HIGH-FREQUENCY DIGITAL TIME-DELAY BEAMFORMING FOR DEMODULATED BANDPASS SIGNALS

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1. INTRODUCTION

In this paper, it is shown that it is possible to realize arbitrary steering angles in high-frequency time-delay beamformers, although working with demodulated signals. First, the concept of "band-pass subsampling" for real and analytic signals is introduced. Then a short description of digital time-delay beamforming for linear antennas is given. The idea of reconstructing band-pass signals from subsampled data, using only a finite number of samples, is dealt with. Examples of computer simulations are shown to illustrate the power of this approach for digital beamsteering with little distortion of the antenna's ideal beam pattern.

2. DEMODULATION BY REAL OR ANALYTIC BANDPASS SUBSAMPLING

In sonar systems, especially in high-frequency active sonars, it is necessary to reduce the sampling rate from twice the complete bandwidth to a lower rate by some kind of demodulation. One method, which will be mentioned here, is bandshifting by "band-pass subsampling". It is based on the fact that if $f(t)$ is a real-valued band-pass signal with bandwidth $< B$ around the center frequency f_0, and if for an integer l the sampling frequency vs satisfies the relation

$$l\,vs \le 2 f_0 - B \le 2 f_0 + B \le (l+1)\,vs, \tag{2.1}$$

then vs is a valid subsampling frequency. This means that the entire information of the signal f is contained in its subsampled version f_{vs}; for details, see Fettweis [3], van Schooneveld [8]. In Fig.1, it is illustrated that in this case the periodically shifted spectra of f do not overlap. In particular, if by adequate system design, the maximum bandwidth B and center frequency f_0 can be related by

$$f_0 = 2\,B\left[k + \frac{1}{4}\right] \quad , \; k \in \mathbf{N} \tag{2.2}$$

then f can be sampled at the rate $vs = 2B$, which is the minimum possible sampling rate.

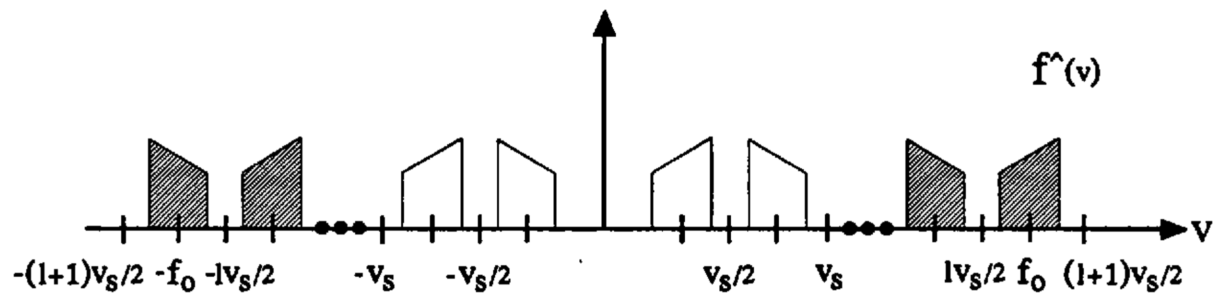

Fig. 1: f subsampled with rate vs

If it is desirable to work with complex signals, then the adjoint analytic signal samples fp_{vs} (fp is obtained by doubling the positive part of the spectrum of f and deleting the negative part, see Whalen [9]) can be generated by one of the following two methods:

-Band-pass subsampling of he high-frequency analytic signal fp at rate vs.

-Generating fp_{vs} from f_{vs} by a digital Hilbert-transformer (see Ries [7]).

Note that the analytic signal is sampled at twice the rate that would be sufficient to sample the complex envelope of f.

3. DIGITAL TIME-DELAY BEAMFORMING

In this section, a short description of digital time-delay beamforming is given. The discussion is restricted to one-dimensional linear antennas, although the methods presented in this paper apply for arbitrary sensor configurations.

