Some Experiments in Adaptive Array Processing for H.F. Applications

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I. Introduction

Adaptive processing has been playing an increasing role in spatial filtering of interfering signals for the last several years. A typical adaptive array processing configuration considered in the literature is shown in Fig. 1. Each antenna element is followed by a receiver and appropriate amplitude and phase weighting. These weights, or coefficients are adjusted periodically using an algorithm which is computed in the adaptive processor, thus effectively realising a spatial filter. The correct choice of the coefficients will ensure the formation of maximum array sensitivity in the direction of interest and of minimum sensitivity in the directions of interfering noise sources.

In this paper we consider some aspects of adaptive array processors which have been treated only partially in the literature, especially for H.F. applications. First, a robust design of the adaptive array is considered with regard to uncertainties in the knowledge of the desired signal direction (e.g. due to fluctuations in the ionosphere or due to random errors in the array parameters) and also against signal cancellation from coherent interferences arising, possibly, from multipath propagation. Secondly, methods of incorporating adaptive processing in some existing electronically scanned arrays without additional hardware are considered. In order to illustrate the need for this, the all-digital realization of Fig. 1 is seen to require a phase coherent receiver to demodulate each element output, which in turn is digitized (thus requiring a separate A/D converter) and processed using a general purpose computer. Besides being expensive, this proposition may not be feasible in certain applications where coherent receivers cannot be used, e.g. when there is no a-priori knowledge of the form of signals to be received by a communication receiving system. We therefore consider some alternatives which implement the adaptive algorithms on an existing, non-adaptive H.F. array antenna system. The aim here is to bring out the possibility of using a variety of modifications to suit the specific practical requirements on system hardware. Finally, results of some experiments with these algorithms both on simulated and real data are presented.

II. Constrained L.M.S. Adaptive Arrays

The constrained least-mean-squares formulation of the adaptive array processing problem was first proposed by Griffiths |1|. In this section we consider the basic optimization problem and some additional constraint systems on it for the design of robust adaptive arrays.

Let C denote the "look direction vector" of the array given by

$$C = \begin{vmatrix} \exp\{j\ (p_1 \cdot u_c)\omega/c\} \\ \exp\{j\ (p_2 \cdot u_c)\omega/c\} \\ \cdot \\ \cdot \\ \exp\{j\ (p_n \cdot u_c)\omega/c\} \end{vmatrix} \qquad (1)$$

where p_j is the 3-dimensional vector of position coordinates of the j'th sensor, u_c is a unit vector in the direction from which the desired signal is propagating and c is the velocity of propagation. Let the signals $\{x_i(k),\ i = 1, 2, \ldots, n\}$ appearing at the n array elements be denoted by a vector $X(k) = \{x_1(k)\ .\ .\ .\ .\ ,\ x_n(k)\}$ and let

$$E\ \{X(k)\} = 0 \qquad (2a)$$

$$E\ \{X(k)\ X^T(k)\} = R \qquad (2b)$$

The output power of the array is given by

$$P_o = E\ \{|y(k)|^2\} = W^T\ RW \qquad (3)$$

where

$$y(t) = \sum_{i=1}^{n} w_i \; x_i(k) \qquad (3a)$$

is the array output, and

$$W = \{w_1, w_2, \ldots , w_n\} \qquad (4)$$

is the weight vector. The problem can now be stated as that of designing the array weight vector in order to minimize the output power P_o, subject to the constraint that the gain in the look direction is held fixed at unity, i.e.

$$C^T W = 1 \qquad (5)$$

The solution of this problem is easily seen to be given by

$$W_{opt} = \frac{R^{-1} \; C}{C^T \; R^{-1} \; C} \qquad (6)$$

This solution is, however, very sensitive to inaccuracies in array parameters like element locations and excitations, and also to the deviation of the signal from the direction specified by the vector C |2|. Hudson |2| and others |3,4,5| have shown that a reasonable control on the sensitivity may be achieved by constraining the length of the optimum weight vector. The problem is then required to be solved under an additional constraint

$$W^T W \leq S \qquad (7)$$

where, roughly speaking, the value of S controls the beamwidth and superdirectivity of the array.

An analytical solution to this modified problem is extremely difficult and not known. Some interesting special cases of the solution are given in a companion paper |2|. An alternative approach based on eigenvalue techniques for a related problem has been given by Gilbert and Morgan |5|.

Finally, consider the case when a coherent interference arriving from a known direction is present in the environment. In order to avoid cancellation of the desired signal by an out-of-phase coherent interference, a simple technique would consist of augmenting the constraint system by an additional linear constraint

$$C_I^T W = 0 \tag{8}$$

where C_I is the coherent interference direction vector. Obviously, however, this approach is inapplicable if no a-priori knowledge of the coherent interference direction is available.

III. Implementation

The most commonly employed technique of implementing the steady state solutions of the problems, considered in the last section, adaptively is by means of the stochastic steepest descent algorithm. Since this still forms the basis of the alternatives considered later in this section, it is described very briefly in the following:

A. The Stochastic Steepest Descent Algorithm

The method of steepest descent uses gradients of the performance surface in seeking its minimum, making each change in the weight vector proportional to the negative of the gradient vector:

$$\begin{aligned} W'_{k+1} &= W_k + \beta(-\nabla_k) \\ &= W_k - \beta R W_k \end{aligned} \tag{9}$$

In its stochastic version, the covariance matrix R is replaced by its sample estimate $X_k X_k^T$ and (9) becomes

$$W'_{k+1} = W_k - \beta\, X_k y_k \tag{10}$$

where y_k is the output signal of the array at the k'th instant. The following additional steps are necessary to incorporate the constraints $W^T C = 1$ and $W^T W \leq S$ of the last section, before going to the next iteration:

(i) W'_{k+1} is projected on the constraint surface $W^T C = 0$ via the projection equation:

$$W''_{k+1} = W'_{k+1} - CC^T W'_{k+1}/C^T C \quad (11)$$

$$= P\, W'_{k+1} \quad (11a)$$

where P is the projection matrix

(ii) W''_{k+1} is then checked to see if it satisfies $||W''_{k+1}||^2 \leq s$. If not, it is suitably normalised to force this constraint (12)

(iii) The constraint vector C is finally added to W''_{k+1} to satisfy (5):

$$W_{k+1} = (W''_{k+1})_n + C/C^T C \quad (13)$$

where $(\cdot)_n$ denotes the normalised version of the argument.

B. <u>Some Salient Features of the H.F. Array System</u>

In an application, with which the author has been involved, and which will serve as a typical example motivating investigations into alternative implementation of the adaptive algorithms, the beamforming system has the following salient features. Some of these, it can be seen, are likely to be typical of most array systems.

