of any active control system, as shown in Figure 1. The control *strategy* used may be either feedforward or feedback, the *disturbance* being controlled may be either deterministic (as is nearly the case with engine noise) or random (as is nearly the case with road noise), and the physical *objectives* may be global or local control of the sound field, or, for example, the blocking of a structural vibration path, which then results in acoustic control.

This latter strategy may be applicable when the primary source of noise is transmitted into the enclosure via a limited number of structural paths. The complexity of the control strategy will depend on the nature of the structural paths and the nature and frequency of the disturbance. In an automotive engine mount at low frequencies only a single direction of motion often has to be controlled. To control the transmission of gear-meshing noise through helicopter struts at higher frequencies, three wave types appear important. These two examples of the active control of structure-borne noise are briefly described at the end of the paper and some preliminary conclusions are drawn about the potential for this strategy of active control.

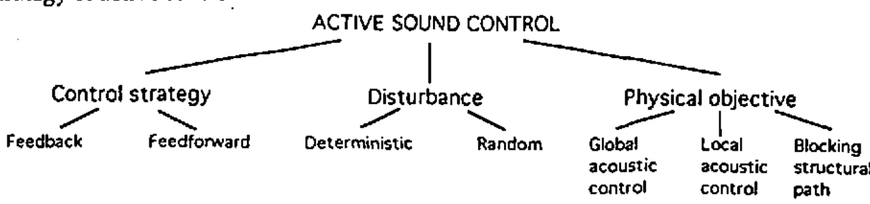

Figure 1. The three important elements which need to be considered in the design of an active sound control system. The physical objectives are examples discussed in the paper.

2. ACOUSTIC CONTROL IN ENCLOSURES

2.1 Global Control

The global control of sound in an enclosure can be theoretically investigated by considering the minimisation of the total acoustic potential energy in the enclosure (Nelson and Elliott, 1992). A computer simulation of such a strategy for a pure tone sound field in an enclosure about the size of a car interior (1.9 m × 1.1 m × 1.0 m), and with a similar acoustic damping is shown in Figure 2, for a range of excitation frequencies. The primary field is generated by a monopole acoustic source in one corner of the enclosure. Either a single acoustic secondary source in the opposite corner or seven acoustic secondary sources in all the other corners are then used to minimise the acoustic energy. With one secondary source, significant reductions in the energy are observed below about 100 Hz, and with seven secondary sources the upper frequency limit is extended to about 250 Hz. The number of loudspeakers required to achieve global acoustic control depends upon the number of significantly excited acoustic modes at the excitation frequency. This can be quantified as being proportional to the average number of acoustic modes within the 3 dB bandwidth of any one mode, the acoustic modal overlap, which is plotted in Figure 3 for the enclosure used in the simulations above. It can be seen that the acoustic modal overlap is less than one below about 100 Hz, but increases rapidly above this frequency, in approximate proportion to the cube of the excitation frequency. A fundamental high frequency limit is thus imposed on global control in an enclosure with a reasonable number of loudspeakers, which occurs at about 250 Hz in an enclosure the size of a car.

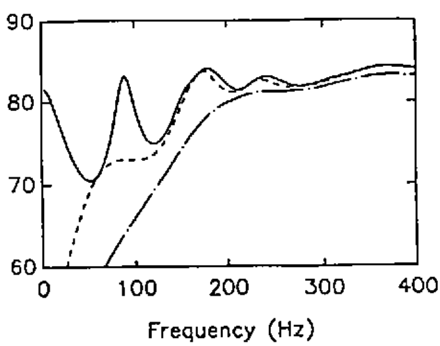

Figure 2 Total acoustic potential energy E_p in a 1.9 m × 1.1 m × 1.0 m rectangular enclosure excited by a pure tone primary source in one corner only (-), and with the addition of one (...) or seven (- - -) optimally djusted remote secondary sources.

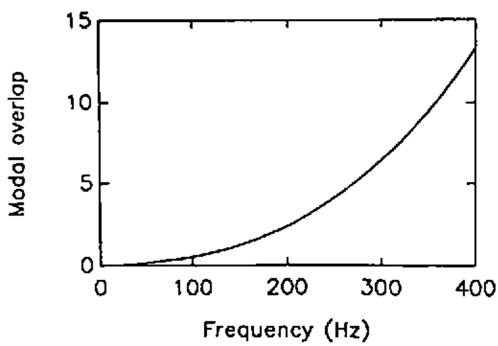

Figure 3 The number of acoustic modes with natural frequencies within the 3 dB bandwidth of any one acoustic mode, the acoustic modal overlap, for the enclosure used to produce the results in Figure 1.

The optimisation of the complex secondary source strengths to minimise the total acoustic potential energy in the enclosure is a quadratic optimisation problem, which has a unique minimum and can be solved analytically in model problems and iteratively, using gradient descent methods, in practical control systems. The optimisation of the *positions* of the secondary sources within the enclosure, however, is a more complicated problem which, in general, does not have a unique solution. For enclosures in which only a few well-defined acoustic modes are excited, secondary source positions can often be identified which efficiently couple into the excited modes, and thus achieve good control. In the more general case, where many ill-defined acoustic modes are potentially excited, such intuitive methods of optimal source placement are more difficult to apply. In this case more sophisticated numerical techniques such as Genetic Algorithms, have been investigated, which efficiently search through a large number of potential secondary source locations for those which give the best global reductions (Baek and Elliott, 1993).

2.2 Local Control

The active control of sound in the local region around a listener's head first appears to have been investigated by Olson and May (1953), and an illustration from this paper is shown in Figure 4. The control objective here is to reduce the sound pressure at the position of a single monitoring microphone. Provided the microphone is close to the secondary loudspeaker, the loudspeaker will not have to drive too hard to achieve this objective, and the sound pressure will not be significantly increased at locations remote from this local active control system (Joseph *et al*, 1993). In the region close to the secondary loudspeaker, a spatial interference pattern is set up between the original, primary, sound field and that due to the secondary source. This pattern defines the fundamental acoustic performance of the local control system, and can be analytically and numerically investigated by calculating the pressure distribution due to a pure tone primary sound field and a loudspeaker driven at the same frequency to cancel the pressure at the location of the monitoring microphone (Joseph *et al*, 1993; David and Elliott, 1993). The region over which the primary pressure field is reduced by more than 10 dB can be termed the zone of quiet. Figure 5 shows the axial size of the zone of quiet calculated for a piston secondary source of diameter 0.1 m, for various excitation frequencies and positions of the monitoring microphone, r_0, in a uniform primary field. With the microphone close to the loudspeaker, the zone of quiet is small but almost independent of frequency up to 1 kHz. With the microphone further away, the zone is larger at low frequencies, but is reduced at higher frequencies, never becoming larger than one tenth of an acoustic wavelength, which can be shown theoretically to be the limiting size of the zone of quiet for a remote microphone (Elliott *et al*, 1988a). The shape of the zone of quiet away from the axis of the secondary source has been investigated by David and Elliott (1993) who found that for a uniform primary field the zones of quiet very nearly formed complete shells round the secondary source even at high frequencies. With the more realistic assumption of a diffuse primary field, however, the zone of quiet still forms a shell at low excitation frequency, but becomes approximately spherical at higher frequencies, with a diameter of $\lambda/10$, as shown in Figure 6.

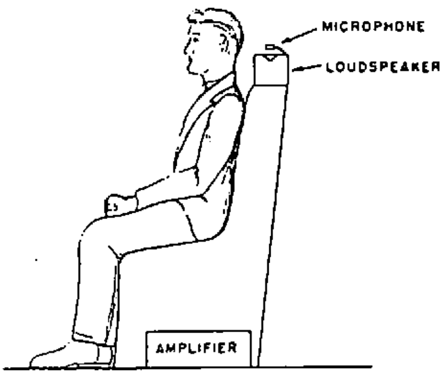

Figure 4. The local control system of Olson and May.

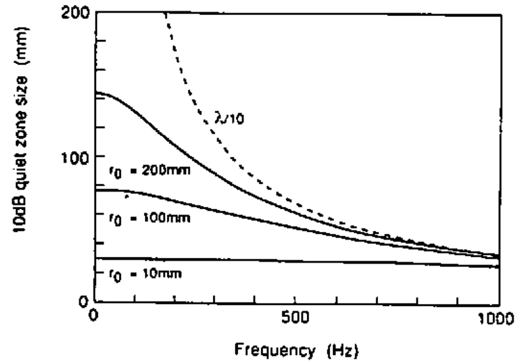

Figure 5. The axial extent of the zone of quiet within which the pressure has been reduced by 10 dB with the secondary piston source .