To start with, time-delay beamforming means appropriately delaying and summing sensor signals. If the antenna is linear with N sensors spaced equidistantly, and if c is the speed of sound, Φ the angle of incidence, Θ the steering angle and d the distance between the sensors, then at sensor j the signal produced by an incident plane wave is

$$s(t - j\tau_{\Phi}), \text{ where } \tau_{\Phi} = (d/c)\sin(\Phi).$$

The ideal time-delay beamformer produces

$$b(t,\Phi,\Theta) = (1/\sum_{j=0}^{N-1} h_j) \sum_{j=0}^{N-1} h_j \, s(t - j\tau_\Phi + j\tau_\Theta), \tag{3.1}$$

where $\tau_\Theta = (d/c)\sin(\Theta)$ are the steering delays, and the h_j shading coefficients.
Now, in a digital realization, only the sample values $s(t - j\tau_\Phi + (k_j/vs))$ are available instead of $s(t - j\tau_\Phi + j\tau_\Theta)$. There is a time difference of

$$\tau_{\varepsilon j} = j\,\tau_\Theta - [\text{floor}\,(j\,\tau_\Theta\, vs)]/vs \tag{3.2}$$

between the ideal steering delay $j\,\tau_\Theta$ and its preceding sample point $k_j(\Theta)/vs$ (floor means nearest integer towards -infinity), see Fig. 2. Especially for subsampled signals, many phase cycles can occur in $\tau_{\varepsilon j}$, such that beamforming without taking some account of $\tau_{\varepsilon j}$ would be useless, since the high-frequency phase is necessary for correct beamforming. Therefore, some way must be found to recover the original signal value $s(t - j\tau_\Phi + j\tau_\Theta)$ from its sampled values around $k_j(\Theta)/vs$.

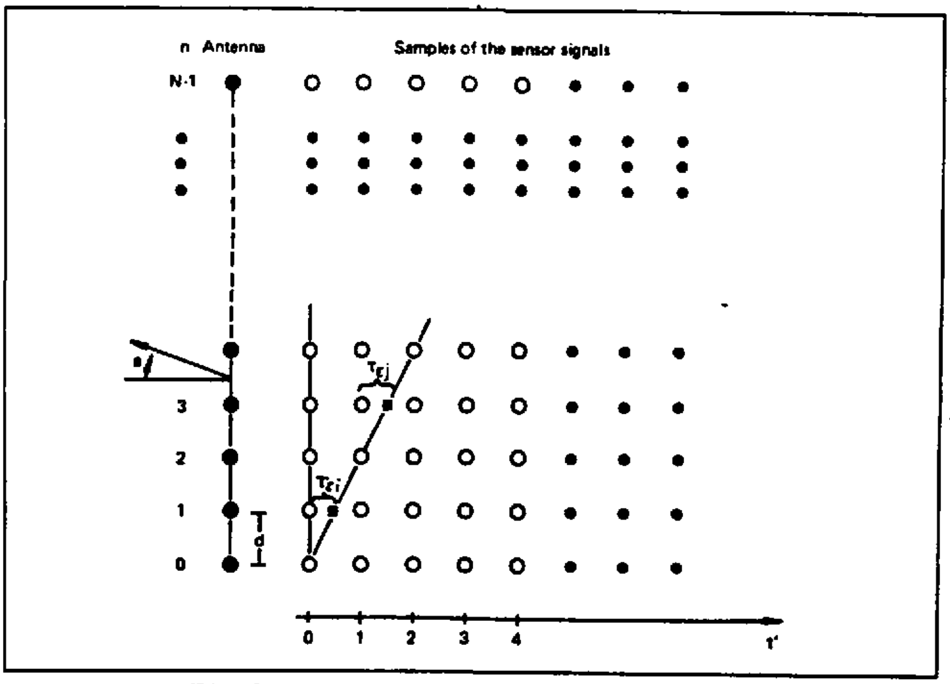

Fig. 2: Digital time delay beamformer samples

Some work that has been done in the past should be mentioned in this connection. Linear interpolation, giving good results in the low-pass case, has been investigated in Rathjen et al.[5]. Digital interpolation filters have received much attention; see for instance Pridham et al.[4]. Still, pure digital interpolation has the disadvantage that arbitrary time delays are not possible, leading to errors in the beam pattern, and the filters need a large number of coefficients to generate high-frequency samples. As an alternate way, a general method which gives good results with low implementation costs is presented in the next section.

