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INVERSE SCATTER IMAGING

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INTRODUCTION

Many new techniques have been proposed for imaging the human body by the processing and manipulation of ultrasound data acquired from the scattering of wave fields incident on some region of interest. The plethora of methods are analysed here to show the following common features: (a) an underlying physical model which prescribes the diversity of interactions thought to be amenable to measurement; (b) a data acquisition configuration that restricts the full complexity of interactions contributing to the measurements, and limits the impact of possible artefacts; (c) a computational model, which is essentially an approximation scheme for linking the measured data to the interaction parameters of the underlying physical model. This approach serves as a unifying basis for classifying the various quantitative scatter imaging methods. These statements are illuminated below by the explicit working of a few telling examples.

SCATTER MAPPING

Consider that, for some hypothetical lossless tissue, the scattering of linear, longitudinal ultrasound waves is known to be caused by elasticity fluctuations only. In this case, we adopt the (correct!) *underlying physical model*:

$$\nabla^2 p(\mathbf{r},t) - p''(\mathbf{r},t)/c^2 = \gamma(\mathbf{r})p''(\mathbf{r},t)/c^2 \qquad (1)$$

Here. p denotes the pressure field at location $\mathbf{r}$ and time t, " denotes $\partial^2/\partial t^2$, c is the (constant) mean ultrasound velocity in the scattering medium, and γ is a function of the elasticity fluctuations (vanishing for a uniform medium). For simplicity, it will be assumed that γ takes on non-zero values only inside some finite region, R, which is embedded inside a uniform medium with wave velocity c.

The following scattering experiment (*data acquisition configuration*) is performed on the region of interest, R. An incident plane wave, of frequency $f=c/\lambda$, and directed along the direction $\mathbf{n}_i$, is allowed to penetrate the region R. The waves scattered into the directions, $\mathbf{n}_s$, are measured at some location quite remote from R (far-field measurement), so that the scattering amplitude, σ, may be deduced. It is easy to show that, in the first Born approximation (*computational model*), the scattering amplitude is given by [1]

$$\sigma(\boldsymbol{\mu}) = k^2\int_R d\mathbf{r}.\gamma(\mathbf{r}).\exp(i\boldsymbol{\mu}\cdot\mathbf{r})/4\pi \qquad (2)$$

with $k=2\pi/\lambda$, and $\boldsymbol{\mu}=(\mathbf{n}_s-\mathbf{n}_i)k$. The three elements (viz. physical model, data model, and computational model) of this inverse scattering technique are now seen to have been identified, and it is clear that, for imaging purposes, the desired output is a mapping of $\gamma(\mathbf{r})$ - to be reconstructed from the set of measurements, $\sigma(\boldsymbol{\mu})$. Given the structure of (2), it is immediately apparent that γ may be obtained by Fourier methods:

$$\gamma(\mathbf{r}) = \int d\boldsymbol{\mu}.\sigma(\boldsymbol{\mu}).\exp(-i\boldsymbol{\mu}\cdot\mathbf{r})/2\pi^2k^2 \qquad (3)$$

In order to effect the inversion in (3), the scattering amplitude σ needs to be measured for all μ-values. The different variants of the diffraction tomography technique may be regarded as nothing but different approaches towards achieving that aim. Given that three experimental variables ($\mathbf{n}_s$, $\mathbf{n}_i$, and k) may be varied in order to sweep μ through its desired range, it is clear that a large number of experimental techniques may be devised. Note, however, that an essential ingredient in all methods will be the recourse to computational methods in order to recover the desired mapping. The aim of any technique will be to produce a quantitative map - i.e. one in which image density relates in a well-defined and known way to the value of the associated tissue interaction parameter. In most applications, mixtures of parameters are more easily mapped: in these cases, only spatially invariant, scalar point functions of interaction parameters are true candidates for quantitative imaging.