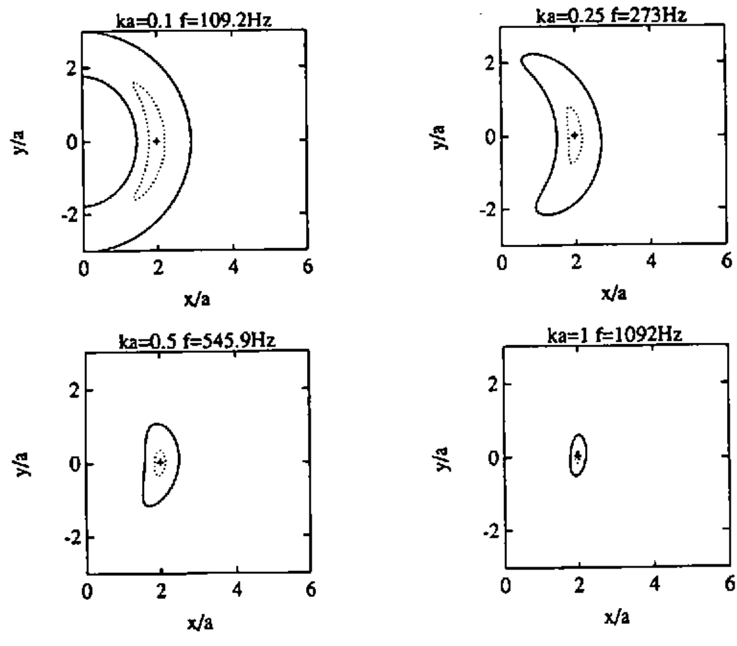

Figure 6. The residual pressure field, averaged over 20 samples, caused by the cancellation of a pure tone diffuse primary field, at the microphone position $x = 2a$, $y = 0$. The piston secondary source, of radius a, is at the origin of the coordinate system in the y,z plane. The solid line represents average reductions in the primary field of 10 dB, the dashed line reductions of 20 dB. The four graphs represent various excitation frequencies for a piston radius of 0.1 m.

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3. CONTROL STRATEGIES

3.1 Feedback Control

The control strategy originally used by Olson and May (Figure 4) was of feeding the microphone signal back to the loudspeaker via an inverting amplifier. A more idealised physical illustration of such a system and its equivalent electrical block diagram is shown in Figure 7(a) and (b).

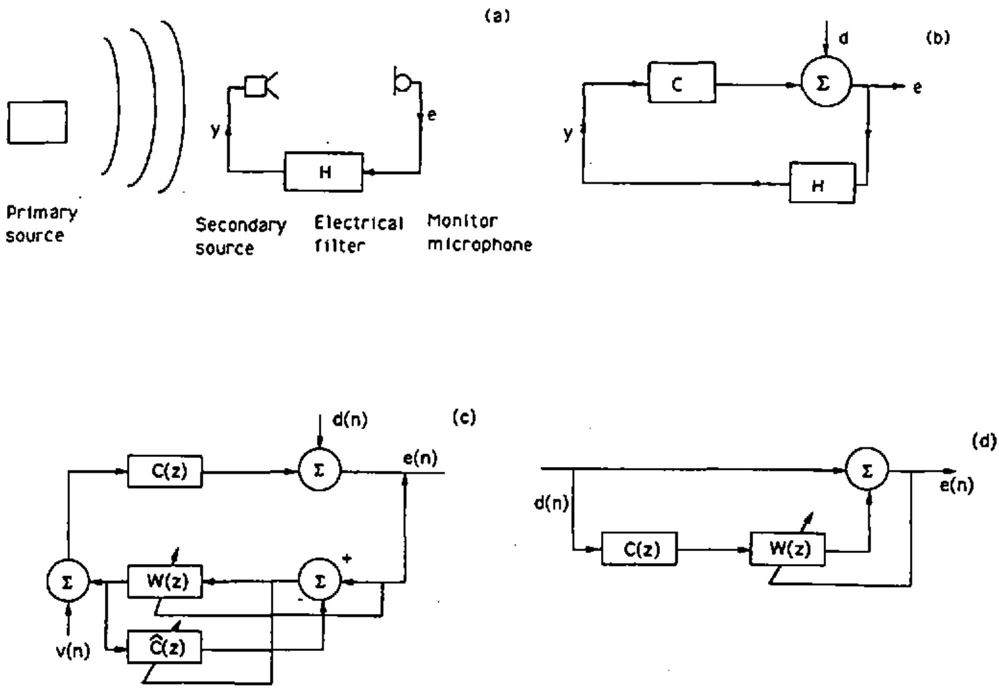

Figure 7 An active control system using feedback control (a), and its equivalent electrical block diagram (b). The block diagram of a possible adaptive controller is shown in (c), together with the effective block diagram if $\hat{C}(z) = C(z)$.

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In this figure, e represents the signal derived from the microphone, due to the combined effect of the primary signal, denoted d, and the feedback loop. The electrical transfer function of the feedback loop is denoted H, which in the system described by Olson and May is a simple gain and inversion. The electrical transfer function from secondary loudspeaker input to microphone output is called the secondary path and is denoted C. This contains the electroacoustic response of the loudspeaker, the acoustic characteristics of the path between loudspeaker and microphone, and the microphone's electroacoustic response. The ratio of the microphone signal's spectrum after control to that before control is thus

$$\frac{E(j\omega)}{D(j\omega)} = \frac{1}{1 - H(j\omega)C(j\omega)} \tag{3.1}$$

If the frequency response of the secondary path, $C(j\omega)$, were relatively flat and free from phase shift, then the gain of an inverting amplifier in the feedback path, $H(j\omega) = -A$, could be increased without limit, causing the overall transfer function of the feedback loop to become arbitrarily small. This is analogous to the virtual earth concept used in operational amplifiers and such a control system is sometimes referred to as an "acoustic virtual earth". The effect of the feedback loop forcing e to be small compared to d, will be to cancel the acoustic pressure at the monitor microphone, as required for active control.

Unfortunately, the frequency response of the secondary path, $C(j\omega)$, can never be made perfectly flat and free of phase shift. The electroacoustic response of a moving coil loudspeaker, in particular, induces considerable phase shift near its mechanical resonance frequency. The acoustic path from loudspeaker to microphone will also inevitably involve some delay due to the acoustic propagation time, and this will also introduce an increasing phase shift in the secondary path with increasing frequency. As the phase shift in the feedback loop approaches 180°, the negative feedback described above becomes positive feedback and the control system can become unstable. Fortunately, as the frequency rises and the phase lag in the secondary path increases, its gain also tends to decrease. It is thus still possible to use an inverting amplifier in the electrical path provided its gain is not large enough to make the net loop gain greater than unity when the total phase shift becomes 180°. This stability criterion can be more formally described using the well-known Nyquist criterion. At lower frequencies the feedback will be negative and the loop gain may still be considerably greater than unity, thus ensuring that some attenuation of the signal from the microphone is produced. The stability problem is compounded in this application, however, because the frequency response of the secondary path, $C(j\omega)$, is significantly altered as the listener changes the position of his head. It is possible to introduce compensating filters into the electrical path to correct for the phase shift in the secondary path to some extent, and increase the bandwidth over which active control is possible. First and second order lead-lag networks, for example, have been successfully used in active headsets by Wheeler (1986) and Carme (1987). It is not, however, possible to design a compensation filter which will minimise the mean square value of the error signal, e, under all circumstances. This is because the spectrum of the primary noise disturbance, d, can change considerably over time, and a compensation filter designed to produce good attenuation in one frequency band will not necessarily produce as good an attenuation in another frequency band. For this reason some authors have suggested that different compensation filters should be used in feedback control systems designed for different noise environments (Veight, 1988).

Apart from simple compensation circuits, the design of feedback controllers can be performed using state space models (see, for example, Wellstead and Zarrop, 1991). It is also possible to formulate the problem from a signal processing viewpoint, assuming that the controller is digital, i.e., it operates on sampled data. The general block diagram can still be represented by Figure 7(b), except that the sampled transfer function of the system under control, C(z), now contains the responses of the data converters and any anti-aliasing or reconstruction filters used. In general, C(z) will not be minimum phase and may contain some bulk delay. We now assume that the controller is implemented as the parallel combination of a "feedback" path, W(z), and a "feedforward" path, $\hat{C}(z)$, as shown in Figure 7(c). The transfer function of such a controller is thus $H(z) = W(z)/(1 + W(z)\hat{C}(z))$. Such a controller arrangement is similar to the echo cancellation architecture used in telecommunications, and the feedback cancellation architecture for feedforward controllers; see, for example, Nelson and Elliott (1991). Its use in feedback control has been suggested by Forsythe *et al* (1991). With such a controller the response of the complete feedback control system becomes $E(z)/D(z) = (1 + W(z)\hat{C}(z))/(1 - H(z)(C(z) - \hat{C}(z)))$.

