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DYNAMICS OF MILLING THIN-WALLED STRUCTURE

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In this paper, the multi pocket structure is regarded as a combination of Kirchhoff plates and the dynamics in milling the multi pocket structure is studied. The equations governing the vibration of the Kirchhoff plates are a set of forth-order partial differential equations with appropriate boundary values, which can be stated in a variational formulation according to the subdomain decomposition method in our former efforts. Continuity conditions between the interface require that the displacement and slope must be continuous, and these can be imposed by the Lagrange Multiplier and least squares weighted residual method. Then the partial differential equation can be transformed into the discretized from. The deduced governing equation is combined with a dynamic milling force model, taking the state-dependent cutting zone, multiple regenerative effect, loss of contact effect and the varying stiffness effect into consideration. The experimental results are used to test the validity of the model.
Keywords: Kirchhoff plates, thin wall milling, vibrations

1. Introduction

The milling dynamics has received considerable attention from researchers, literatures mostly concentrate on the stability prediction since the chatter is undesired vibrations needs to be avoided. The governing equation for milling process is usually the Delay Differential Equation with time delay. Altintas and Budak developed the frequency method for stability prediction[1], Insperger *et al*. developed the semi-discretization method[2] for the stability analysis of delayed system, Long[3] used the semi-discretization method to analysis the stability of milling process. These methods for the milling stability prediction mainly based on the static modal parameters and the stability is determined by the initial cutting parameters including the radial and axial cutting depth, the spindle speed, the teeth number, and so on. However, the milling process is a dynamic process, where the cutting zone is obviously a state-dependent parameter. Balachandran and Zhao[4] developed a mechanics based model to cover these dynamic effects, they[5] indicated that besides the regenerative effect, the loss of contact effect is another mechanism responsible for the milling dynamics, especially with the low radial immersions.

The traditional models usually considered only one vibration mode of the structure, where the governing equation is an Ordinary Differential Equation. For the milling of thin walled structure, the equation governing the vibration of the workpiece is usually the Partial Differential Equation with time delay. Due to the complex structural dynamics of the thin-walled workpiece[6], Finite Element Method is an alternative way to model the milling dynamics[7,8] with high computational cost.

The thin-walled multi pocket structure is typically used in the aerospace industry[9]. For the structural dynamics of this thin-walled workpiece, Meshreki *et al*. [9] described the plate dynamics in terms of the beam mode functions, where the continuity conditions require that the displacement, the rotation, the bending moment and the shear force must be continuously. In this paper, a dynamic model for the milling of pocket structures is developed based on the subdomain decomposition method in our former efforts[10]. The pocket structure is regarded as a combination of Kirchhoff plates. Each plate can be divided into a series of smaller subdomains advance. The continuity conditions between the interfaces require that displacement and slope must be continuous. This is guaranteed by the Lagrange Multiplier and the least squares weighted residual method. Then, the discretized form of the governing Equation for the pocket structure based on the Kirchhoff hypothesis can be obtained by integrating with various milling force models. In addition, the state-dependent cutting zone, multiple regenerative effect, and loss of contact based on the mechanics based model presented by Balachandran and Zhao[4] are also considered in the cutting force model. The reminder of this paper is organized as follows. In Sect. 2, the theoretical modelling is presented, including the subdomain decomposition model of the pocket and the state dependent milling force model. In Sect. 3, the comparison between the numerical simulation results and the experimental results are given. concluding remarks are given in Sect. 4.

2. Theoretical Modelling

A typical multi pocket structure is shown in Fig. 1(a), where the thin-walled pocket structure is regarded as a combination of plates by the appropriate boundary conditions. Fig. 1(b) is a schematic diagram of the milling process.