IMAGE FUZZINESS

Consider now the case that, unknown to the experimenter, scattering from density fluctuations also occurs in the region of interest, R. Provided that the "tissue" remains sensibly lossless, then the experiment outlined above yields the scattering amplitude [1]

$$\Sigma(\mu) = k^2\int_R d\mathbf{r}[\gamma(\mathbf{r}) + \mathbf{n}_s\cdot\mathbf{n}_i\beta(\mathbf{r})].\exp(i\mu\cdot\mathbf{r})/4\pi \tag{4}$$

where $\beta(\mathbf{r})$ is a function containing the density fluctuations. $\Sigma(\mu)$ represents the measured data set, and is the input to the image reconstruction algorithm.

At this stage, the correctness of the underlying physical model becomes crucial. If the model in (1) is unwittingly assumed, then the inversion technique is embodied in (3), since the true structure of the scattering amplitude will be unknown. In this case, however, the data set Σ will not produce a quantitative γ - map when substituted into an algorithm of this type. Instead, the desired mapping will be contaminated by β- and frequency dependent $\nabla^2\beta$- contributions. This circumstance, that an image displaying a desired interaction parameter is corrupted in some unpredictable manner by other interaction features not included in the original physical model, was first identified by the the name "fuzziness" [2]. Note that a fuzzy image may well display an apparently high resolution - this last feature being more dependent on the quality of measured data (particularly sampling and range in μ-space) and finesse of the inversion algorithm employed. Clearly, however, a fuzzy image will not, in general, be a truly quantitative one. In the example shown here, the incorporation of the spatial derivatives of β in the final mapping results in this fuzzy image being essentially non-quantitative: the image density at a point does not depend, in general, only on the values of the interaction parameters (γ, β) at the corresponding point in the object.

The critical importance of the physical model for all aspects of quantitative scatter imaging is illustrated by further consideration of the example (4). Consider now that it is known that density fluctuations also contribute to the scattering, and that the following (correct) physical model is assumed:

$$\nabla^2 p(\mathbf{r},t) - p''(\mathbf{r},t)/c^2 = \gamma(\mathbf{r})p''(\mathbf{r},t)/c^2 + \nabla\cdot(\beta(\mathbf{r})\nabla p(\mathbf{r},t)) \tag{5}$$

Now, the correct scattering amplitude is known by the experimenter to have the structure (4), and it becomes possible to devise data acquisition strategies for obtaining quantitative scatter images. Thus, a data set $\Sigma_P(\mu)$ may be acquired in a scattering experiment for which the scattered waves are measured only orthogonally to the incident direction. For these data, $\mathbf{n}_s\cdot\mathbf{n}_i=0$, and (4) shows that substitution of Σ_P into an inversion algorithm of the type (3) will yield a quantitative mapping of γ. On the other hand, accumulating a data set $\Sigma_B(\mu)$ under backscattering conditions ($\mathbf{n}_s=-\mathbf{n}_i$) will lead to a quantitative mapping of the parameter γ-β, which is related to characteristic impedance fluctuations. Thus, the underlying physical model aids in devising an appropriate data acquisition configuration, in order to minimise the artefacts associated with fuzziness. It is important to appreciate that when the physical model contains two interaction parameters, then two independent experiments, and hence data sets, are required in order to achieve (two) truly quantitative images. The concept is easily generalised to any number of interaction parameters [3].

In practical applications to imaging human tissues, it should be remembered that physical models such as (1) and (5) are not adequate, since they fail to take into account the significant, frequency dependent, absorption processes occurring. Moreover, additional scattering terms may be present, which arise from fluctuations in the *absorption* parameters [4]. It is clear, therefore, that quantitative inverse scatter imaging of human tissues will call for accurate physical modelling and carefully designed data acquisition schemes, at the very least.

A CLASSIFICATION OF TECHNIQUES

A number of inverse scatter imaging methods have been proposed and implemented either with phantoms or real tissue data in recent years [5]. While each of these techniques may be seen to embody the three components (physical, data, and computational models) discussed above, it is on the basis of their data acquisition configurations that they may be classified and distinguished.

Computerised ultrasound tomography (CUT) [6] These "reconstruction-from-projection" methods for producing attenuation and velocity maps may be interpreted as inverse scatter imaging techniques that rely on measurements of the *forward scattered* fields only.