1. The system is capable of simultaneously forming a number of independently controlled beams from signals received from an array of 24 elements, in the H.F. band.

2. The system is computer controlled so that the computer calculates the appropriate phase shift and gain for each element of the array on appropriate instructions of frequency, required beam direction and the required beam shape. The computer stores data on the location of each element and the beam forming system can be used, in principle, for any array geometry.

3. The system is required to work with only two, preferably non-coherent receivers.

4. Array weights are to be implemented with three bits of amplitude and four bits of phase quantization.

5. The nominal switching speed of the weights is of the order of 100 beams/sec.

In view of these features, it is clear that the stochastic steepest descent algorithm as discussed above is not applicable directly. In particular, the restriction of at most two receivers makes it impossible to make all the element signals available simultaneously for adaptive processing. In the following we discuss two alternatives which incorporate the above features of the array. Both of these require the formation of two beams (or channels) simultaneously, one of which is made adaptive while using the other in a search mode around the former's current setting. Furthermore, it is assumed that the weights of the two beams are to be switched synchronously. Finally, for obvious reasons, the weights of the array are adapted serially (i.e. cyclically), one at a time.

C. <u>First Alternative: The Perturbation Algorithm</u>

A schematic diagram of the set-up for this algorithm is shown in Fig. 2 and consists of the following steps.

1. Set the adaptive and search beam weights, denoted by W_A and W_S respectively, to conventional values. Set $k = 1$.

2. Find the corresponding receiver output in the adaptive beam. Let this be y_A.

3. Perturb the i'th ($i = k \bmod 2n$, k = iteration number, n = number of weights) weight component of the search beam by one level of quantisation and let the corresponding search beam output be $y_S(i)$. Form the gradient

$$g = \frac{y_S(i) - y_A}{\Delta W(i)} \tag{14}$$

where $\Delta W(i)$ is the perturbation in the i'th component.

4. Update the i'th component of the adaptive beam using

$$W_i(k+1) = W_i(k) - \beta g \tag{15}$$

where β is a constant which controls the rate of adaptation and steady state properties of the processor.

5. Apply the linear (look direction) and nonlinear constraints using (11) - (13) and quantise to obtain the new adaptive weight vector W_A. Set W_S equal

to W_A, $k = k+1$.

6. Take the next input sample and go to (2).

Remarks: This implementation requires only two non-coherent receivers, and is a close approximation to the steepest descent technique of section III-A. A possible source of error may be the mismatch in the gains of the two receivers. But this can be estimated from the outputs of the two receivers during the interval when W_S is set equal to W_A, and used for suitably modifying the value of g in (15). The form of the incoming signals is not important since the gradient is estimated by a differential perturbation with both the beams having the same input. Finally, if the data rate is higher than the beam switching speed, the intervening samples may be suitably integrated to yield an average intensity of the signals for use in the algorithm.

D. Second Alternative: Multiplexing

When the two receivers available for use with the two beams are coherent, with the capability of yielding information regarding the amplitude and phase of the carrier, it is preferable to use this second alternative. In this, the search beam channel is used simply to sample each element signal cyclically as the data arrives, by switching the search beam weights in a multiplexer mode. Each sample is then used to update the corresponding weight component in the adaptive beam in accordance with eqn. (10). After a complete cycle of such updates (stored in the computer), the constraints (11) - (13) are applied and the weights quantised for use in the next cycle.

The problem of receiver gain mismatch again exists but is more difficult to compensate for, in this case. One way of doing it would be to compare the outputs of the two receivers periodically by using them in the same mode.

E. Convergence Properties

The two algorithms proposed above are essentially suboptimal implementations of the steepest gradient algorithm of section III - and hence have very similar features regarding their convergence behaviour. Some of the important points are summarised below:

(i) Convergence is assured for all values of β lying in the range

$$0 < \beta < \frac{2}{\mathrm{tr}\ (PRP)} \qquad (16)$$

where P is the projection matrix defined in (11).

An important difference, however, now exists in the use of the feedback coefficient β. In the case when all the element signals are available simultaneously, this can be normalised against signal power variations by the factor $X_k^T X_k = \mathrm{tr}\ (X_k X_k^T)$ at each instant. In III-C and III-D, however, this will have to be achieved by prior signal normalization i.e., before feeding to the adaptive processor.

(ii) The rate of convergence is directly proportional to the value of β. However, larger values of β in the range given in (16) yield large amounts of loop or gradient noise in the steady state. It can be shown that the steady state gradient noise power is given by

$$G_n = \beta P_{so}\ \mathrm{tr}\ (PRP)$$
$$= \beta P_{so}\ (\mathrm{tr}\ R - P_c) \qquad (17)$$

where P_{so} = Desired signal output power from the optimal array

P_c = Output power of a conventional array

Thus the output SNR is given by

$$(S/N)_o \doteq \frac{1}{\beta\ \mathrm{tr}\ (PRP)} = \frac{1}{\beta\ (\mathrm{tr}\ R - P_c)}$$

IV. Experimental Results

Two sets of experiments have been conducted using the above algorithms. In the first, an 8-element circular array was considered and fed with data simulating various conditions. The second set of experiments have been performed on some real data from a 7-element, Λ-shaped array and provided by

Christopher Thomas of Leicester University. Some of these results are briefly described in the following.

A. Experiments with Simulated Data

(i) Radiation Pattern Typical radiation patterns of the array, before and after adaptation are shown in Fig. 3, in which a 0 dB source at 0^{o} and a 20 dB interference at 100^{o} were simulated. The null depth achievable in these experiments was seen to be strongly related to the strength of the interfering source. The stronger the interfering source, faster the rate of convergence and larger the null-depth. Typically, about 50 cycles of iterations in implementation of III-C and III-D are necessary to reach steady state.

Fig. 4 illustrates some typical results for output SNR as a function of the input SNR, obtained by freezing the weights of the array after 50 iterations, again for an 8-element circular array with $\lambda/2$ spacing.

(ii) Effect of Random Errors The degradation in performance due to random errors in the array parameters is seen to depend on the model employed for these errors. If the errors are assumed to be constant (but unknown) over all iterations (e.g. errors due to inaccurate locations of the array elements of a rigidly fixed array, or due to imprecise knowledge of the signal direction), there is seen to be little degradation in the null-depth in the direction of the interference. The effective SNR, however, may be highly degraded due to loss of gain in the signal direction. But incorporation of the superdirectivity constraint (7) with suitable choice for the value of S, can control this effect by desensitizing the response of the array in a small sector around the look direction. Typically, about 10-15% errors in the effective weights can be tolerated with the help of this constraint.