Clearly, if the "feedforward" part of the controller is adapted to have the same transfer function as the system under control (the "plant"), so that $\hat{C}(z) = C(z)$, then the error signal becomes $E(z) = (1 + W(z)C(z))D(z)$, as shown in Figure 7(d). The feedback control problem has thus been transformed into an entirely feedforward problem. In the special case of the plant response C(z) corresponding to a pure delay, Figure 7(d) is that of an adaptive line enhancer (Widrow and Stearns, 1985) and to minimise the mean square value of the error, W(z) acts as an optimal *predictor* for the filtered disturbance signal. This action is similar to the prediction achieved in minimum variance controllers (Wellstead and Zarrop, 1991). In practice the two parts of the controller W(z) and $\hat{C}(z)$ could be implemented by adaptive digital filters. For example, $\hat{C}(z)$ could initially be adapted to model C(z) using the LMS algorithm with the identification signal v(n), and W(z) could then be adapted using the filtered-x LMS algorithm using a copy of $\hat{C}(z)$ to generate the filtered reference signal. It may also be possible to simultaneously adapt the two parts of the filter using the LMS algorithm, with or without the identification noise, v(n). Alternatively, it may be possible to use the RLMS algorithm to adjust both filters in a similar way to that described by Billout *et al* (1991). The feedback control architecture illustrated in Figure 7(c) can be readily extended to plants with multiple inputs and multiple outputs. The practicality of such an architecture for a feedback controller remains to be investigated. The consequences of the model do, however, provide an interesting signal proessing insight into the behaviour of a feedback controller.

3.2 Feedforward Control

Feedforward methods of active noise control were originally developed for broadband noise in ducts (Lueg, 1936) and for pure tone noise generated by transformers (Conover, 1956). A generic block diagram for such systems is shown in Figure 8a. The difference between this and the feedback approach is that a separate *reference signal*, x, is now used to drive the secondary source, via the electrical controller, W. This reference signal must be well correlated with the signal from the primary source. In systems for the control of broadband random noise, the reference signal gives advance information about the primary noise before it reaches the monitor microphone, which enables a causal controller to effect cancellation. In systems for the control of noise with a deterministic waveform, such as harmonic tones, this "advanced" information has little meaning since the controller only has to implement the appropriate gain and phase shift characteristics at each frequency.

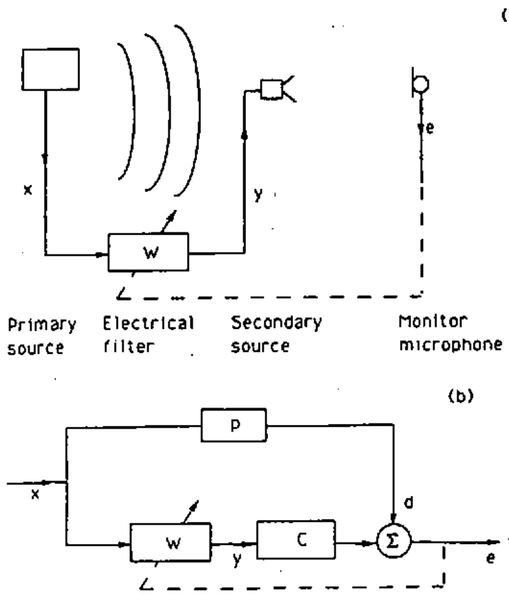

Figure 8 An active control system using adaptive feedforward control (a), and its equivalent electrical block diagram (b).

The electrical block diagram of a purely feedforward controller is shown in Figure 8b. We denote the frequency response of the secondary path from secondary source input to monitor microphone output as $C(j\omega)$, the frequency response of the feedforward controller as $W(j\omega)$ and the frequency response of the primary path from reference signal to monitor microphone as $P(j\omega)$, thus

$$\frac{E(j\omega)}{D(j\omega)} = 1 + W(j\omega)C(j\omega)/P(j\omega) \qquad (3.2)$$

Because the spectrum of the error signal, $E(j\omega)$, is linearly related to the response of the electrical controller, $W(j\omega)$, this can, in principle, be adjusted at each frequency to model the response of the primary path, $P(j\omega)$, and invert the response of the secondary path, $C(j\omega)$, and thus give complete cancellation of the error spectrum. The frequency response required of the controller in this idealised case is thus $W(j\omega) = -P(j\omega)/C(j\omega)$, and for pure tone disturbances this equation only has to be true at a single frequency for active control to be accurately implemented. In the broadband case the problem becomes one of practical filter design, so that the coefficients of the electrical filter used in the controller are designed to give a frequency response which best approximates that required. Another complication in the broadband case is that often measurement noise is present in the reference signal due, for example, to the air flow over the microphone in a duct. The frequency response of the controller which best minimises the power spectral density of the error signal in this case is a compromise between cancellation of the primary noise signal and amplification of the measurement noise through the controller (Roure, 1985). A feedforward control system also has to be very accurately adjusted; to within ±0.6 dB in amplitude and ±5° of phase for a 20 dB reduction of a pure tone primary signal. Because the properties of the primary noise and, to a lesser extent, the characteristics of the secondary path will probably change with time in a practical system, the controller in active feedforward systems is often made adaptive in order to maintain this delicate balance. The most convenient method of implementing an adaptive filter is using digital techniques.

3.3 Adaptive Filters for Feedforward Control

In Figure 9, the differences in the block diagram of a conventional, electrical noise canceller (Figure 9a) and that for a single channel active noise control system (Figure 9b) are illustrated. The well-known LMS algorithm is widely used to adapt the coefficients of an FIR digital filter for electrical noise cancellation (Widrow and Stearns, 1985), and this can be written as

$$w(n+1) = w(n) - \alpha\, x(n)e(n) \qquad (3.3)$$

in which $w(n)$ is the vector of filter coefficients at the n-th sample time, $x(n)$ is the vector of past reference signals, and $e(n)$ is the instantaneous error signal and α is a convergence coefficient. If this algorithm is used without modification in an active control application, however (Figure 9b), the result is likely to be an unstable system. This is because the signal from the adaptive filter, W, suffers a phase shift in passing through the secondary path, C. The instantaneous measurement of the gradient of the mean square error with respect to the coefficient vector, $x(n)e(n)$ in equation (3.3), is thus no longer an unbiased estimate of the true gradient. The solution to this problem, first proposed by Morgan in 1980 and independently by Widrow *et al*, and Burgess in 1981, is to introduce a similar phase shift into the reference signal path, before the gradient estimate is formed. This is achieved by using an electrical filter ($\hat{C}$), which models the response of the secondary path (C), to

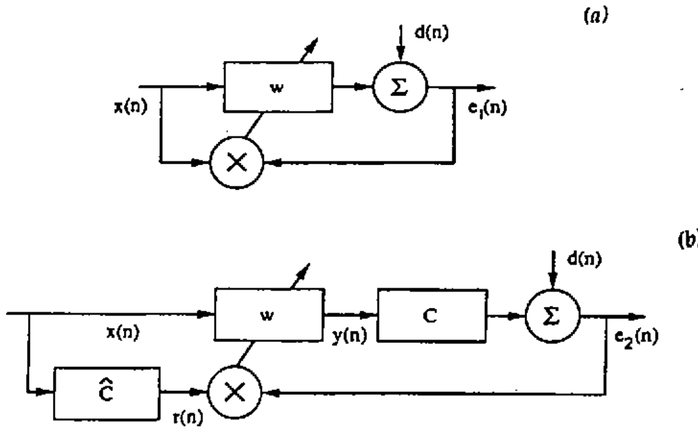

Figure 9 Block diagrams of (a) an electrical noise canceller adapted using the LMS algorithm and (b), a single channel active control system adapted using the filtered-x LMS algorithm.

generate a filtered reference signal, r(n), which is then multiplied by the error to form the gradient estimate. The resulting update equation is called the "filtered-x LMS" algorithm, which may be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \alpha\, \mathbf{r}(n)e(n) \tag{3.4}$$

where $\mathbf{r}(n)$ is now the vector of past filtered reference signals. The maximum convergence coefficient which can be used in the filtered-x LMS algorithm has been found empirically (Elliott *et al*, 1989) to be approximately

$$\alpha_{max} \approx 1/[\overline{r^2}(I + \delta)] \tag{3.5}$$

where $\overline{r^2}$ is the mean square value of the filtered reference signal, I is the number of filter coefficients, and δ is the overall delay in the secondary path (in samples). This compares with the limit on the convergence coefficient for the normal LMS algorithm, which is approximately (Widrow and Stearns, 1985)

$$\alpha_{max} \approx 1/[\overline{x^2}\, I]. \tag{3.6}$$

The delay in the secondary path, which usually forms the most significant part of the dynamic response of this system, thus reduces the maximum convergence coefficient in the filtered-x algorithm, but only to the extent that the speed of response is comparable with the delay, δ. In actively controlling the sound in an enclosure with dimensions of a few metres, this delay is typically of the order of 10 ms and the initial convergence speed of the algorithm is fairly rapid. It is well known that the LMS algorithms can exhibit other, slower, modes of convergence whose time constants are determined by the eigenvalues of the autocorrelation matrix $E[x(n)x^T(n)]$. Similar slow modes are observed for the filtered-x LMS algorithm due to the eigenvalue spread of the autocorrelation matrix of the *filtered* reference signal, $E[r(n)r^T(n)]$ (Elliott *et al*, 1989).