Reflectivity tomography techniques employ *back-scattered* fields only. A wide variety of different approaches have been proposed, but, to date, superposition tomography [7] and flow imaging [8] are the two variants that have actually proceeded beyond the theoretical stage. Both have already been implemented in a clinical setting, providing informative "in vivo" images in a number of different medical contexts.

Diffraction tomography was originally suggested by Mueller and his colleagues [9], and has been extensively developed since then, spawning a large family of variants, whose members are occasionally introduced as "new" techniques. All approaches are, however, characterised by their reliance on measurements of the *angle-scattered* field. Despite its great flexibility (especially to accomodate a wide spectrum of physical and computational models) and undoubted potential, diffraction tomography has lagged significantly behind the two previous categories in respect of its "in vivo" applications.

Despite the practical emphasis on tomographic imaging, inverse scatter techniques are true three dimensional imaging methods, in principle. However, ultrasound penetrability in many human tissues will severely limit the possible applications of even two dimensional imaging by this approach. Clearly, given the somewhat simplistic physical and computational models occasionally invoked, it will be some time before *quantitative* scatter imaging is widely available. On the other hand, the CUT approach has probably adequately achieved this goal, albeit with only modest resolution.

THE PROPAGATOR-REFLECTOR FORMALISM

There is another approach towards systematising the structure of the inverse scatter imaging techniques that we are considering here (viz. those utilising linear, longitudinal waves) and has been called the propagator-reflector formalism [3]. To introduce this, it is noted that the structure of physical models in general, and of (1), (5) and (8) in particular, may be written symbolically as

$$\Pi p(\underline{r},t) = R p(\underline{r},t) \qquad (6)$$

where Π is a (linear) operator describing how the wave p *propagates* through a uniform (or slowly varying) medium with interaction parameters taking on the (regional) mean values of the fluctuations actually occuring in the true medium, and R is a linear operator describing the *reflections* (≡ scattering) by the small-scale variations in interaction parameters. A precise definition, and a more detailed investigation of the significance, of these operators may be found in [3], but we note here that the following three major problems of inverse scatter imaging may be related to the components of this formalism.

Fuzziness has been described above, and is seen to be dependent on the accuracy of the *reflector* model.

Distortion of the recovered image may occur, if the *propagator* model is incorrect. (Consider: amplitude distortion if absorption is neglected in Π, or geometric distortion if refraction effects are omitted).

Resolution is determined ultimately by the sampling of the measured fields and the adequacy of the data set(s). However, it is also dependent on the computational model (as are the fuzziness and distortion). In this sense, resolution is influenced by the physical model (≡ Π + R), which determines the inversion algorithm itself.

THE BORN APPROXIMATION

The first Born approximation ("1BA") is of some considerable interest, since the inverse problem can be solved exactly when it is valid. This is demonstrably so in (3) and (4). Clearly, it is important to have some idea of whether the 1BA applies in any given imaging situation, and much effort has been devoted to investigating its validity conditions.

In practice, computational models are checked, in an inverse scattering context, by considering a relatively simple physical model. The *exact* scattered field from a very simple (usually two-dimensional, rotationally symmetric) object is laboriously computed, for the case of a simple input field, such as a continuous plane wave. The emphasis on simplicity is dictated by the extreme difficulty and computational intensity demanded by calculations of this type. This exact, computed, field is then utilised to provide the data set from which the object is recovered via the computational model under test. The success of the inversion algorithm is assessed by its ability to faithfully recover the original input object.

Virtually all realistically tested computational schemes depend on the Born or Rytov approximations, and the validity of the former is discussed here. It is, unfortunately, difficult to extrapolate from results obtained in computer experiments of the type outlined above: these are limited to very simple, two-dimensional objects, and their generality is not clear.

In the following, we propose a somewhat different approach to this problem, based on the seemingly simple observation that the 1BA cannot be expected to be valid, in general, unless the full Born-Neumann expansion converges.