On the other hand, if the errors are truly random and vary from iteration to iteration (e.g. errors in the attenuators and phase shifters' settings), the degradation in SNR is more serious since it is caused by losses both in the signal gain and null-depth. Fig. 5 shows the null-depth achieved in an 8-element circular array for a 20 dB interference at 45^{o} bearing, as a function of the r.m.s. errors while constraining the norm of the weights to unity. The performance is seen to degrade very much after about 20% errors.

(iii) Coherent Interference If an interference is fully or partially coherent with the signal from the look direction, there is once again a loss in the output SNR. This loss is a function of the correlation coefficient between the two. In a typical ionospheric, multipath environment at H.F., this coefficient is a function of the path length difference between the look direction and interfering signals. Fig. 6 shows a typical behaviour of the null-depth against correlation coefficient.

In general, the effect of adaptive processing in such an environment is seen to be that of combining the two signals in nearly equal strength, although in arbitrary phase. The result is a loss in the desired signal power. This phenomenon is seen to disappear when the constraint (8) is included in the algorithm.

B. Experiments with Real Data

Data was recorded from a 7-element, Λ-shaped array whose geometry is shown in Fig. 7. The data at each element was recorded from the outputs of coherent receivers (with I.F. bandwidth of 100 Hz) with in-phase and quadrature phase channels on to cassettes through A/D converters and via a PDP-11 computer. The data was then transferred onto a disc-file of a Modular-1 computer for further processing and experiments.

Two sets of data were recorded from a source (frequency = 1.34 MHz) at different times of the day and are quite typical of situations which may be encountered in H.F. applications. The total recorded interval in each case was approximately 3 minutes and 15 seconds, containing about 470 data samples. Although from the same source, the two sets of data have an interesting difference. While the first set had mainly a ground wave component, a significant contribution in the second set also came from skywave due to a decrease in the D-region absorption at the time of its recording.

In order to conduct the adaptive processing experiments on this data, a number of beams looking in different directions were simulated on the computer and the output power of each was displayed. The source therefore, acts as an interference for all but the beam pointed in its direction. The results are summarised in Figs. 8 and 9.

Fig. 8 shows the result of conventional and adaptive processing on the first set of data by forming 90 beams distributed over the entire azimuth and an elevation of 0° (groundwave). The outputs of adapative processors using all the three algorithms are illustrated in the figure. The signal is clearly seen to arrive from the dominant peak at about 130° - the outputs of all other beams being considerably lower than those of the corresponding non-adaptive beams. The second dominant peak seen in this figure is ascribed to the grating lobes.

For the second set, an extensive search both in azimuth and elevation showed that the skywave component arriving at an angle of 70° was significantly stronger. This was demonstrated by the fact that efforts to receive a signal by an adaptive array looking at the groundwave component resulted in about 10 dB loss in signal power. The corresponding effort at 70°, however, resulted in a loss of only about 1.5 dB. The reason for this loss, of course, is the presence of the coherent interference, the weaker signal getting more suppressed than the stronger signal in adaptive array processing. Thus Fig. 9 shows the results obtained with this data at 70° elevation.

Both these experiments confirm the possibility of obtaining deep nulls in the interference directions using the modified algorithms presented in these papers. Except in the case when a correlated signal arrives from a widely different, unknown direction as in case 2 above, the performance of these algorithms seems to be entirely satisfactory.

V. Acknowledgements

The author gratefully acknowledges helpful discussions with Dr. J.E. Hudson and Professor J.W.R. Griffiths of Loughborough University and Dr. Williams and Dr. Bailey of the G.C.H.Q. Thanks are also due to Chris Thomas of Leicester University for recording data for our use and to G.C.H.Q. for permission to publish the paper.

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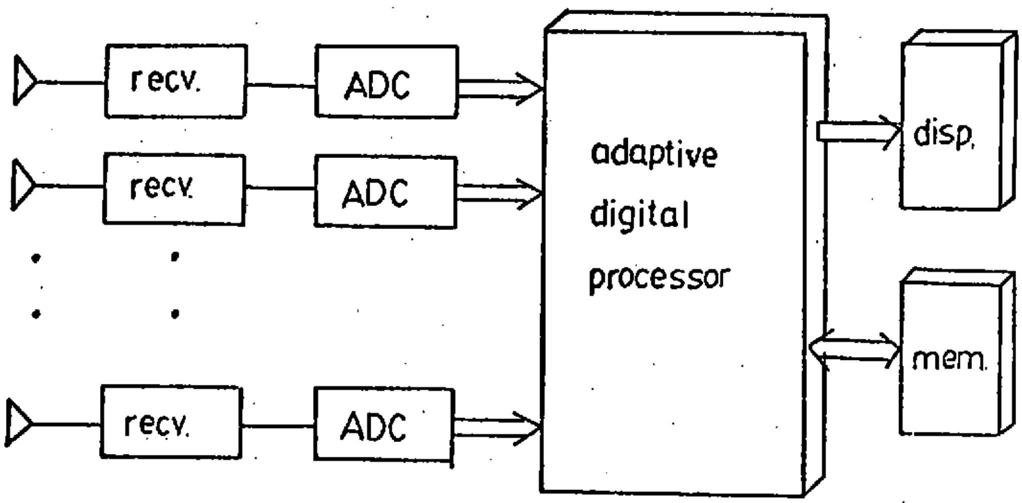


