Proceedings of The Institute of Acoustics

INVESTIGATION OF EFFECTS OF OBJECT SUPPORT ESTIMATE ON MICROWAVE PHASE RETRIEVAL BASED ON PLANE TO PLANE DIFFRACTION

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1. INTRODUCTION

The phase retrieval problem arises in microwave imaging applications where the phase is either lost or impractical to measure. In the majority of electromagnetic scattering problems, the object size or support will be known, a priori. In some circumstances however the support of an object may not be defined by a simple geometrical shape or the support may not be known at all. Such cases of phase retrieval may have arisen in the original fields of optics, electron microscopy and crystallography. However they will occur in microwave applications, for example, single view imaging of concealed objects and diffraction tomography.

The complex nature of the ambiguity problem between the phase and the modulus of the Fourier transform implies that the direct analytical solution to this problem is not possible. In certain one-dimensional scattering problems the ambiguity between transform phase and modulus may be eliminated. This situation may also be the case for some two dimensional scattering problems, but generally speaking, these cases are very restricted and not practically relevant. For these reasons, iterative techniques which rely upon the availability of additional information in the form of a second intensity measurement have become attractive alternatives. Two commonly known iterative techniques (Gerchberg and Saxton, and Misell) have already been studied by the authors on data obtained from microwave antenna measurements [1]. It was found that the data requirements from the Gerchberg and Saxton algorithm was not only impractical but that the results obtained were not satisfactory. The Misell algorithm was, on the other hand, much superior but its use is again practically limited to a few microwave diagnostic situations. To eliminate the severe limitations of these two algorithms the new technique, which is particularly relevant to the wavelength regime of microwave diagnostics, was introduced. Studies on the convergence as well as the practicality of the algorithm were found to be promising. In this present paper the dependence of this algorithm on the object support has been investigated using both simulated and measured data. The extension of this approach to tomographic imaging has also been investigated.

2. THE EFFECT OF OBJECT SUPPORT ON THE CONVERGENCE OF THE PLANE-TO-PLANE DIFFRACTION ALGORITHM

The plane-to-plane diffraction algorithm given in Appendix 1 involves the spectral propagation of a complex estimate formed at the object domain back and forth between two arbitrary data planes. The algorithm does not only eliminate the need for the measurement at the object domain, but offers versatility in the selection of the two planes anywhere in the near-field, which means that far-field measurements are not essential.

A simple proof of the convergence of the algorithm based on the reduction of error energy is given in the Appendix 2 and it is seen that the object domain constraints are an important feature of the algorithm. Therefore studies of the performance of the algorithm are interesting when the object support is either not known at all or only a crude estimate of the object support is available. The results of some numerical studies for these cases are presented in section 4.

3. APPLICATION OF THE PLANE-TO-PLANE DIFFRACTION ALGORITHM TO MICROWAVE TOMOGRAPHY

The investigations reported hitherto have been directed towards the reconstruction of effectively planar objects, e.g. an antenna aperture. Consider now a typical diffraction tomographic measurement of an object which is assumed to be cylindrical and hence the field along its axis is ideally invariant. The scattered field measurement at line ℓ_o in rotated axis is given by [2]

$$\tilde{u}(\alpha;\ell_o) = \frac{j}{2\sqrt{k_o^2-\alpha^2}} e^{\sqrt{j\, k_o^2-\alpha^2\ell_o}}\, \tilde{O}(\alpha,\sqrt{k_o^2-\alpha^2} - k_o) \qquad \text{for } |\alpha| < k_o \tag{1}$$

Equation (1) is the mathematical expression for the Fourier diffraction theorem and relates the two dimensional Fourier transform of the object $\tilde{O}(\alpha,\sqrt{k_o^2-\alpha^2} - k_o)$ to the one dimensional Fourier transform of the scattered field $\tilde{u}(\alpha;\ell_o)$ at the scan line. Note that the two dimensional Fourier transform of the object is confined to the spatial frequencies, α, within a semi-circular arc at a distance $\sqrt{2}\, k_o$ from the origin at the frequency domain.

Equation (1) can be employed with the plane-to-plane diffraction algorithm for retrieving the phase from two sets of one-dimensional modulus data taken at arbitrary measurement planes for the number of views considered. Having obtained these data for each view angle at two arbitrary measurement planes, the algorithm starts with a complex initial trial function and involves the following steps.

i. Reduce the 2D complex object estimate to 1D and propagate the first data plane for every view according to equation (1)

ii. Apply the constraint and back propagate the resulting estimate back to the object domain from each measurement plane for every view

iii. Reconstruct the new two dimensional object estimate by using either space [3] or frequency domain [2] techniques

iv. Constrain the new two dimensional complex object estimate

v. Go back to the first step and repeat the process for the second measurement plane

This process is repeated until a satisfactory convergence is obtained. Note that in reducing the two dimensional object to one dimension the largest object constraint is retained. For this reason the algorithm can be shortened significantly since the 1D instead of full 2D object support is used. Although such a modification to the algorithm is ill-defined in that the phase of each