We choose to work within the framework of a relatively straightforward, but much used, physical model, viz: the inhomogeneous Helmholtz equation. This describes a loss-less medium with a fluctuating, spatially varying, (acoustic) velocity, $c(\underline{r})$, such that

$$c^2(\underline{r}) = c_0^2(1 + n(\underline{r}))^{-1} \tag{7}$$

with c_0 constant, and all velocity variations incorporated into $n(\underline{r})$. A linear acoustic wave, $p(\underline{r},t)=\psi(\underline{r})\exp(i\omega t)$, with $\omega=c_0k$, will propagate through the above medium according to

$$\nabla^2\psi(\underline{r}) + k^2\psi(\underline{r}) = -n(\underline{r})k^2\psi(\underline{r}) \tag{8}$$

It is convenient to express the Helmholtz equation in its integral formulation:

$$\psi(\underline{r}) = \psi_0(\underline{r}) + k^2\int d\underline{r}' G(\underline{r},\underline{r}')n(\underline{r}')\psi(\underline{r}') \tag{9}$$

where ψ_0 denotes the incident field (i.e. the wave that would exist in the absence of the velocity fluctuations), and G denotes the Green's function appropriate for the scattering problem,

$$G(\underline{r},\underline{r}') = \exp(ik|\underline{r} - \underline{r}'|)/4\pi|\underline{r} - \underline{r}'| \tag{10}$$

In an entirely symbolic way, the integral equation can be written

$$\psi = \psi_0 + K\psi \tag{11}$$

with ψ and ψ_0 denoting the appropriate functions, and K denoting the *operator* appropriate to the kernel

$$K(\underline{r},\underline{r}') \equiv k^2G(\underline{r},\underline{r}')n(\underline{r}) \tag{12}$$

The underlying structure of the basic integral equation is now apparent, and it may clearly be solved by iteration to yield the so-called Born-Neumann expansion for the field:

$$\psi = \psi_0 + K\psi_0 + K^2\psi_0 + \ldots\ldots \tag{13}$$

The full series solution for the field ψ is extremely cumbersome to evaluate in any realistic case. In practice, therefore, the series is terminated, in order to give an approximate solution. In particular, the first Born approximation is given by

$$\psi_B \equiv \psi_0 + K\psi_0 \tag{14}$$

It may be shown [10] that the Born-Neumann expansion converges provided that

$$k^4\int d\underline{r}\int d\underline{r}'|n(\underline{r})|.|G(\underline{r},\underline{r}')|^2|n(\underline{r}')| < 1 \tag{15}$$

Some manipulation of this last expression leads to the following *sufficient* condition for the convergence of the series solution to ψ:

$$(1/4\pi)k^2\sup_r\int d\underline{r}'|n(\underline{r}')|.|\underline{r} - \underline{r}'|^{-1} < 1 \tag{16}$$

where $\sup_r$ denotes that the least upper bound of the succeeding function be taken, as the variable $\underline{r}$ ranges over its entire domain.

Consider now a specific example, viz. the scattering of an incident plane wave by a uniform sphere of radius R. Let the acoustic velocity difference between the sphere and its surrounding medium be such that

$$|n(\underline{r})| = \Delta^2 \quad \text{for } |\underline{r}| \leq R$$
$$= 0 \quad \text{otherwise} \tag{17}$$

Computation of the sufficiency conditions derived above, lead to the conclusion that the convergence of the Born series is assured if

$$\tfrac{1}{2}\Delta kR < 1 \tag{18}$$

In other words, if this condition is violated, the 1BA is unlikely to be a good approximation. Under such circumstances, the utilisation of an inversion algorithm which assumes the validity of the 1BA might well lead to unacceptable images.

Although we have chosen a simple example only, the treatment presented here is perfectly general, three (or even more!) dimensional, readily applicable to quite complex scattering structures, and even capable of handling more sophisticated physical models - all without the need for recourse to extensive computer simulations.

CONCLUSIONS

We have indicated that physical modelling underpins much of inverse scatter imaging, and gives rise to the notion of image fuzziness. Moreover, physical models enable data acquisition strategies to be devised, which minimise artefacts arising from this source.

All inverse scattering techniques consist of three major components: a physical model, a data model, and a computational model. Available methods may be classified according to their data acquisition configurations.

The Born approximation is an important element in many inversion schemes, and we have proposed a powerful but simple approach towards stating its validity conditions, in very general terms.

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