Fig. 1   An all Digital Adaptive Array

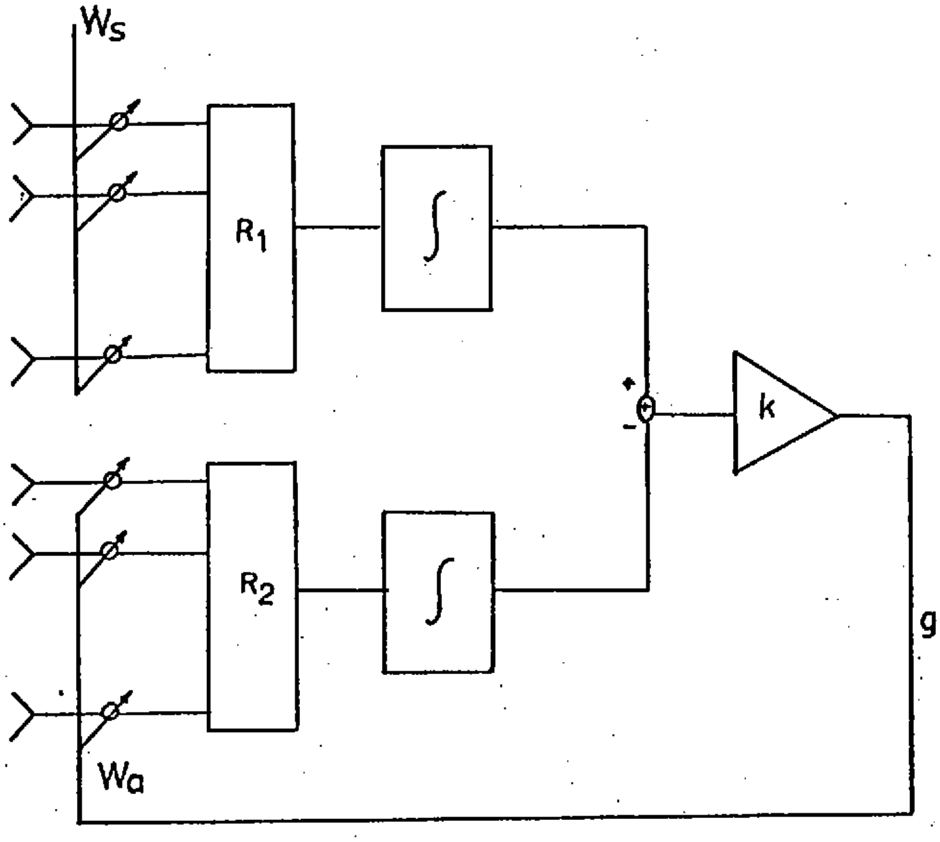
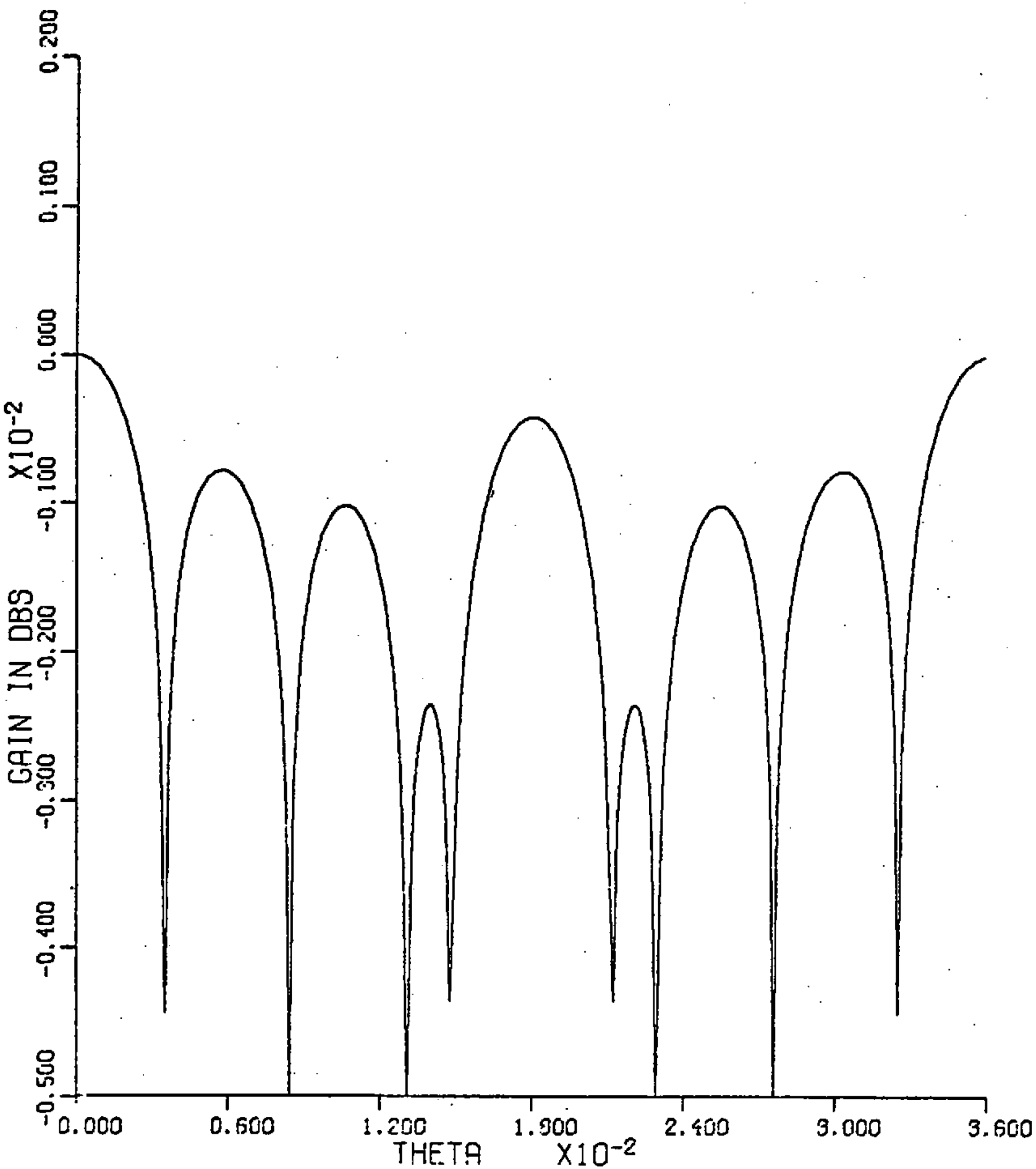


Fig. 2 The Perturbation Algorithm



0.200
0.100
0.000
-0.100
-0.200
-0.300
-0.400
-0.500
GAIN IN DBS X10^{-2}
0.000
0.600
1.200
1.800
2.400
3.000
3.600
THETA X10^{-2}

Fig. 3(a)