The stability of the filtered-x LMS algorithm is also affected by the accuracy of the filter ($\hat{C}$) modelling the true secondary path (C). The estimate of the gradient vector does not have to be exact, however, and the filtered-x algorithm is surprisingly robust to errors in $\hat{C}$. Morgan (1980) has shown that for pure tone reference signals the phase of $\hat{C}$ at the excitation frequency only has to be within ±90° of that of the true error path, C, for the system to converge slowly. Numerical results (Boucher *et al*, 1991) also suggest that phase errors of 40° do not significantly affect the maximum convergence speed of the algorithm. Similarly, Widrow and Stearns (1985, p.292) remark, in the context of using the filtered-x LMS algorithm for adaptive inverse control, that this model "need not be very precise", and that its most important attribute is that "its impulse response has at least as great a transport delay" as the secondary path.

The implementation of the filtered-x LMS algorithm is somewhat more complicated than that of the normal LMS, because of the need to generate the filtered reference signal. The filter used to model the secondary path is often created in an identification phase, prior to control, during which a training signal is fed to the secondary source. This filter could be another FIR filter which is adjusted, during identification, using a separate LMS algorithm. Because the response of this filter does not have to exactly match that of the secondary path, it is often only necessary to use relatively few coefficients in this filter.

When the single channel active control systems described above is used to control a deterministic primary waveform, the signal from the single monitor microphone can be driven to zero. Such a controller could, for example, be used to produce a zone of quiet around a monitor microphone using a closely spaced secondary source, as discussed in Section 2.2. If, however, such a system were used in an attempt to achieve global active control in an enclosure, by placing the microphone some distance away from the secondary source, the consequence may well not be that desired. This would be the case, for example, if the secondary source happened to be in a position in the enclosure in which, at the excitation frequency, it was only weakly coupled, acoustically, to the monitor microphone.

The secondary loudspeaker would thus have to drive very hard to cancel the primary field at the microphone, and although a small zone of quiet would be generated at this point, the sound pressure at other points in the enclosure would tend to rise significantly.

Clearly what is required is a practical measurement which gives a better estimate of the acoustic quantity which it is desired to control with a global system: the total acoustic potential energy. This quantity is proportional to the volume integral of the mean square acoustic pressure throughout the enclosure. A single pressure measurement is obviously a poor estimate of this volume integral if the driving frequency is high enough for a number of acoustic modes to be significantly excited. A better estimate of the volume integral would be the sum of the mean square pressures at a number of locations throughout the enclosure volume. This practical requirement motivates the development of a generalisation of the filtered-x LMS algorithm in which the filter coefficients are adjusted to minimise the sum of the mean square values of multiple error signals. In fact, further generalisation is then possible to include the practically important cases of multiple secondary sources, and its use with multiple reference signals. Figure 10 shows the block diagram of an active control system with K reference signals, M secondary sources and L monitor microphones.

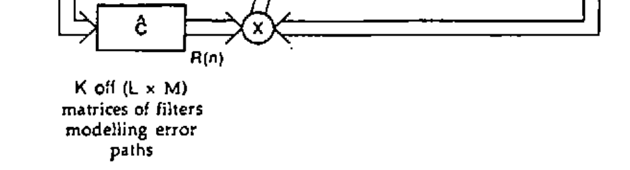

Figure 10 Electrical block diagram of a general multichannel feedforward control system, adapted using the multiple error LMS algorithm.

There are now M × L different acoustic paths between each secondary source and each monitor microphone, all of which have to be modelled and used K times to generate the array of filtered reference signals required for the adaptive algorithm. This algorithm adjusts each of the coefficients of each of the K × M adaptive filters in the controller, which drive every secondary source from every reference signal. The resulting "Multiple Error LMS" algorithm (Elliott and Nelson, 1985) is described, in its simplest form, by the equation

$$w(n+1) = w(n) - \alpha \, R(n) e(n) \tag{3.7}$$

in which $w(n)$ is the vector of KMI control filter coefficients at the n-th time sample, α is a convergence coefficient, $R(n)$ is a KMI $\times$ L matrix of filtered reference signals and $e(n)$ is an L $\times$ 1 vector of error signals. Each coefficient of the adaptive filter driving each secondary source from each reference signal is thus adjusted every sample by an update term composed of the sum of the products of each error signal with the corresponding filtered reference signal.

The theoretical analysis of the behaviour of the Multiple Error LMS algorithm is not well developed. It is, however, possible to analyse some aspects of the convergence of the algorithm using similar methods to those used by Widrow and Stearns (1985), for example, in the analysis of the LMS algorithm, as outlined below. For generality in this analysis, all sampled signals are taken to be complex, and so could represent transformed variables. It is convenient to express the equations for the control filter outputs and error sensor outputs, derived above, in matrix form (Elliott *et al*, 1987, 1988) such that

$$\mathbf{y}(n) = \mathbf{X}(n)\mathbf{w} \tag{3.8}$$

where the vector of control filter outputs is $\mathbf{y}(n) = [y_1(n),\ y_2(n) \ldots y_M(n)]^T$, $\mathbf{w}$ is the MKI $\times$ 1 vector containing the control filter coefficients and $X(n)$ is an M $\times$ MKI matrix of reference signals. Similarly we can write

$$\mathbf{e}(n) = \mathbf{d}(n) + \mathbf{R}(n)\mathbf{w} \tag{3.9}$$

where the vector of error signals is $\mathbf{e}(n) = [e_1(n),\ e_2(n) \ldots e_L(n)]^T$, $\mathbf{d}(n)$ is $\mathbf{e}(n)$ prior to control, and $R(n)$ is the matrix of reference signals filtered by the true secondary paths. We now define a more general cost function than that discussed above, which is similar to that used in optimal feedback control theory and includes both error and "effort" terms, as

$$J = E[\mathbf{e}^H(n)\mathbf{Q}\mathbf{e}(n) + \mathbf{y}^H(n)\mathbf{R}\mathbf{y}(n)] \tag{3.10}$$

in which the superscript ^{H} denotes the Hermitian (complex conjugate transpose) and E denotes an expectation operator. $\mathbf{Q}$ is an error weighting matrix, which is Hermitian and positive definite but not necessarily diagonal, and $\mathbf{R}$ is an effort weighting matrix which is also Hermitian and positive definite but not necessarily diagonal. A cost function as complicated as this may be required for the active control of sound radiation, for example (Elliott and Rex, 1992). In general, the effort term in the cost function prevents the control algorithm from using very high secondary source amplituders to achieve very small reductions in the error signals (Elliott *et al*, 1992), and plays a similar role here to that of Tikhonov regularisation in the solution of ill-posed problems. Using the equations for $\mathbf{e}(n)$ and $\mathbf{y}(n)$ above, this cost function can be expressed in the complex quadratic form (Nelson and Elliott, 1992)

$$J = \mathbf{w}^H\mathbf{A}\mathbf{w} + \mathbf{w}^H\mathbf{b} + \mathbf{b}^H\mathbf{w} + c \tag{3.11}$$

where in this case

$$\mathbf{A} = E[\mathbf{R}^H(n)\mathbf{Q}\mathbf{R}(n) + \mathbf{X}^H(n)\mathbf{R}\mathbf{X}(n)]$$
$$\mathbf{b} = E[\mathbf{R}^H(n)\mathbf{Q}\mathbf{d}(n)]$$
$$c = E[\mathbf{d}^H(n)\mathbf{Q}\mathbf{d}(n)].$$