4. RECONSTRUCTION OF REAL AND ANALYTIC BANDPASS SIGNALS

It is not so well known that error-free reconstruction of real and analytic band-pass signals from samples at a correct subsampling rate is possible by appropriate modifications of Shannon' sampling theorem, as

$$f(t) = \sum_{k=-\infty}^{+\infty} f\left[\frac{k}{vs}\right] \cos\left\{2\pi \frac{f_0}{vs} (vs\ t - k)\right\} \operatorname{sinc}\left\{\frac{1}{2}(vs\ t - k)\right\}, \tag{4.1}$$

in the real case (see Fettweis [3] and van Schooneveld [8]), and

$$fp(t) = \sum_{k=-\infty}^{+\infty} fp\left[\frac{k}{vs}\right] e^{-i\,2\pi\frac{f_0}{vs}(vs\,t-k)} \operatorname{sinc}(vs\ t - k) \tag{4.2}$$

in the analytic signal case (see Ries [7]). Using these formulae, the original high-frequency signal f could be reconstructed at arbitrary time-delays for beamforming. However, formulae of this type are known to be impractical, since the sinc-function decays very slowly, such that many terms of the series have to be evaluated for accurate reconstruction. Hence, it is interesting to replace the sinc-function by other, more appropriate functions, called "reconstruction kernels" with better properties. A direct approach is to look for reconstruction kernels that are time-limited, and so only a finite number of samples have to be considered.

Such kernels have been derived in Engels et al. [2], Ries [6] for the low-pass case and Ries [7] for the band-pass case. Since the underlying theory is too lengthy to be presented here, only two important properties of these kernels are mentioned here:

- Their Fourier transform is a Hermite-type interpolation of the rect function at all integers ± k.

- The reconstruction is error-free for CW-signals with frequency f_o.

A good choice is to take reconstruction kernels based on B-Splines, because they are easy to implement. B-splines of order n are piecewise polynomials, defined by

$$B_n(v) = \int_{-\infty}^{+\infty} \left[\frac{\sin(v/2)}{v/2}\right]^n e^{ivx}\, dv \tag{4.3}$$

or, equivalently, the n-fold convolution of the rect function with itself; see Engels et al. [2] and Ries [6] for details.

As typical examples, the kernels

$$R_1 := B_1,\ R_2 := B_2 \text{ and } R_3 := 4B_3 - 3\,B_4\,,$$

needing 1, 2 and 4 sample points, respectively, will be inserted in the reconstruction formula instead of the sinc function. In the analytic signal case, the R_1 and R_2 reconstruction have the following interpretation from their low-pass meaning:

-R_1 reconstruction is phase compensation with respect to center frequency ("sample and hold" in the low-pass case).

-R_2 reconstruction is linear interpolation with phase compensation with respect to center frequency.

In the next section, some typical applications are shown.

5. APPLICATIONS TO DIGITAL TIME-DELAY BEAMFORMING

Examples of applications of the methods mentioned are shown here for a linear antenna with 65 sensors, 1000 Hz sampling frequency and a frequency band of 1250 ± 200 Hz to be considered. There is a Dolph-Chebysheff shading, aiming at a sidelobe reduction to -25 dB. The signals arrive at an angle of 90° , and the steering angle is varied from -90° to +90°. Beam pattern for the center frequency of 1250 Hz are not shown, because they are without distortion for all three methods.

In the Figures 3A to 7B, $|b(t,\Phi,\Theta)|^2$ is shown for CW-signals of frequencies 1050 Hz and 1150

Hz, respectively, and for different complex or real interpolation methods to show the influence of interpolation errors. It is noted that the so called "Pseudo Grating Lobes (PGLs)", studied in detail in Bödecker et al. [1] for linear lowpass interpolation, are also present here, since they are inherent in any non ideal interpolation scheme. Furthermore, the less costly methods based on R_1 and R_2 give acceptable results for a deviation of 100 Hz from the center frequency, but a deviation of 200 Hz leads to severe degradation of the beam pattern and destroys the desired effect of the antenna's shading. However, the superior (but more costly) R_3 method reduces the PGLs below the significant level for all these frequencies and allows Dolph-Chebysheff shading up to -25 dB in our example.

Last but not least, in Figs. 6A and B as well as 7A and B, the real band-pass formula based on R_1 or R_2, respectively, is shown, which is a digital time-delay beamformer with purely real processing of subsampled data.