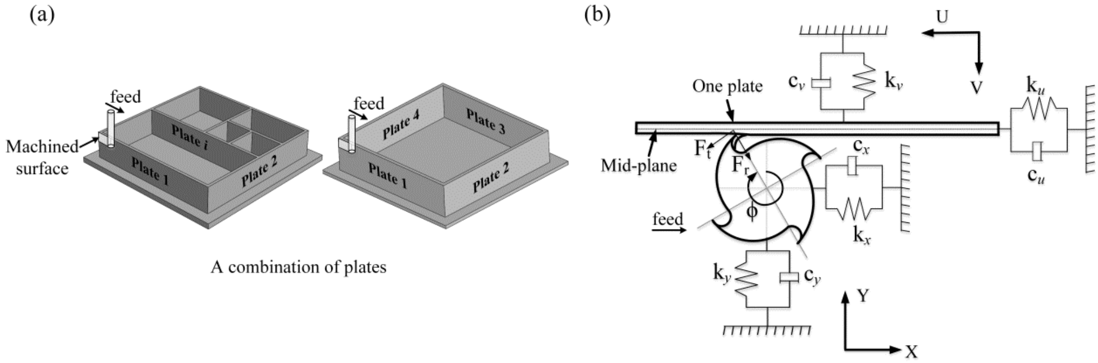


Fig. 1 Pocket milling (a) pocket structure; (b) the schematic diagram of milling process

Different from the traditional milling models, for the plate dynamics based on the Kirchhoff hypothesis, the governing equation for the milling process takes the following form

$$
\begin{aligned}
&m_x\ddot{q}_x + c_x\dot{q}_x + k_x q_x = F_x(t,\tau) \\
&m_y\ddot{q}_y + c_y\dot{q}_y + k_y q_y = F_y(t,\tau) \\
&m_u\ddot{q}_u + c_u\dot{q}_u + k_u q_u = F_u(t,\tau) \\
&D\left(\frac{\partial^4 q_v}{\partial \hat{x}^4} + 2\frac{\partial^4 q_v}{\partial \hat{x}^2 \partial \hat{y}^2} + \frac{\partial^4 q_v}{\partial \hat{y}^4}\right) + \rho h \frac{\partial^2 q_v}{\partial t^2} = F_v(t,\tau) + f(\hat{x},\hat{y},\dot{q}_v)
\end{aligned} \tag{1}
$$

where m_i, c_i, k_i, q_i $(i = x, y, u)$ is the modal mass, modal damping, modal stiffness and vibration displacement in *i* direction respectively. q_v is the vibration displacement in *v* direction. The last equation describes the motion of plate along the transverse direction with the excitation of cutting force. $F_x = F_u$, $F_y = F_v$ is the milling force in the corresponding direction.

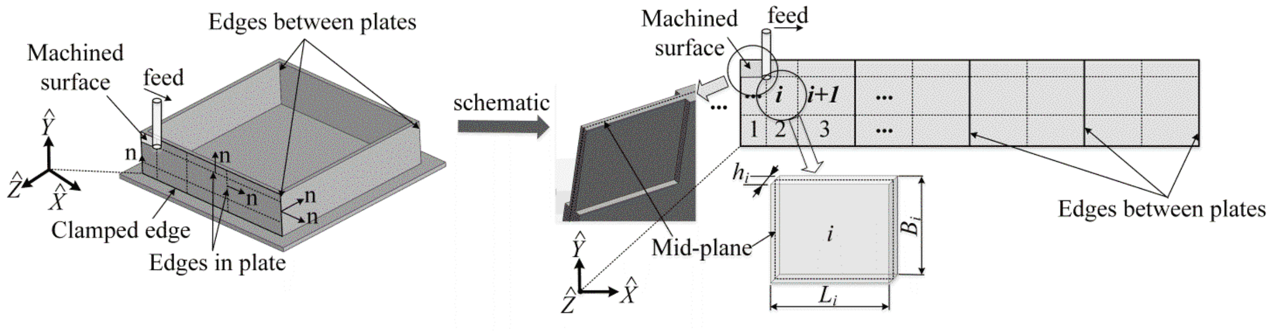


Fig. 2 the subdomain decomposition model

According to the subdomain decomposition method in our former efforts[10], the pocket structure can be regarded as a combination of Kirchhoff plates. the continuity conditions require that the displacement and slope must be continuous along the interface. There exist three kinds of interface In Fig. 2. Namely, the clamped edge, the edges between plates and the edges in plate.