This equation has a unique global minimum, assuming **A** is positive definite, for a set of control filter coefficients given by

$$\mathbf{w}_{opt} = -\mathbf{A}^{-1}\mathbf{b} = -E[\mathbf{R}^H(n)\mathbf{Q}\mathbf{R}(n) + \mathbf{X}^H(n)\mathbf{R}\mathbf{X}(n)]^{-1}\, E[\mathbf{R}^H(n)\mathbf{Q}\mathbf{d}(n)] \qquad (3.12)$$

which result in the least squares value of the cost function given by

$$J_{min} = c - \mathbf{b}^H\mathbf{A}^{-1}\mathbf{b}. \qquad (3.13)$$

The vector of derivatives of the cost function with respect to the real and imaginary components of the vector of control filter coefficients, $\mathbf{w}_R$ and $\mathbf{w}_I$, can be written as (Haykin, 1986; Nelson and Elliott, 1992)

$$\mathbf{g} = \frac{\partial J}{\partial \mathbf{w}_R} + j\frac{\partial J}{\partial \mathbf{w}_I} = 2[\mathbf{A}\mathbf{w} + \mathbf{b}] \qquad (3.14)$$

which in this case can be written as

$$\mathbf{g} = 2E[\mathbf{R}^H(n)\mathbf{Q}\mathbf{e}(n) + \mathbf{X}^H(n)\mathbf{R}\mathbf{y}(n)]. \qquad (3.15)$$

In practice only an approximation to each of the paths from secondary source to error sensor can be measured and used to generate the practically implemented filtered reference signals, the matrix of which may be denoted $\hat{\mathbf{R}}(n)$. Using the instantaneous estimate of **g**, with $\hat{\mathbf{R}}(n)$, to update all the control filter coefficients at every sample, yields the generalised version of the Multiple Error LMS algorithm:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \alpha[\hat{\mathbf{R}}^H(n)\mathbf{Q}\mathbf{e}(n) + \mathbf{X}^H(n)\mathbf{R}\mathbf{y}(n)]. \qquad (3.16)$$

A convergence analysis of this algorithm can be performed in a similar manner to that generally used for the LMS algorithm (Widrow and Stearns, 1985). The algorithm, if stable, converges to a solution which can be found by setting the term in square brackets in equation (3.16) to zero and using equations (3.8) and (3.9) to give

$$\mathbf{w}_\infty = -E[\hat{\mathbf{R}}(n)\mathbf{Q}\mathbf{R}(n) + \mathbf{X}^H(n)\mathbf{R}\mathbf{X}(n)]^{-1}E[\hat{\mathbf{R}}^H(n)\mathbf{Q}\mathbf{d}(n)] \qquad (3.17)$$

which is not, in general, equal to the optimal solution, equation (3.12), since $\hat{\mathbf{R}}(n) \neq \mathbf{R}(n)$.

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Using this expression for $\mathbf{w}_\infty$, substituting for $\mathbf{e}(n) = \mathbf{d}(n) + \mathbf{R}(n)\mathbf{w}(n)$, and making the usual assumption that the filter weight vector is statistically independent of the reference signals, the update equation (3.16) can be written as

$$E[\mathbf{w}(n+1) - \mathbf{w}_\infty] = [\mathbf{I} - \alpha E[\hat{\mathbf{R}}^H(n)\mathbf{Q}\mathbf{R}(n) + \mathbf{X}^H(n)\mathbf{B}\mathbf{X}(n)]]E[\mathbf{w}(n) - \mathbf{w}_\infty], \qquad (3.18)$$

the convergence of which depends on whether the real parts of the eigenvalues of the generalised autocorrelation matrix, $E[\hat{\mathbf{R}}^H(n)\mathbf{Q}\mathbf{R}(n) + \mathbf{X}^H(n)\mathbf{B}\mathbf{X}(n)]$, are positive. (Morgan, 1980). Note that the eigenvalues of $\hat{\mathbf{R}}^H(n)\mathbf{Q}\mathbf{R}(n)$ are, in general, complex since $\hat{\mathbf{R}}(n)$ is not necessarily equal to $\mathbf{R}(n)$, and the real parts of these eigenvalues are not guaranteed positive (as they would be in the normal LMS analysis) for the same reason. The effort term in this expression, $\mathbf{X}^H(n)\mathbf{B}\mathbf{X}(n)$, is guaranteed to be positive definite (assuming the control filters are persistently excited), and thus will have positive real eigenvalues. This effort term can thus have the effect of stabilising an otherwise unstable system (Elliott *et al*, 1992).

Although every stage in the implementation of the single channel filtered-x LMS algorithm must now be replicated many times in the implementation of the Multiple Error LMS algorithm, the same basic elements (of secondary path model estimation, filtered reference generation and multiplication of error signals with these delayed filtered reference signals) are present in the multiple channel algorithm. In fact the implementation of quite large systems is often not as difficult as it would first appear, principally because the low frequency sound fields which one often wants to control in practice are periodic. Examples of such periodic sound fields are the engine firing noise inside cars and the blade-passing noise due to the propellers inside aircraft. For example, a practical active control system built in 1987 for investigating the active control of propeller noise in a 50 seat aircraft (Elliott *et al*, 1990) had three reference signals ($K = 3$), at the fundamental blade-passing frequency and its first two harmonics, sixteen secondary sources ($M = 16$) and thirty two monitor microphones ($L = 32$). The computational burden of implementing the Multiple Error LMS algorithm at a sample rate of about 700 Hz was not, however, excessive. This is because each of the reference signals was a sinusoid and so only two coefficients were required for each of the $K \times M = 48$ individual control filters and $K \times M \times L = 1536$ individual filters used to generate each of the filtered reference signals. In fact an array of 16 DSP chips (TMS 320C20) were used to implement the algorithm (one for each secondary source). A number of other monitoring functions were also implemented, however, and the processors were not working at anything like their full capacity.

In practical systems for the active control of engine noise in cars the problem is further reduced since the enclosed volume is much less than that of a 50 seat aircraft. Typically, a 6 loudspeaker-8 microphone control systems can be used to control up to 3 harmonics, as shown in Figure 11 (Elliott *et al*, 1988b; McDonald *et al*, 1991). In this application, however, the control filters must adapt to changes in excitation, due to changing engine speed and load for example, which occur on a much shorter time scale to those occurring during steady cruise in an aircraft. Practical implementations of the Multiple Error LMS algorithm used to control the engine noise in cars have a convergence time of the order of one tenth of a second. This rapid adaptation is important subjectively, so that the control system is not heard to lag behind the noise from the engine during gear changing, for example. The result of using such a system in a small four cylinder car is shown in Figure 12, which indicates reductions of 10-15 dB in the noise at the engine firing frequency in the front of the car above 3,000 rpm (100 Hz firing frequency), and somewhat smaller reductions in the rear even at low engine speeds.

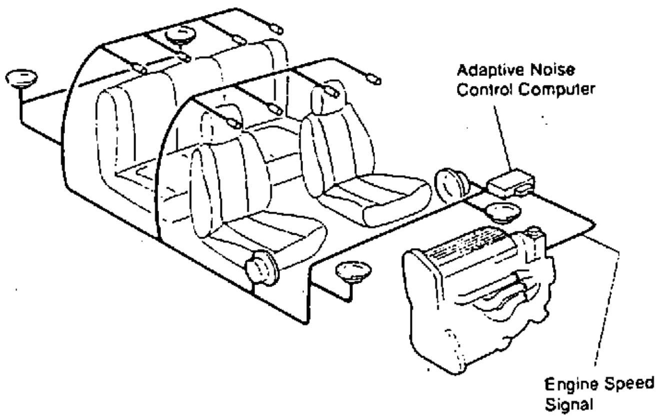

Figure 11 Schematic diagram of a six loudspeaker, eight microphone active noise control system for reducing engine boom inside a car.

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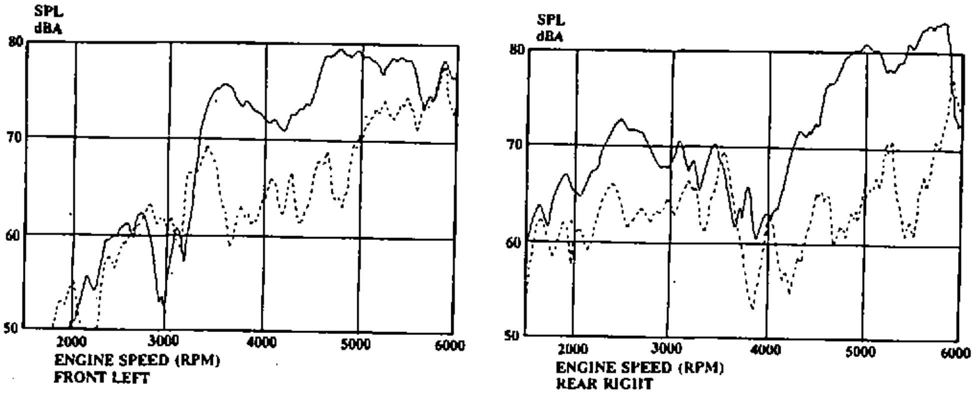

Figure 12 A-weighted sound pressure level due to the engine firing frequency alone, measured at head height in the front (left hand graph) and rear (right hand graph) of a small four cylinder car when accelerating hard in second gear with the active control system off (solid line) and on (dashed line).