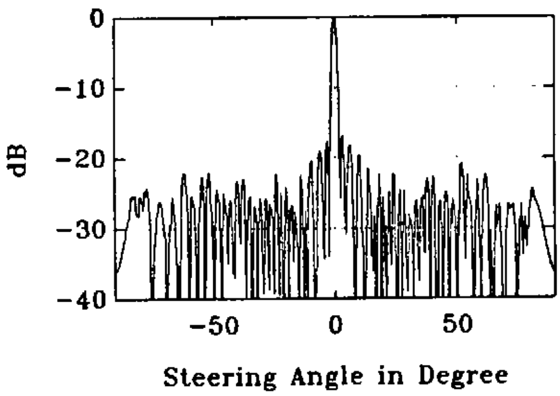

Fig. 3A: R_1 complex, CW 1150 Hz

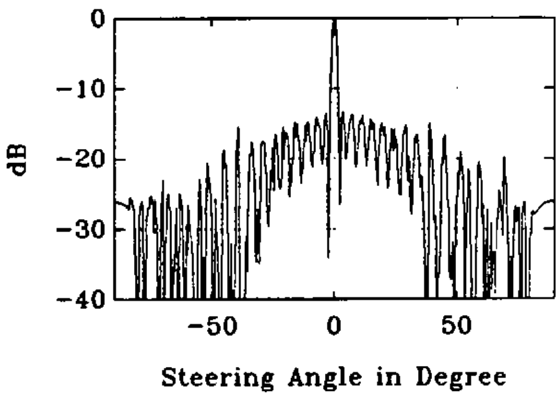

Fig. 3B: R_1 complex, CW 1050 Hz

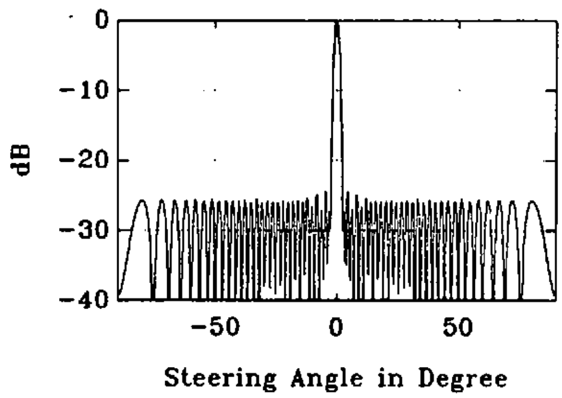

Fig. 4A: R_2 complex, CW 1150 Hz

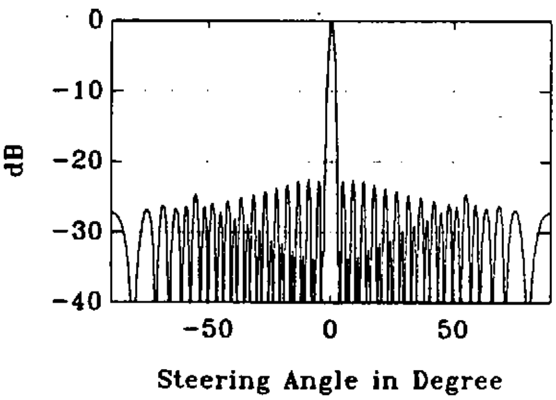

Fig. 4B: R_2 complex, CW 1050 Hz

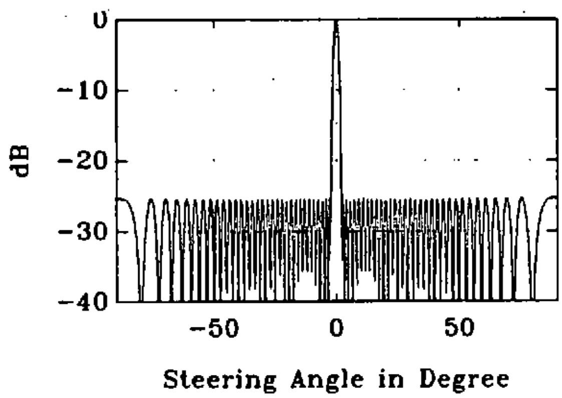

Fig. 5A: R_3 complex, CW 1150 Hz

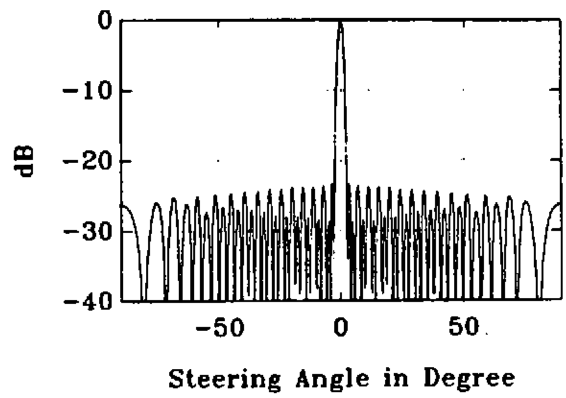

Fig. 5B: R_3 complex, CW 1050 Hz

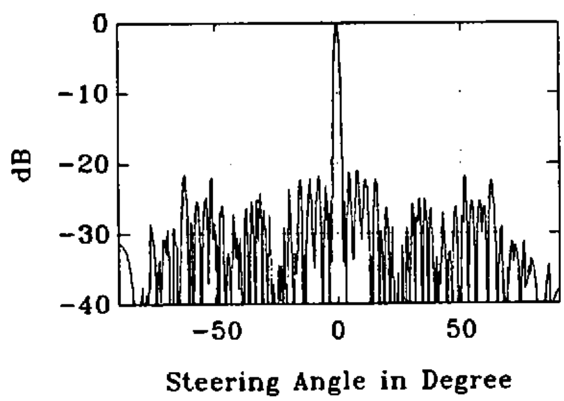

Fig. 6A: R_1 real, CW 1150 Hz

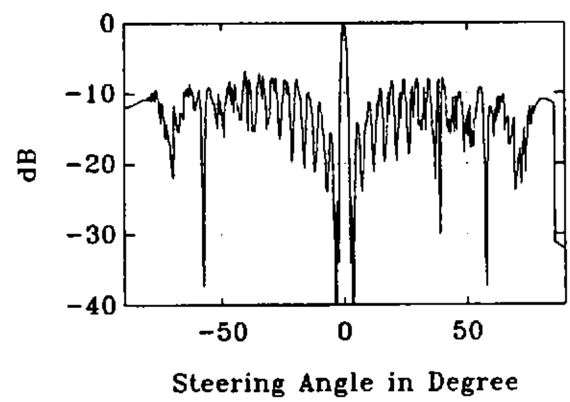