For the pocket structure in Fig. 2, the modified variational principle relaxing the boundary conditions is

$$\Pi = \int_{t_0}^{t_1} \sum_i (T_i - U_i + W_i) dt + \int_{t_0}^{t_1} \sum_{i,i+1} \Pi_{\lambda\kappa} dt \tag{2}$$

where T_i and U_i is the kinematic energy and strain energy in the *i*-th subdomain respectively, W_i is the work done by external force. The second term of the right side of Eq. (2) comes from the modified variational principle and the least squares weighted residual method[10].

Since all the interface and boundary conditions in the pocket structure have been relaxed in Eq. (2), any complete basis of functions can be used as displacement function for the plate. We choose the Chebyshev orthogonal polynomials of the first kind as the admissible function for expanding the displacement field. By the similar procedures in[10], setting the generalized energy functional in Eq. (2) to be zero, the governing equation in Eq. (1) becomes

$$\begin{aligned}
& m_x\ddot{q}_x + c_x\dot{q}_x + k_x q_x = F_x(t,\tau) \\
& m_y\ddot{q}_y + c_y\dot{q}_y + k_y q_y = F_y(t,\tau) \\
& m_u\ddot{q}_u + c_u\dot{q}_u + k_u q_u = F_u(t,\tau) \\
& m_{v1}\ddot{q}_{v1} + c_{v1}\dot{q}_{v1} + k_{v1} q_{v1} = \Phi_1(\hat{x},\hat{y}) F_v(t,\tau) \\
& m_{v2}\ddot{q}_{v2} + c_{v2}\dot{q}_{v2} + k_{v2} q_{v2} = \Phi_2(\hat{x},\hat{y}) F_v(t,\tau) \\
& \cdots \\
& m_{vn}\ddot{q}_{vn} + c_{vn}\dot{q}_{vn} + k_{vn} q_{vn} = \Phi_n(\hat{x},\hat{y}) F_v(t,\tau)
\end{aligned} \tag{3}$$

where q_{vi} is the *i*-th mode coordinate. The governing equation in Eq. (1) has became the discretized form in Eq. (3). By considering different milling force models, Eq. (3) can be transformed into different forms to describe the thin walled pocket milling dynamics. We consider a state-dependent milling force model based on the research in[4].

By considering the multiple regenerative effect, the relative displacement in *x* and *y* direction is given as

$$\begin{aligned}
A(t) &= q_x(t) - q_x(t - l_x\tau) + q_u(t) - q_u(t - l_x\tau) + l_x f\tau \\
B(t) &= q_y(t) - q_y(t - l_y\tau) + q_v(t) - q_v(t - l_y\tau)
\end{aligned} \tag{4}$$

where $\tau = 60/(NR)$ is the tooth passing period, *N* the number of teeth, *R* the spindle speed (rpm). l_x and l_y denote the multiple regenerative effect, given by

$$
\begin{aligned}
&q_x\left(t-l_x\tau\right)-l_x f\tau+q_u\left(t-l_x\tau\right)=\max\begin{Bmatrix} q_x\left(t-\tau\right)-f\tau+q_u\left(t-\tau\right), \\ q_x\left(t-2\tau\right)-2f\tau+q_u\left(t-2\tau\right),\cdots \end{Bmatrix} \\
&q_y\left(t-l_y\tau\right)+q_v\left(t-l_y\tau\right)=\max\left\{q_y\left(t-\tau\right)+q_v\left(t-\tau\right),q_y\left(t-2\tau\right)+q_v\left(t-2\tau\right),\cdots\right\}
\end{aligned}
\tag{5}
$$

The relative displacement in Eq. (5) represent the state-dependent cutting zone. Therefore, the milling force can be determined as

$$
\begin{Bmatrix} F_x(t) \\ F_y(t) \end{Bmatrix}=\sum_{i=2}^{N}\begin{Bmatrix} F_x^i(t) \\ F_y^i(t) \end{Bmatrix}=\begin{bmatrix} \kappa_{11}(t) & \kappa_{12}(t) \\ \kappa_{21}(t) & \kappa_{22}(t) \end{bmatrix}\begin{Bmatrix} A(t) \\ B(t) \end{Bmatrix}
\tag{6}
$$

Substituting Eq. (6) into the discretized Eq. (3), one can obtain the governing equations of motion of milling processes. Numerical simulation can be carried to get structural responses. Noting that due to the state-dependent cutting zone in Eq. (4), the milling force in Eq. (6) must be recalculated at each integration time step during the time domain simulation.