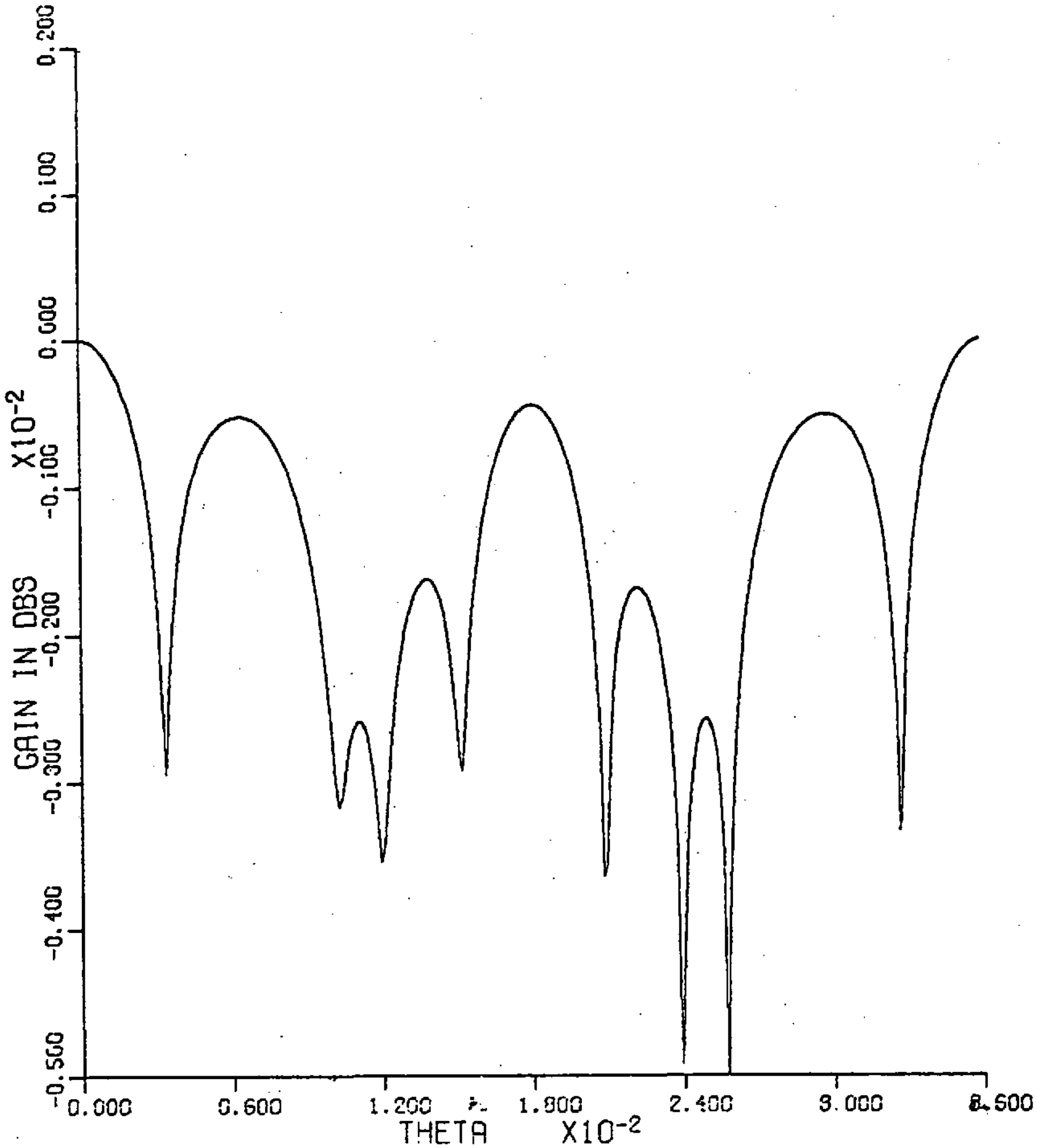

0.200
0.100
0.000
-0.100
-0.200
-0.300
-0.400
-0.500
GAIN IN DBS X10-2
0.000
0.600
1.200
1.800
2.400
3.000
3.600
THETA X10-2

Fig. 3(b)