Fig. 6B: R_1 real, CW 1050 Hz

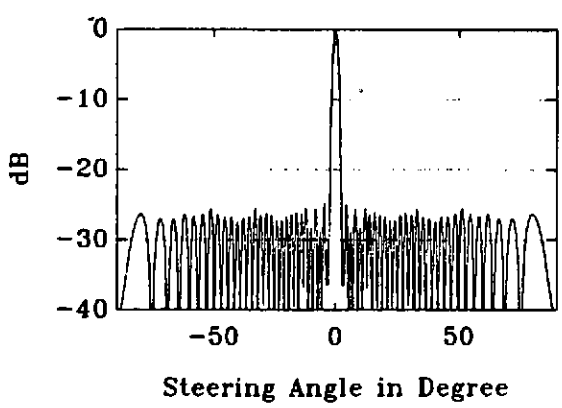

Fig. 7A: R_2 real, CW 1150 Hz

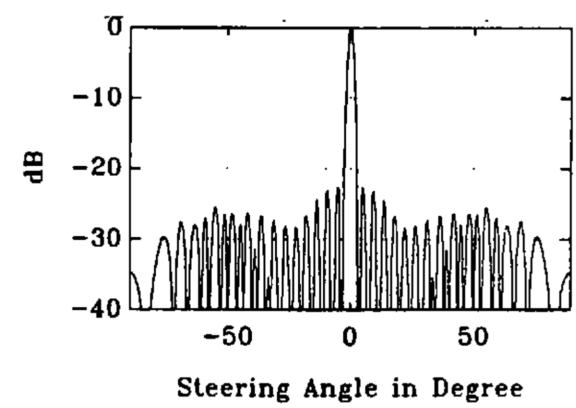

Fig. 7B: R_2 real, CW 1050 Hz

6. CONCLUSION

After a short introduction to demodulation by "bandpass subsampling" and to digital time-delay beamforming, the application of a signal reconstruction method for beamsteering is dealt with. The simulation examples show that these methods give good results with low implementation cost.

7 REFERENCES

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