An automotive application which presents a greater challenge than controlling engine noise, in terms of both designing and implementing a practical control system, is the active control of low frequency road noise (Sutton *et al*, 1989). An important distinction between this case and that of engine noise control is that the multiple reference signals which must be used have a random rather than a sinusoidal waveform. This means that the control filters, and those used to generate the filtered reference signals, must model a broadband response, and so have many more than two coefficients. This considerably increases the convergence time of the algorithm and the computational burden. Additional complications also arise because the delay through the active control system must now be carefully controlled so that it is not greater than the delay due to the propagation of the physical disturbances through the car body. Experimental systems to actively control road noise in cars have, however, been successfully demonstrated (McDonald *et al*, 1991; Saunders *et al*, 1992).

4. ACTIVE CONTROL OF STRUCTURAL PATHS

The primary source of noise is sometimes transmitted into an enclosure via a limited number of structural paths. It may thus be possible to reduce the noise inside the enclosure by actively controlling the vibration transmitted through these path. The complexity of this active control strategy will depend on the nature of the structural paths and the waveform and frequency range of the disturbance. We illustrate the use of this strategy below, using automotive engine mounts and helicopter gearbox struts as examples. In both of these examples the disturbance being controlled is periodic. The active control of broadband random vibration using feedforward methods is more difficult, partly due to the problems of reference signal generation and noise, referred to above in connection with the active control of road noise in cars. If random disturbances are being transmitted by wave motion in a beam-like structure, the problem is somewhat analogous to the active control of sound in ducts. One major difference is that the speed of propagation of structural waves can be very much higher than in the acoustic case, particularly for compressional waves, and so it is often difficult to implement a practical controller due to causality constraints. With flexural waves in structures, the speed of propagation increases with frequency and there is also an evanescent component to the wave motion which must be taken into account at low frequencies. Both of these effects are discussed in more detail by Elliott and Billet (1993), who also report experiments in which flexural waves with a random waveform, propagating along a beam were actively attenuated by up to 30 dB.

4.1 Active Engine Mounts

The design of an engine mount is a compromise between good vibration isolation, and acceptable mounting rigidity (the latter condition implying that the static displacement is small). By introducing an actuator within the mount to actively control the engine vibrations this trade-off can, to some extent, be overcome. In principle six components of vibration (3 translational and 3 rotational) must be actively controlled to provide perfect isolation through the mount. In order to reduce the complexity of the active component of the mount, however, the passive components can be designed to provide good isolation in respect to all but one of these vibration components. Jenkins *et al* (1991), for example, investigated the use of an intermediate air mount to remove shear and rotational components. Another design has been developed by Freudenberg, based on a hydromount, and is illustrated in Figure 13 (Quinn, 1992). Below about 20 Hz this design behaves like a conventional hydromount, with damping being provided by the fluid being pumped back and forth between the central (4) and lower (7) chambers. Above this frequency the inertia of the fluid in the narrow connecting passages becomes high enough to block this flow. An electromagnetic actuator (1) produces a movement in a metal diaphragm (2) which can then act on the fluid in the central chamber directly, causing a larger motion at the bottom of the mount, because of the hydraulic amplification inherent in the design. Results reported by Quinn (1992) indicate that significant reductions in vibration at the engine firing frequency can be achieved in a car with a control system using a *single* active engine mount only, whose position is shown in Figure 14. Figure 15 for example shows the vibration of the floor and steering column measured at the engine firing frequency in a car with and without a single active mount operating to cancel the vertical vibration of the mounting point under the mount. This reduction in vibration also leads to a reduction in noise in the car as illustrated in Figure 16, which shows the A-weighted sound pressure at the engine firing frequency in the front and rear of the car when the single engine mount was used. It can be seen that in this case reductions of up to 12 dB in the sound pressure level have been achieved by actively controlling a single structural path.

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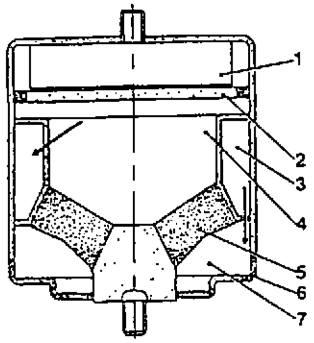


Figure 13 The main features of an automotive hydromount incorporating an electromagnetic actuator for active vibration control.

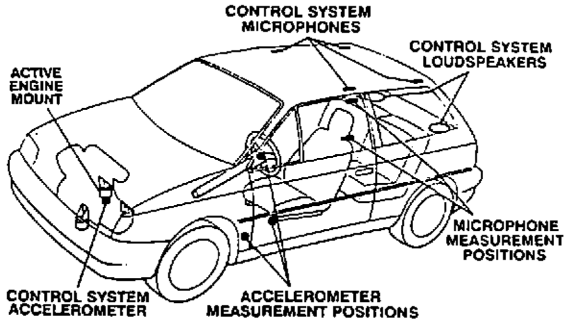


Figure 14 A combined system for the active control of engine noise and vibration in a car.

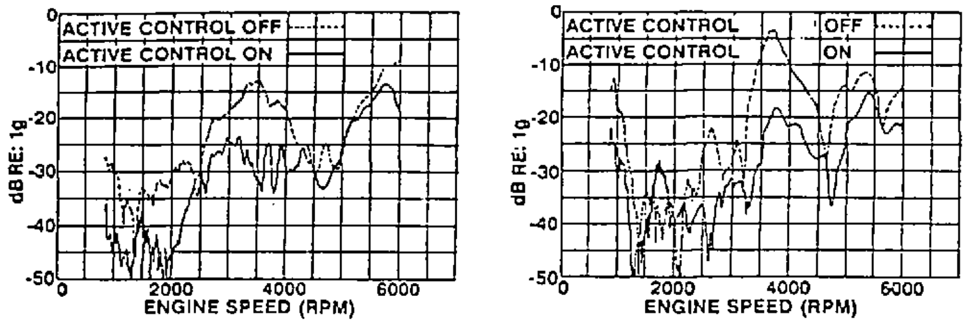


Figure 15 Vibration level at the engine firing frequency, measured on the floor (left) and on the steering column (right) of a car, with and without an active control system using a single active engine mount.

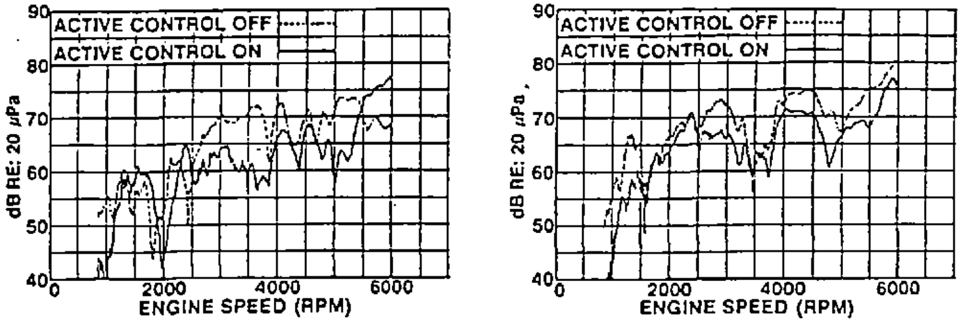


Figure 16. A-weighted sound pressure level at the engine firing frequency, measured at the front (left) and rear (right) seat positions in a car, with and without an active control system using a single active engine mount.