3. Experimental Validation

In order to verify the proposed model for the dynamics of milling pocket structure, an experiment is set up, such as shown in Fig. 3. The initial dimensions (unit in mm) are shown in Fig. 3(a). The pocket was machined on a five-axes CNC milling machine with a three-fluted cutter.

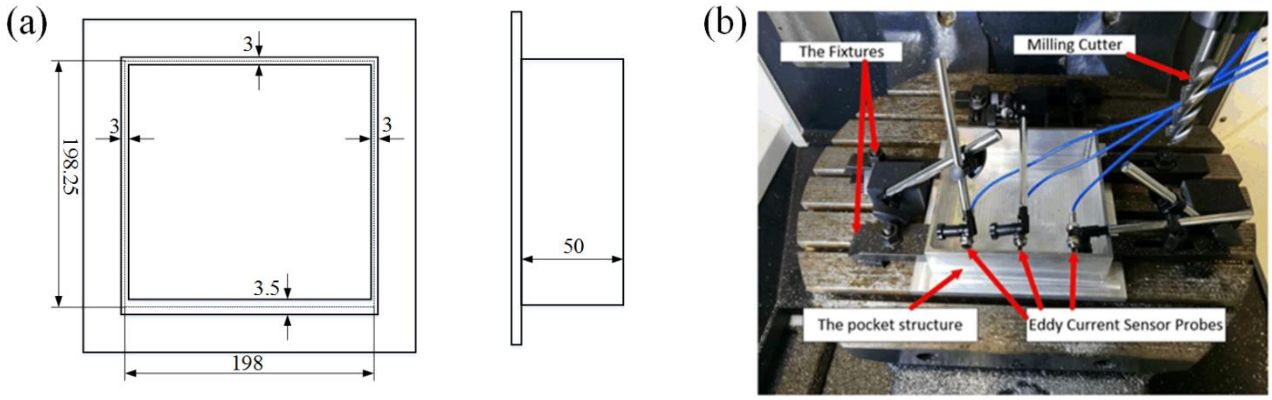


Fig. 3 (a) the pocket dimensions (b) experimental setup

To guarantee the clamped boundary conditions, the pocket is fixed to the moving table of the machining center. Three eddy current sensors are located along the direction of length of one side of the pocket structure at points A (40mm, 50mm), B (100mm, 50mm), and C (160mm, 50mm) to measure the displacements, as shown in Fig. 3(b). The pocket was machined from outside. An up milling operation with the spindle speed 4000rpm was conducted, with the radial depth of cut 0.5mm and axial depth of cut 2mm. The measured displacements by these three eddy current sensors are shown in Fig. 4.

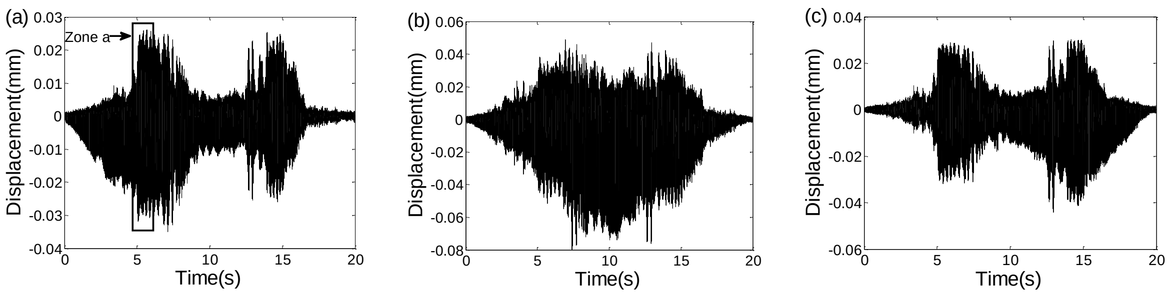


Fig. 4 Experimental Measurements (a) Sensor A (b) Sensor B (c) Sensor C

By the forth-order Runge-Kutta method, the numerical simulation results through the proposed model at these three points corresponding to the eddy current sensors are given in Fig. 5.