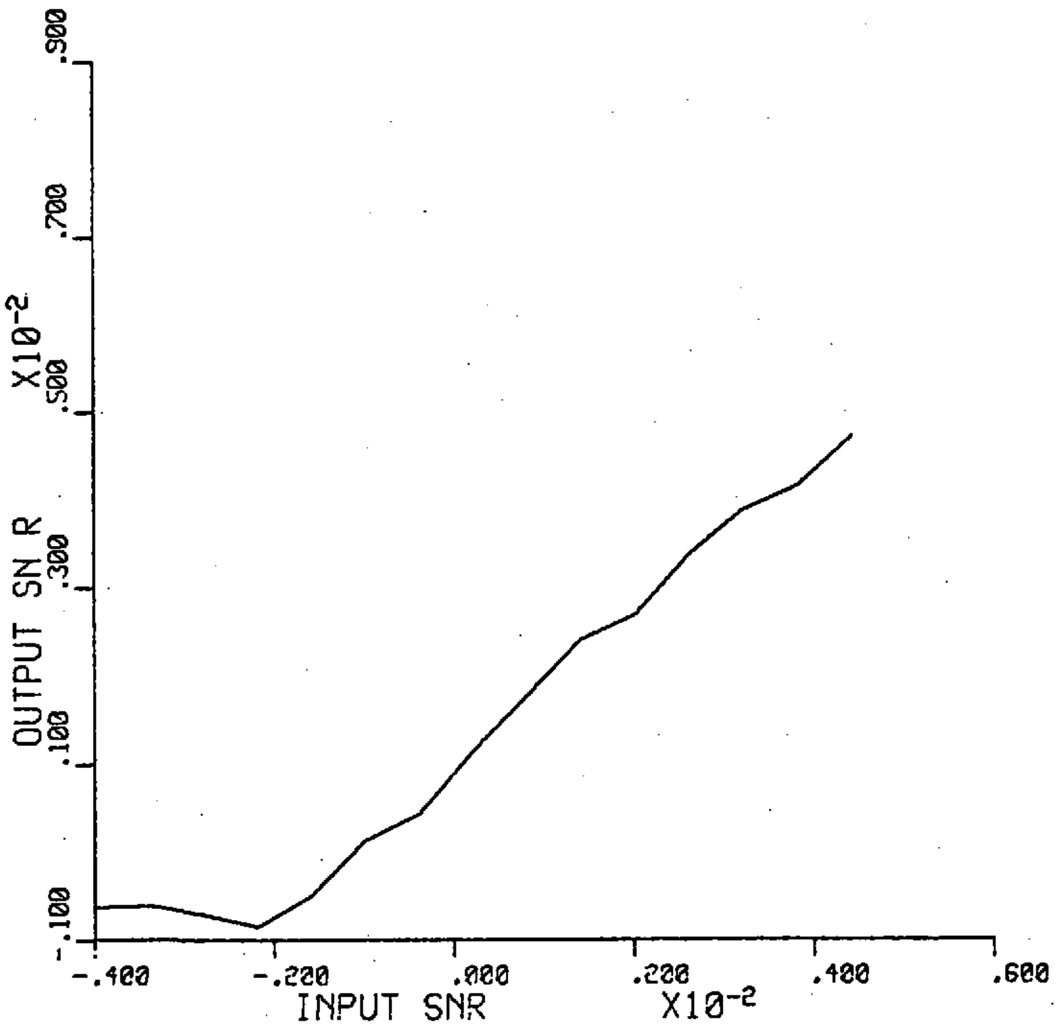
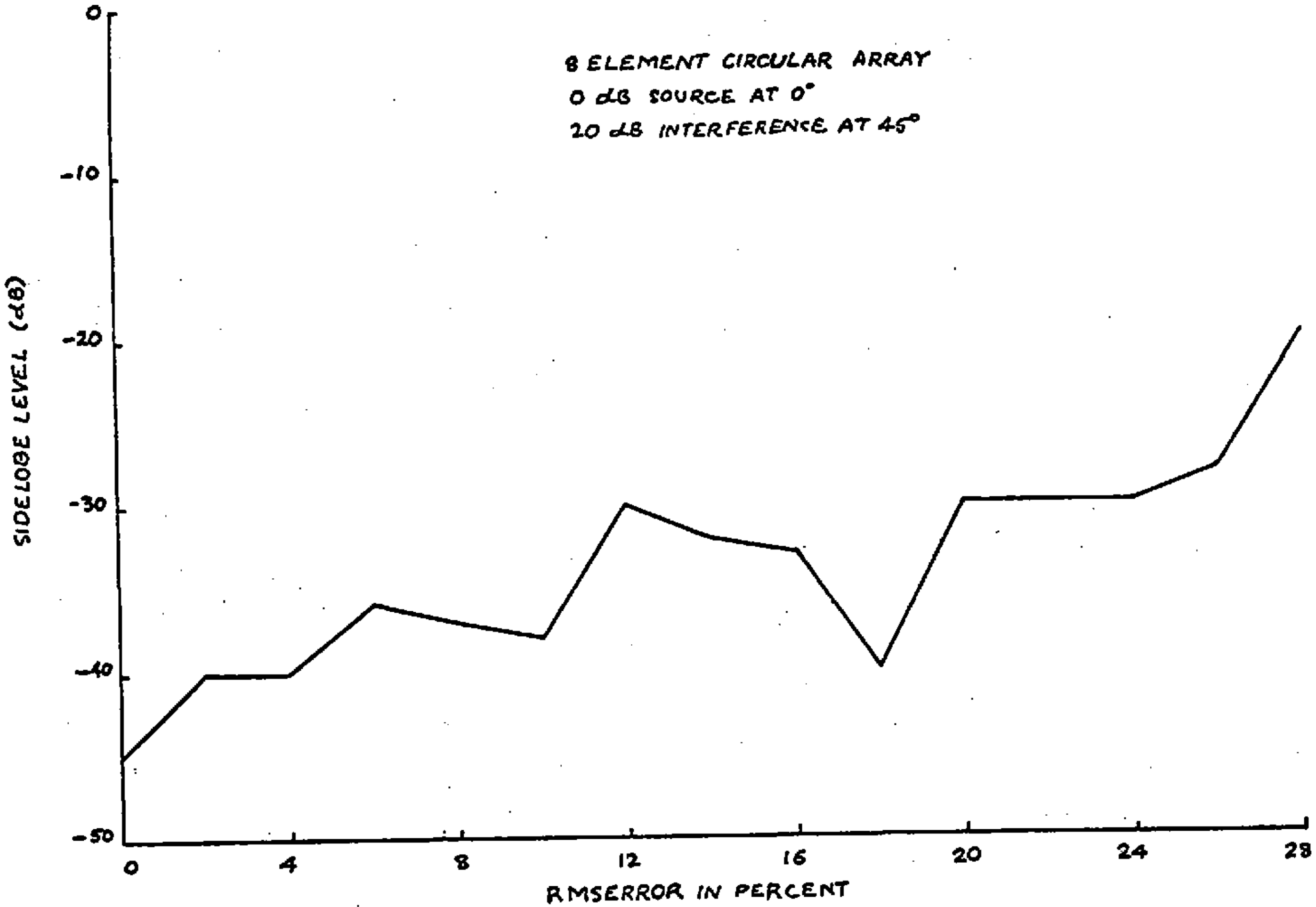
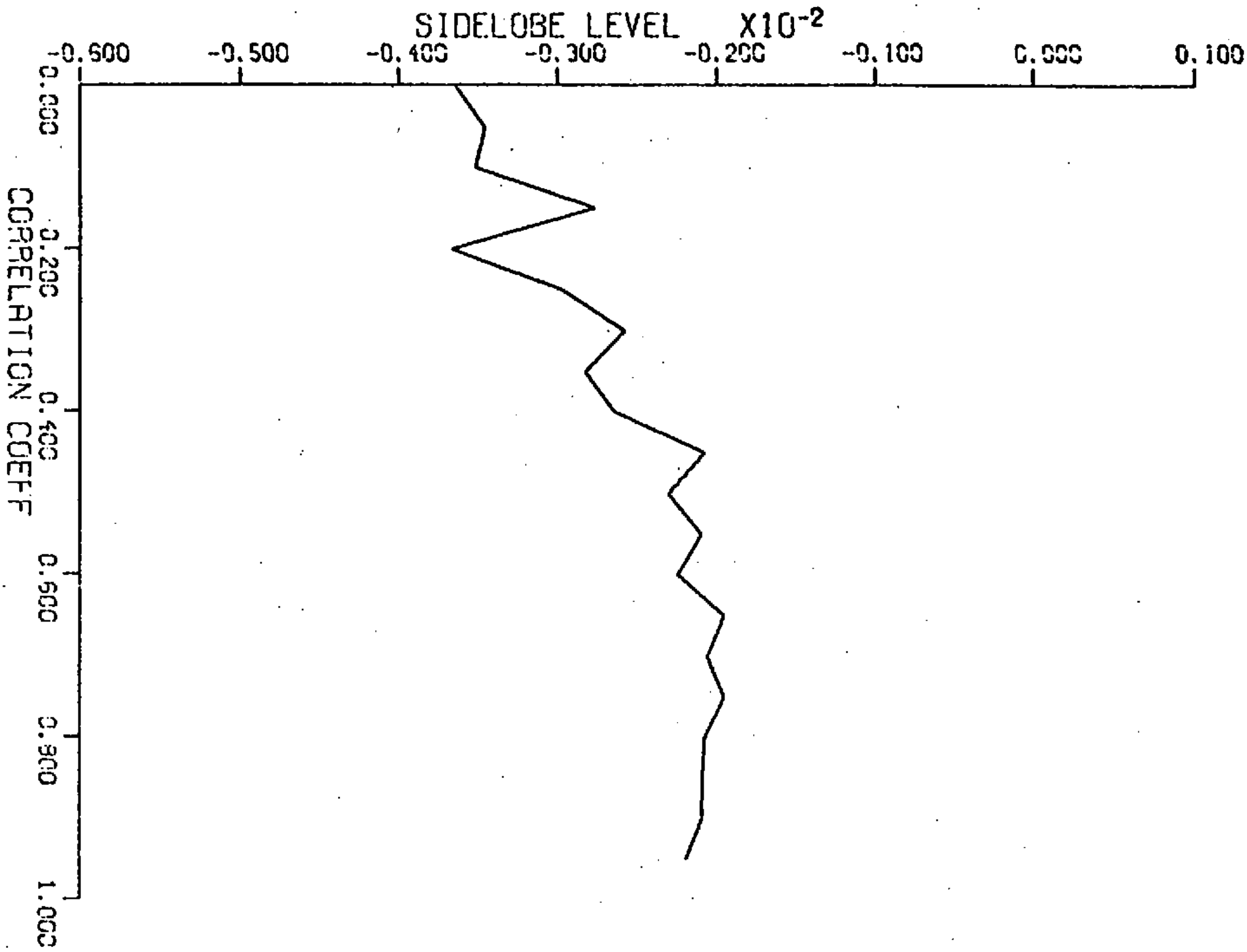