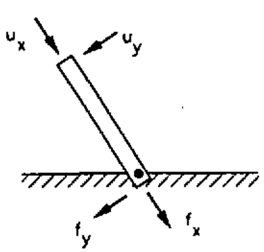
The engine noise in the car is not, however, entirely caused by vibration transmission via the engine mounts. Flanking paths generally exist via other mecanical attachments to the engine and acoustic transmission from exhaust and intake. It would be very difficult to actively control all of these flanking paths directly, but a combined acoustic and vibrational active control system can be used, with both active mounts and internal loudspeakers as actuators, and structural accelerometers and microphones as error sensors, which will control the vibrations due to the dominant path and acoustically control the remaining sound due to the flanking paths. Such a system described by Quinn (1992), and also illustrated in Figure 14, used a single active mount and, two internal loudspeakers, all controlled using the Multiple Error LMS algorithm to minimise the squared outputs from a single accelerometer and four internal microphones. Measurements of the resulting internal noise show that although the loudspeakers give only slight improvements in the noise reduction in the front of the car, the improvement in the rear is significant. Another important aspect of such a combined control systems is that the total power requirements can be significantly reduced, since the control system can control the noise using whichever actuator is most efficiently coupled into the enclosure under different conditions.

4.2 Active Gearbox Struts

Gear meshing tones are a dominant source of noise inside helicopters, and can be transmitted from the gearbox to the fuselage via the struts which often connect these components. The gearbox struts must be rigid, so as not to impair the flight characteristics of the helicopter, and consequently have poor vibration isolation performance. The active control of fuselage vibration at the blade passing frequency, about 17 Hz, in a helicopter has been accomplished at Westlands (Staples and Wells, 1990) using a hydraulic actuator acting between the two sides of a compliant ring incorporated within the gearbox strut . At the much higher frequencies of the gear meshing tones (500 Hz - 1 kHz) such hydraulic actuators are less effective. The transmission of vibration along the strut also becomes more complicated at these higher frequencies.

In general, there are again six components of vibration associated with each position along a structure such as a strut which maintains a constant cross-sectional shape, and six actuators would be required to suppress this motion. If the transmission of vibration is viewed as a wave propagation problem at these high frequencies, then we can identify 6 wave types which combine together to give the 6 vibrational components discussed above. On a uniform structure which conforms to the assumptions of the Euler-Bernoulli beam theory, these wave types propagate independently and can be identified as compressional, torsional and two orthogonal planes of flexural waves. Each of the flexural waves has propagating and nearfield components. Providing the length of the strut is longer than a flexural wavelength, however, the evanescent components of the two flexural waves will transmit very little energy, and the number of wave components which must potentially be controlled is reduced from 6 to 4. A number of authors have previously considered the simultaneous active control of some, or all, of these wave types. Gibbs and Fuller (1992), for example, used pairs of piezoelectric transducers to independently measure and control compressional and flexural waves on a thin beam. Pan and Hanson (1991) have analysed the active control of all four types of wave in a thick beam, and some experimental results on a thick beam have been presented by Clark *et al* (1992). In the experiments of Gibbs and Fuller (1992), and of Clark *et al* (1992) each of the different structural waves was independently sensed and generated. A number of independent single channel controllers could, in principle, then be used to control each of the wave types.

The active control of the compressional and two components of flexural wave, in a tube which is reasonably representative of a helicopter strut, has been reported by Brennan *et al* (1992). Figure 17 shows a simple model of such a strut which is assumed to be pinned at some angle to a rigid body. The strut is excited by a constant velocity source with longitudinal and transverse components u_x and u_y and transmits longitudinal and transverse forces f_x and f_y through the pinned joint. The longitudinal and transverse mechanical transfer impedance calculated for such a system are shown in Figure 18 for an aluminium tube with internal and external diameters of 70 and 93 mm and a length of 1m.



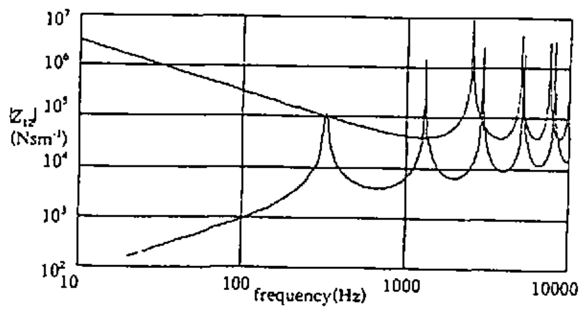


Figure 17 The strut modelled as a pinned beam driven by longitudinal and transverse imposed velocities u_x and u_y and transmitting longitudinal and transverse forces f_x and f_y.

Figure 18 The force at one end of a pinned beam when excited by a velocity at the other end for longitudinal (upper) and transverse (lower) excitation.

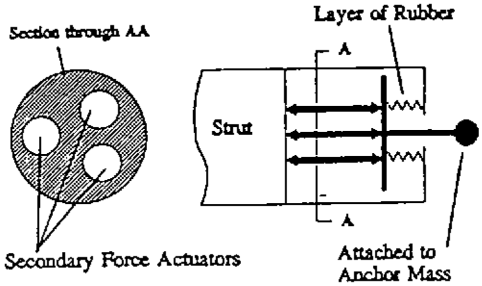


Figure 19 Arrangement of three magnetostrictive actuators mounted within a tube to control the longitudinal wave and two planes of flexural wave.

It is clear that the dominant transmitted force is that due to longitudinal motion at low frequencies, for which the strut acts as a stiff spring. At higher frequencies however, the mass of the strut allows a transverse force to be transmitted, and this is greatly amplified, at about 1.5 kHz for example, by flexural resonances within the strut, unless some passive method can be used to attenuate the transverse motion. Both compressional and flexural waves must be controlled in this case to achieve vibration isolation at the frequencies of interest. Brennan *et al* (1992) also report a method of exciting these three wave types using three magnetostrictive actuators mounted *within* the hollow tube used to model the strut, as shown in Figure 19. A multi-channel feedforward control system was used to minimise the sum of the squared outputs from three accelerometers mounted on a receiving structure by simulaneously controlling the amplitude and phase of the signals driving the three magnetostructure actuators. It should be noted that no attempt was made to *independently* measure or control the three wave types which were known to be present. The actuators had the *capability* of independently exciting each of the wave types, and it was only possible for the vibrations at all three accelerometers to be cancelled by *controlling* all three wave types. A fully coupled multi-channel control system, in which the effect of each actuator is accounted for at each sensor, will thus have the same final effect as three single-channel control systems independently detecting and controlling the three wave types. Initial laboratory measurement of such an active isolation system for the strut are promising, although nonlinearity of the magnetostrictive actuators appears to limit the performance at high amplitudes. Feedforward control methods which are capable of compensating for this nonlinearity are currently under investigation (Sutton and Elliott, 1993).

5. CONCLUSIONS

Many factors need to be taken into account if active control methods are to be successfully used in practical applications. These can be categorised as being to do with the physical objectives of the control system, the type of disturbance caused by the primary source, and the electrical control strategy used to implement the controller. Both feedback and feedforward control strategies are briefly reviewed in this paper, although the use of adaptive digital filters in feedforward control has been considered in more detail. Such adaptive feedforward control has been used in the control of engine noise (a nearly deterministic disturbance), and road noise (a nearly random disturbance) in cars. Some understanding of the range of applicability of various physical objectives can be obtained by considering their fundamental physical limitations. Two purely acoustic control strategies have been discussed, together with one concerned with the control of transmitted vibration to reduce the structure-borne noise.

The number of secondary loudspeakers required to achieve global control of an enclosed soundfield increases with the acoustic modal overlap. Because this rises is proportion to the cube of the excitation frequency in a three dimensional enclosure, the upper frequency limit of such a strategy, using a reasonable number of secondary loudspeakers, is fundamentally limited. Similarly, the number of actuators required to control the vibrations of the flexible boundaries of an enclosure generally increases with the structural modal overlap, which can become large at higher frequencies. Local control methods can be used to generate zones of quiet around monitoring microphones. If the secondary loudspeakers are close to these microphones, the sound field remote from such local systems will not be significantly changed. The zone of quiet forms a uniform shell around the secondary source at low frequencies, which, in a diffuse sound field, breaks up as the wavelength becomes comparable with the distance from loudspeaker to microphone, and becomes a sphere centred around the microphone at higher frequencies, with a diameter of about one-tenth of an acoustic wavelength.

In some applications the sound is transmitted into the enclosure via a limited number of structural paths, and actively controlling the vibration in each of these paths may be possible. Although in principle we need to control six components of vibration in a lumped element, or six structural wave components in a distributed component, good passive design can simplify the dominant excitation mechanism, reducing the number of actuators required to actively control each path. A single actuator appears sufficient for a well designed engine mount operating at low frequencies, and even for high frequency disturbances propagating through a strut, it may be possible to control the vibration transmission with three actuators. With a limited number of structural paths, the total number of actuators used to control the vibrational input to an enclosure can thus be kept within reasonable bounds. One danger of this philosophy is that any flanking paths not originally accounted for, due to acoustic transmission for example, will still excite the enclosure. This has been observed when using active mounts to control the noise inside cars, in which case a combined active control system using structural actuators and loudspeakers has been shown to improve the performance.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

K.H. BAEK & S.J. ELLIOTT, 'Genetic algorithms for choosing source locations in active control systems'. *Proceedings Acoustics '93* (1993).