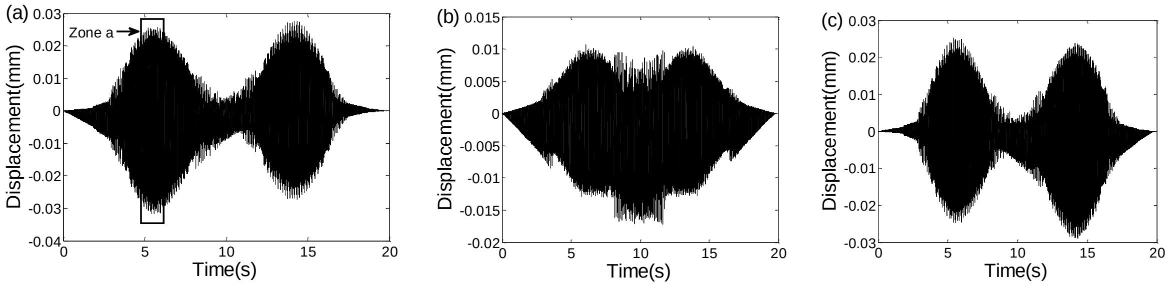


Fig. 5 Numerical Simulation (a) Point A (b) Point B (c) Point C

By these figures, one can find that both of the experimental and the simulation results indicate the amplitude of response of point A (40mm, 50mm) increase with the cutting tool moving and up to the maximum displacement when the cutting point near about the measured point first and then decrease with the tool moving to the middle point. However, when the cutting point near about the point C (160mm, 50mm), the amplitude of measured displacement up to a local maximum value. This may be because the excited mode shape is symmetrically about the middle point of structure. For the position B, the numerical results showed a relative smaller amplitude compared with the experimental results.

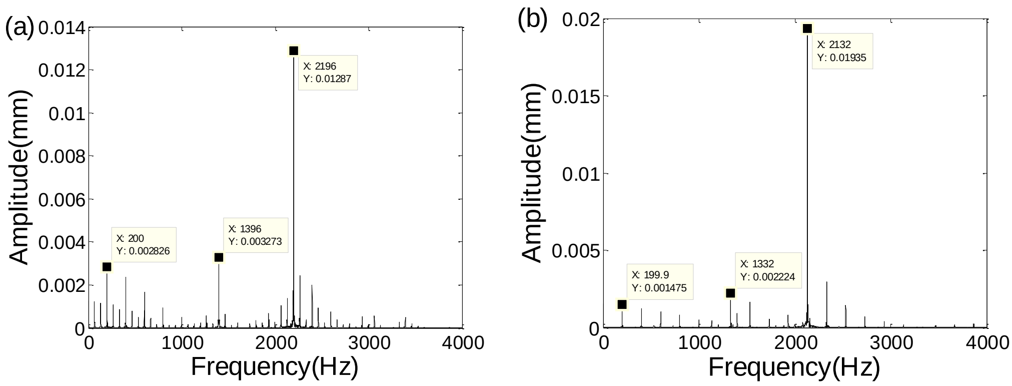


Fig. 6 The FFT spectrum in zone *a* (a) Experiment (b) Numerical Simulation

The frequency components in the time zone *a* for the sensor A in Fig. 4a and for the point A in Fig. 5a are given in Fig. 6a and 6b respectively. By Fig. 6, one can find both the experiment and the numerical results showed the typical tooth passing frequency 200Hz in the case of spindle speed 4000rpm, and the higher-order harmonics. However, the spindle rotation harmonics, which is shown in the results obtained by experiment, does not appear in the simulation results. This is because in the numerical simulation, the tool eccentricity and unbalanced excitation are ignored. Besides, there is slight difference on two chatter frequency components. In the experimental results, the two components are 1396Hz and 2196Hz, while the numerical ones are 1332Hz and 2132Hz. This may be due to the difference between the theoretical model and the experiment. The comparisons in both the time domain and frequency domain revealed that, the proposed theoretical model is valid to predict the vibrations of cutting, though there exist some difference between the results obtained by simulation and the results obtained by the experiment.