OUTPUT SN R X10⁻²
.100
.100
.300
.500
.700
.900
-.400
-.220
.000
.220
.400
.600
INPUT SNR X10⁻²

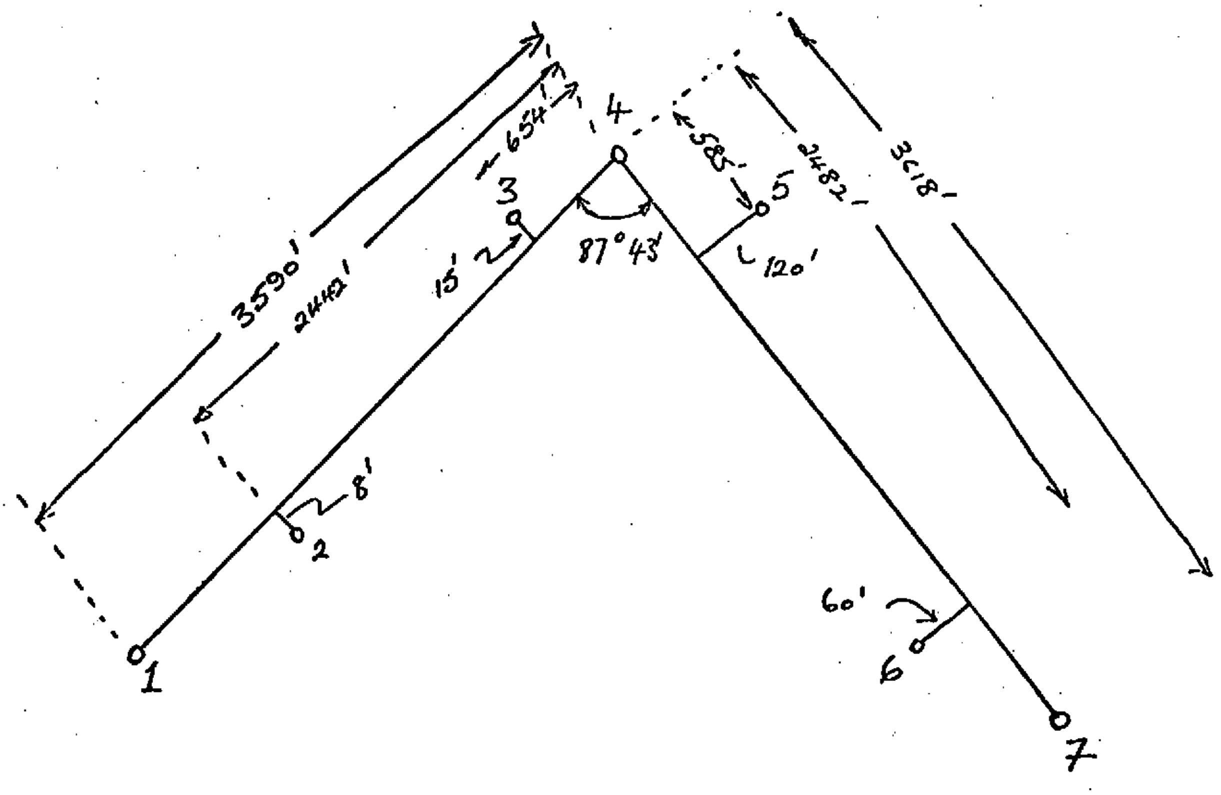
Fig. 4



8 ELEMENT CIRCULAR ARRAY
0 dB SOURCE AT 0°
20 dB INTERFERENCE AT 45°
0
-10
-20
-30
-40
-50
SIDELOBE LEVEL (dB)
0
4
8
12
16
20
24
28
RMSERROR IN PERCENT

Fig. 5


SIDELOBE LEVEL X10^-2
-0.600
-0.500
-0.400
-0.300
-0.200
-0.100
0.000
0.100
CORRELATION COEFF
0.000
0.200
0.400
0.600
0.800
1.000



Metric coordinates (ms.)

1.	-773.738	-773.738
2.	-524.591	-528.039
3.	-144.185	-137.721
4.	0.0	0.0
5.	151.946	-100.218
6.	522.00	-547.86
7.	779.773	-779.773

(Assuming exact North apex direction)