BILLOUT, M.A. GALLAND, C.H. HUU & S. CANDEL, 'Adaptive active control of instabilities'. *Proc. Recent Advances in Active Control of Sound and Vibration*, 95-107 (1991).

C.C. BOUCHER, S.J. ELLIOTT & P.A. NELSON , 'The effect of errors in the plant model on the performance of algorithms for adaptive feedforward control'. *Special Issue of the Proceedings of the Institution of Electrical Engineers* 138, 313-319. Pt. F on Adaptive Filters (1991).

M.J. BRENNAN, S.J. ELLIOTT & R.J. PINNINGTON, 'Active control of vibrations transmitted through Struts', *Proc 1st Int. MOVIC conference,* 605 - 609 (1992).

J.C. BURGESS, 'Active adaptive sound control in a duct: a computer simulation'. *Journal of the Acoustical Society of America* 70, 715-726 (1981).

C. CARME, 'Absorption acoustique active dans les cavites'. *These presentée pour obtenir le titre de Docteur de l'Université D'Aix-Marseille II, Faculté des Sciences de Luminy* (1987).

R.L. CLARK *et al*, ' An experimental study of the active control of multiple-wave types in an elastic beam'. *Journal Acoustical Society of America* 92, 871-876 (1992).

W.B. CONOVER, ' Fighting noise with noise'. *Noise Control* 2, 78-82 (1956).

A. DAVID & S.J. ELLIOTT, 'Numerical studies of actively generated quiet zones'. To be published *Applied Acoustics*.

S.J. ELLIOTT and P.A. NELSON, 'Algorithm for multichannel LMS adaptive filtering'. *Electronics Letters* 21, 979-981 (1985).

ACTIVE CONTROL OF STRUCTURE-BORNE NOISE

S.J. ELLIOTT & P.A. NELSON, 'Multichannel active sound control using adaptive filtering'. *Proc. ICASSP* 88, 2590-2593 (1988).

S.J. ELLIOTT & P.A. NELSON, 'Multiple-point equalisation in a room using adaptive digital filters'. *J. Audio Eng. Soc.* 37, 899-907 (1989).

S.J. ELLIOTT, I.M. STOTHERS & P.A. NELSON, 'A multiple error LMS algorithm and its application to the active control of Sound and vibration'. *IEEE Trans.* ASSP - 35 1423-143 (1987).

S.J. ELLIOTT, P. JOSEPH, A.J. BULLMORE & P.A. NELSON, 'Active cancellation at a point in a pure tone diffuse sound field'. *Journal of Sound and Vibration* 120, 183-189 (1988a).

S.J. ELLIOTT, I.M. STOTHERS, P.A. NELSON, A.M. McDONALD, D.C. QUINN & T. SAUNDERS, 'The active control of engine noise inside cars'. *Proc. Inter-Noise '88*, 987-990 (1988b).

S.J. ELLIOTT, P.A. NELSON, I.M. STOTHERS and C.C. BOUCHER, 'In-flight experiments on the active control of propeller-induced cabin noise'. *Journal of Sound and Vibration* 140, 219-238 (1990).

S.J. ELLIOTT, P. JOSEPH, P.A. NELSON & M.E. JOHNSON, 'Power output minimisation and power absorption in the active control of sound'. *J. Acoust. Soc. Am.* 90, 2501-2512 (1991).

S.J. ELLIOTT, C.C. BOUCHER & P.A. NELSON, 'The behaviour of a multiple channel active control system'. *IEEE Transactions on Signal Processing* 40, 1041-1052 (1992).

S.J. ELLIOTT & J. REX, 'Adaptive algorithms for underdetermined active control problems'. *Proc. ICASSP* II, 237-240 (1992)

S.J. ELLIOTT & L. BILLET, 'Adaptive control of flexural waves propagating in a beam'. To be published in *Journal of Sound and Vibration* 161(1) (1993).

S.E. FORSYTHE et al. 'Stabilization of a digitally controlled Active Isolation System. *Proc. Recent Advances in Active Control of Sound and Vibration* , 879-889, (1991).

C.R. FULLER, S.J. ELLIOTT & P.A. NELSON, '*Active Control of Vibration*'. To be published by Academic Press, 1993.

G.P. GIBBS & C.R. FULLER, 'Experiments on active control of vibrational power flow using piezoceramic actuators/sensors'. *AIAA Journal* 30, 457-463 (1992).

ACTIVE CONTROL OF STRUCTURE-BORNE NOISE

M.D. JENKINS, P.A. NELSON & S.J. ELLIOTT, 'Active isolation of periodic machinery vibration from resonant substructures'. *Proc. Winter Meeting of the American Society of Mechanical Engineers* (1990).

P. JOSEPH, S.J. ELLIOTT & P.A. NELSON, 'Near field zones of quiet'. To be published in *Journal of Sound and Vibration*.

M.C. JUNGER & D. FEIT, '*Sound, Structures and their Interaction*', 2nd Edition, MIT Press (1986).

P. LUEG, 'Process of silencing sound oscillations'. *US Patent No.* 2,043,416 (1936).

A.M. McDONALD, S.J. ELLIOTT & M.A. STOKES, 'Active noise and vibration control within automobiles'. *Proc. Int. Symosium on Active Control of Sound and Vibration*, Tokyo, 147-157 (1991).

D.R. MORGAN, 'An analysis of multiple correlation cancellation loops with a filter in the auxiliary path'. *IEEE Trans. on Acoustics, Speech and Signal Processing* **ASSP-28**, 454-467 (1980).

P.A. NELSON & S.J. ELLIOTT, 'Active minimisation of acoustic fields' *Journal of Theoretical and Applied Mechanics, Special Issue, supplement to* **6**, 39-89 (1987).

P.A. NELSON & S.J. ELLIOTT, '*Active control of sound*'. Academic Press (1991).

H.F. OLSON & E.G. MAY, 'Electronic sound absorber'. *Journal of the Acoustical Society of America* **25**, 1130-1136 (1953).

J. PAN & C.H. HANSON, 'Active control of total vibratory power flow in a beam. I: Physical system analysis'. *Journal of the Acoustical Society of America* **89**, 200-209 (1991).

D.C. PERRY, S.J. ELLIOTT, I.M. STOTHERS & S.J. OXLEY, 'Adaptive noise cancellation for road vehicles'. *Proc. IMech E conference Auto. Electronics* 150-163 (1989).

D.C. QUINN, 'Automotive active engine mount system'. *AVL Conference*, 3-8 (1992).

A. ROURE, 'Self adaptive broad band active sound control system'. *Journal of Sound and Vibration*, **101**, 429-441 (1985).

T.J. SAUNDERS, T.J. SUTTON & I.M. STOTHERS, 'Active control of random sound in enclsoures'. *Proc. ATA 2nd Int. Conference on Vehicle Comfort* (1992).

ACTIVE CONTROL OF STRUCTURE-BORNE NOISE

A.E. STAPLE AND D.M. WELLS, 'The development and testing of an active control of structural response system for the EH101 helicopter'. *Proc. 16th European Rotorcraft forum*, III 6, 1-11.

I.M. STOTHERS, T.J. SAUNDERS, A.M. McDONALD & S.J. ELLIOTT, 'Adaptive feedback control of sunroof flow oscillations'. *Proc. Acoustics '93* (1993).

T.J. SUTTON, S.J. ELLIOTT, P.A. NELSON & I. MOORE, 'Active control of multiple-source random sound in enclosures'. *Proc. Institute of Acoustics* 12, 689-693 (1989).

T.J. SUTTON & S.J. ELLIOTT, 'Frequency and time domain controllers for the attenuation of harmonic vibrations in nonlinear structural systems'. *Proc. Acoustics '93* (1993).

I. VEIGHT, 'A lightweight headset with an active noise compensation'. *Proc. Inter-Noise '88*, 1087-1090 (1988).

P.E. WELLSTAND & M.B. ZARROP, '*Self tuning systems*'. John Wiley (1991)

B. WIDROW & S.D. STEARNS, '*Adaptive Signal Processing*'. Englewood Cliffs, New Jersey: Prentice Hall (1985).

B. WIDROW, D. SHUR & S. SHAFFER , 'On adaptive inverse control'. *Proc. of the 15th ASILOMAR Conference on Circuits, Systems and Computers* , 185-195 (1981).