As explained in the aforementioned section, the dynamics in milling thin-walled structure is characterized by the multimode effect, this imply that the structural responses include the multiple mode components. For the closed shape structure, for example the pocket structure in our study or the circular shells [11], the higher modes may be closed to each other. The densely distributed eigen modes make the dynamics analysis more difficult. To investigate the contribution of each mode to the vibration displacement, considering the first eight modes, the displacement components by numerical simulation for point A are shown in Fig. 7. It is to be noted that the physical displacement equals the superposition of these modal components.

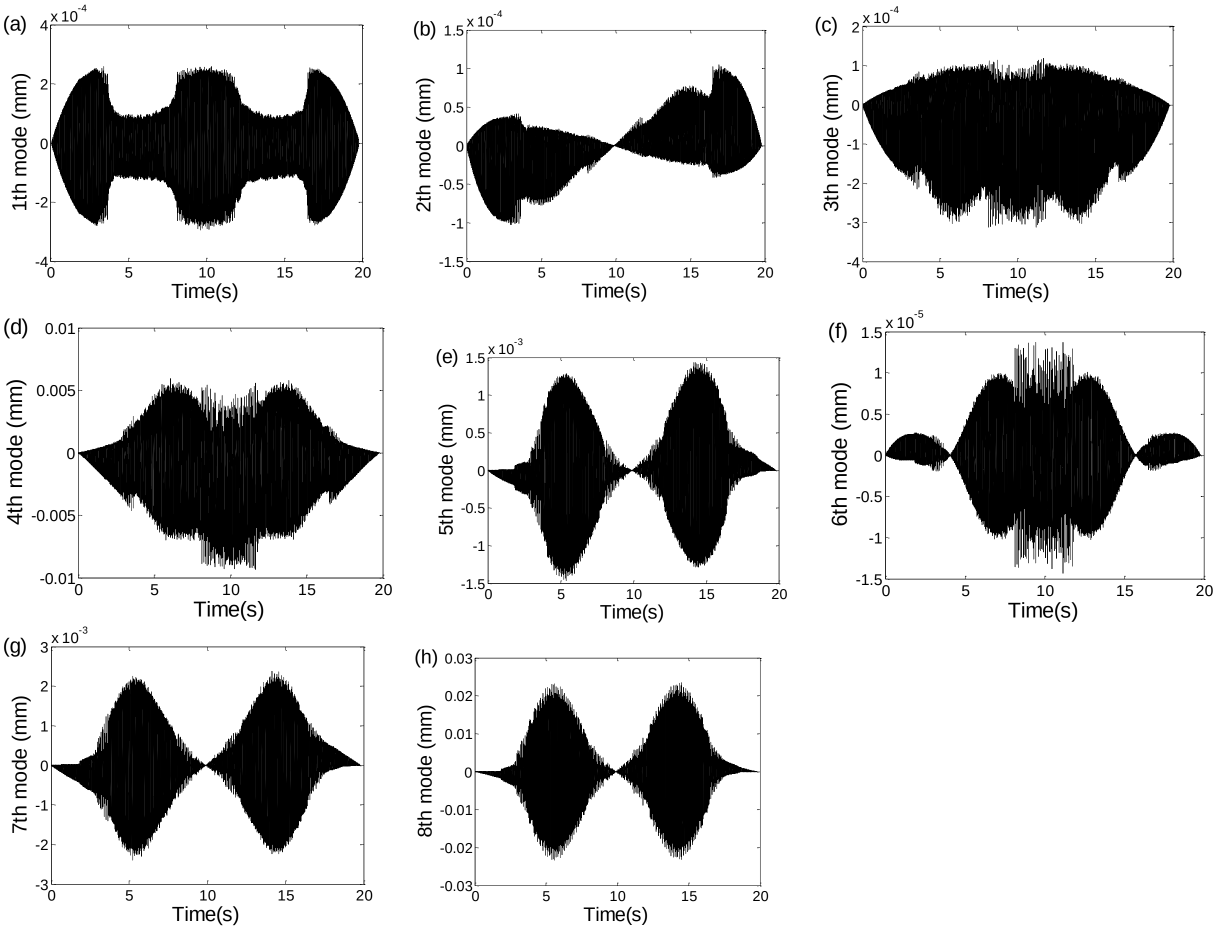
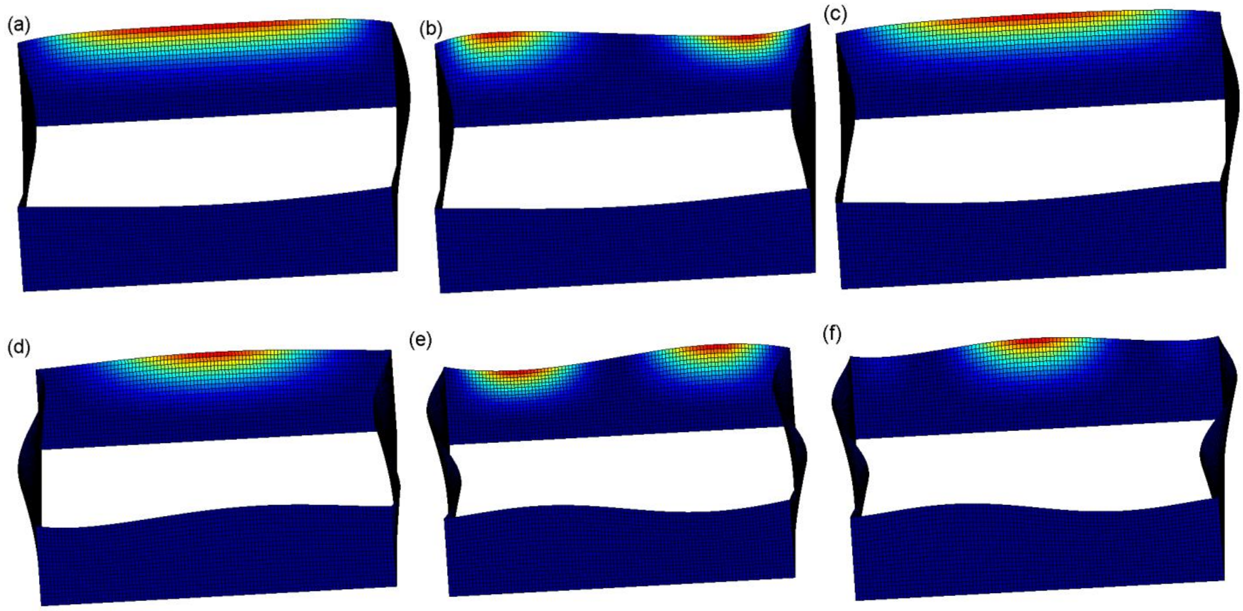


Fig. 7 The displacement components in each mode

By the order of magnitude, one can conclude that the forth and eighth mode are the dominate modes at point A. It is observed that the main trend of change about the vibration amplitude at point A and C was determined by the eighth mode. While the vibrations of point B is mainly determined by the forth mode. To understand the mechanisms of modal effects, the modal shapes obtained by the subdomain decomposition method are given in Fig. 8. Noting that to give a clearer view, only the machined plate is coloured and the figures are viewed in the back.