Fig. 7 The Blake Hill Array

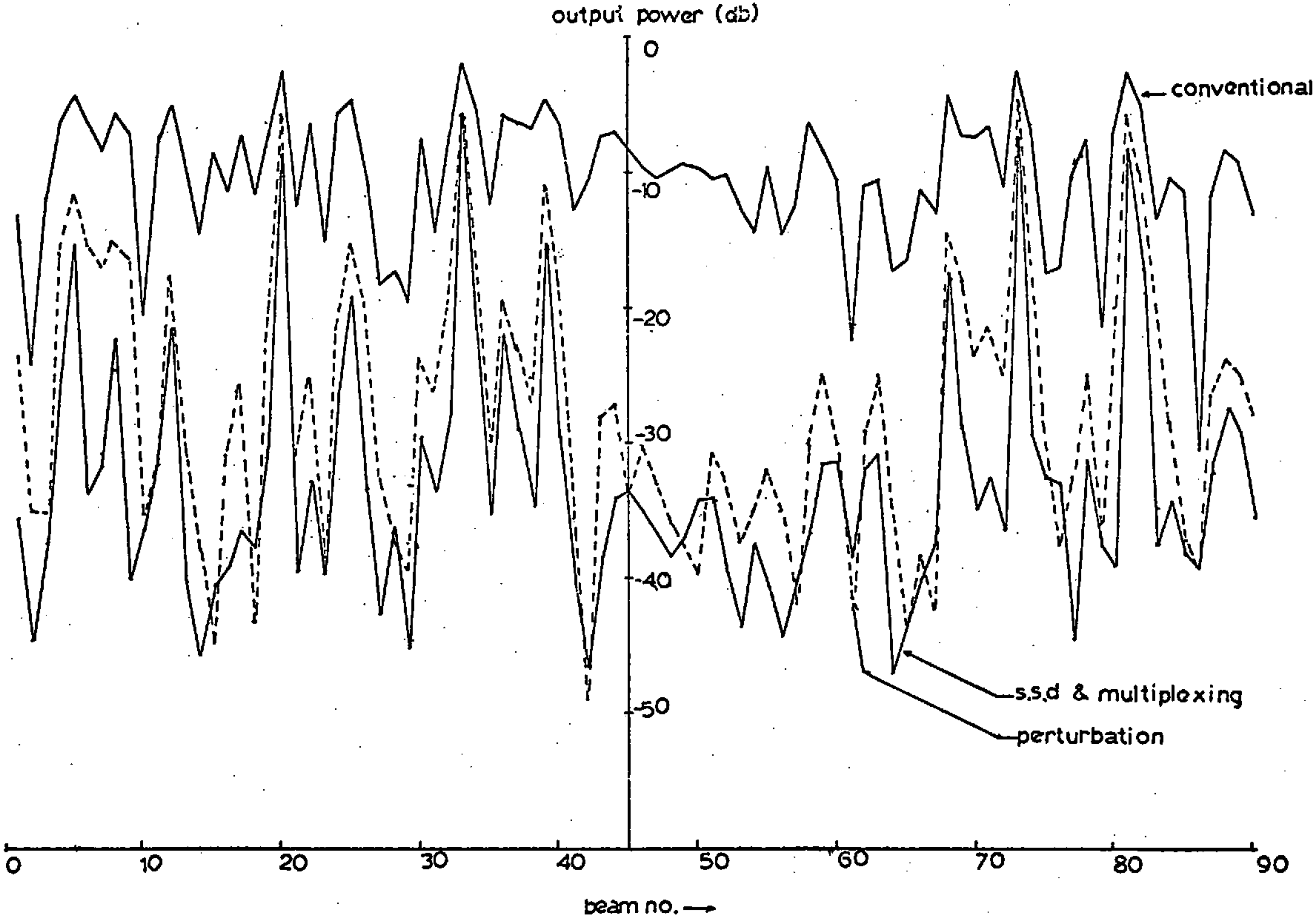


Fig. 8 Experimental Results

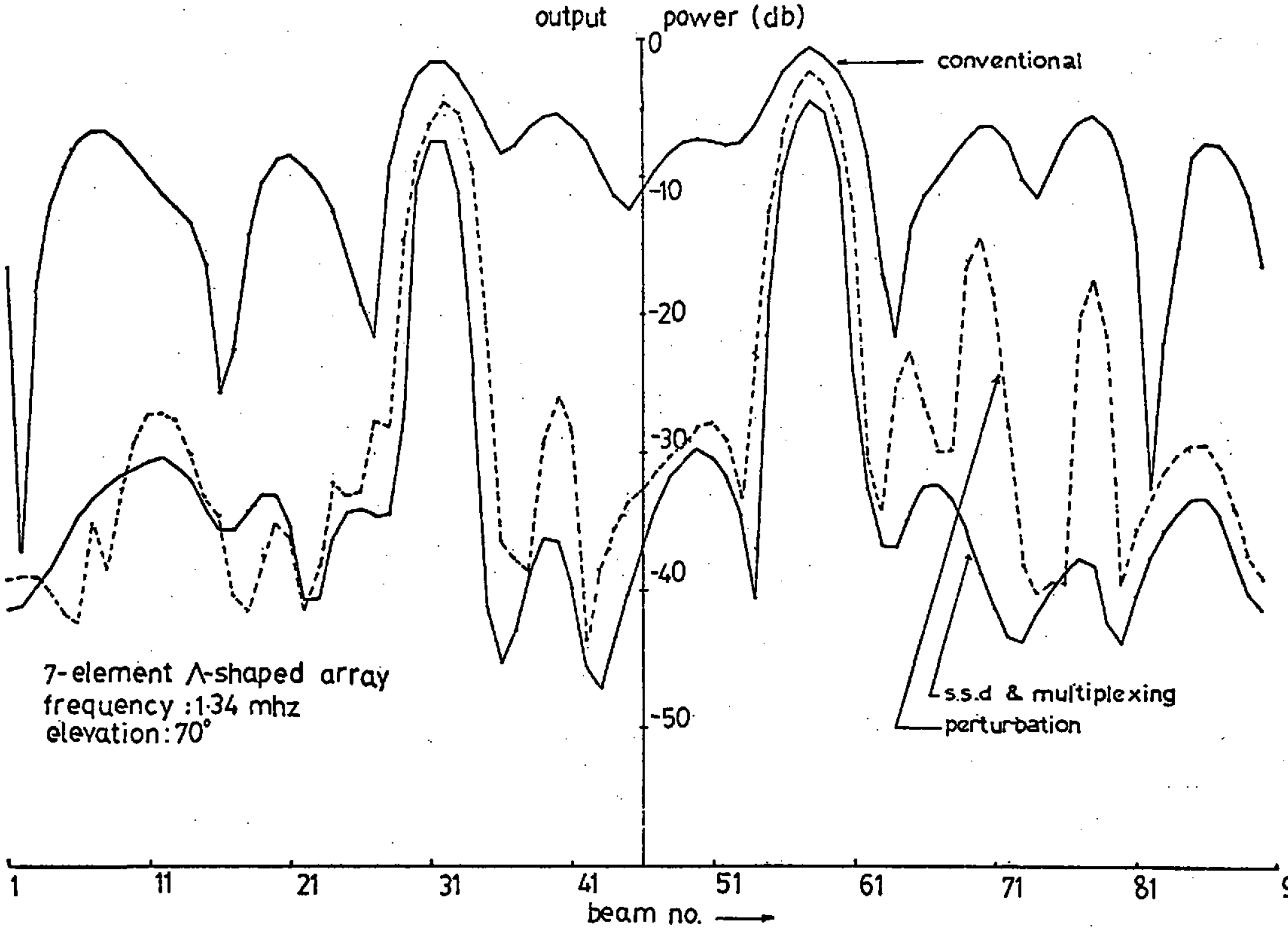


Fig. 9 Experimental Results