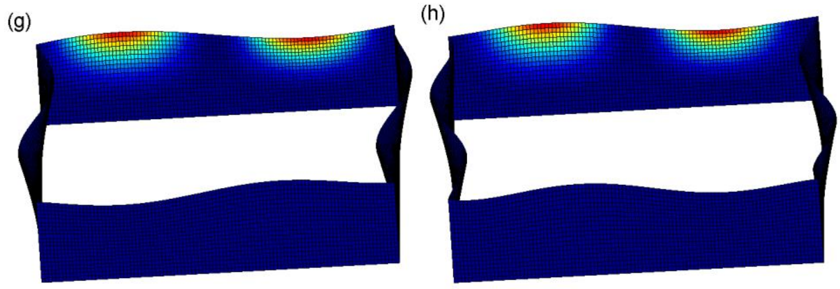


Fig. 8 The modal shape (a)~(h) the first mode to the eighth mode

Observing the eighth modal shape in Fig. 8h, along the cutting path, one can find the antinode, node, and antinode successively. This can explain that the point A and C shown the similar varying vibration amplitude. For the forth mode, there only exist an antinode at the midpoint, this can also explain that point B have the largest vibration amplitude at nearly 10s when the tool cutting at about the middle span.

The modal shapes in Fig. 8 also indicate that the displacement component of the forth mode at point A and C is in phase since no node exist between these two locations, while the displacement component of the eighth mode should be out of phase since there exist a node nearly at point B. The 4th and 8th modal displacements at point A and C are given in Fig. **9**.

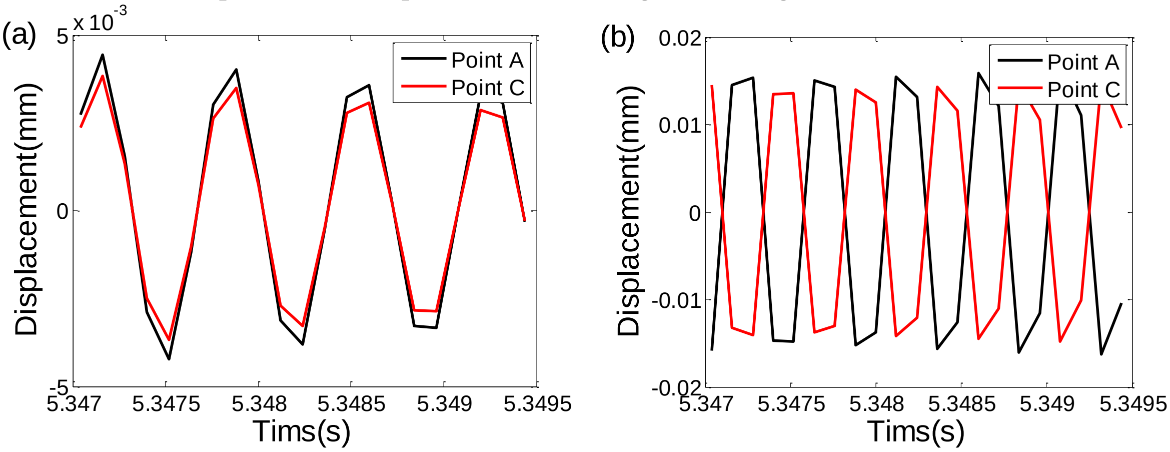


Fig. 9 Modal displacement components at Point A and C (a) the 4th mode (b) the 8th mode

One can conclude from Fig. 9 that the displacement components of the fourth mode at point A and C are in phase, while it are out of phase for the 8th mode, this is consistent with the modal shape in Fig. 8.

4. Conclusions

In this paper, the dynamics of milling the thin-walled pocket structures are investigated. The main conclusions are drawn as:

1. The subdomain decomposition method in our former efforts is used to study the milling dynamics in pocket structure. It regards the pocket as a combination of Kirchhoff plates. By considering the state-dependent milling force model, the numerical results consistent with experimental results in which a pocket with four plate is machined. Noting that even though the experiment only considered a pocket with four plates, this work can be easily extended to incorporate the structure with more plates as shown in Fig. 1(a)
2. Different from model presented in the traditional milling dynamics, which usually consider only one mode of the workpiece, the multimode effects in thin wall milling are important which should be taken into consideration.

3. During the finish machining operations for the thin-walled structure, there exist different dominate vibration modes at different positions. At the node of one mode, the corresponding modal contribution to the physical displacement is zero. For one mode, the modal displacement component at different positions may be in phase or out of phase.

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