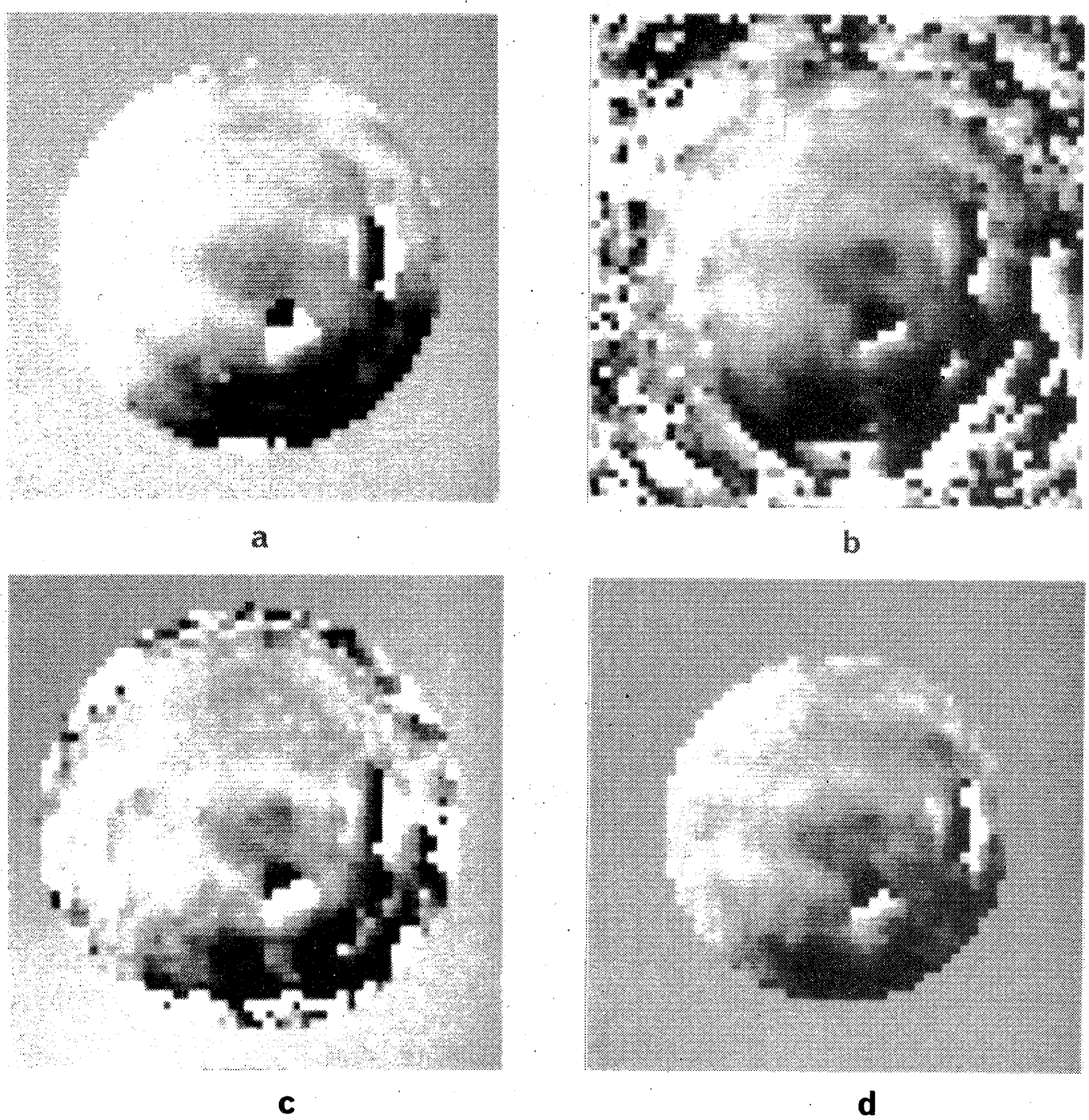

Figure 1 :

(a) : Aperture phase distribution reconstructed to 2.5% accuracy by using the plane-to-plane diffraction algorithm with exact object size support after 20 iterations.

(b) : Amplitude only reconstructed phase distribution by using plane-to-plane diffraction algorithm with no size constraint after 20 iterations.

(c) : Amplitude only reconstructed phase distribution with 1.2 times the true aperture size used as the constraint.

(d) : Amplitude only reconstructed phase distribution with 0.8 of the true aperture size used as the constraint.

projection must now be recovered in 1D for every view, some satisfactory results have been obtained.

4. RESULTS OF SUPPORT SIZE VARIATION ON RECONSTRUCTION OF ANTENNA APERTURE PHASE

To demonstrate the performance of the plane-to-plane diffraction algorithm against the support, the aperture phase distribution consisting of simulated phase aberrations representing Gaussian type surface profile deformities superimposed on a background phase is used. The measured data were obtained from holographic radiation pattern measurements performed at a frequency of 10GHz on a 3.66m reflector antenna [1]. Using the modulus information at two measurement planes at 20m and 150m away from the antenna, the results in Figure 1b, 1c and 1d were obtained. Further numerical studies have also been carried out to investigate the performances of two other iterative algorithms when the precise support of the object is not known, and these were not successful.

5. PRELIMINARY INVESTIGATION OF MODULUS ONLY TOMOGRAPHIC RECONSTRUCTION

An interesting application of phase retrieval techniques may arise in tomography. Presently, the plane-to-plane algorithm is the only one which could be applied practically to reconstruct 3D objects from 2D scans.

Shown in figure 2 are the reconstructed phase and amplitude of a coeccentric dielectric cylinder of 10 wavelengths radius from 36 views of complex scattered field at scans 15 wavelengths away from the centre of the cylinder. The fields are computed from the exact analytical expressions and used in the reconstruction process [4]. Figure 3 shows the reconstructed cylinder from modulus only data at measurement planes 12 and 20 wavelengths from the centre of the cylinder by using the modified plane-to-plane diffraction algorithm described in section 3 for 60 views. Although only a simple largest object support (i.e. diameter) is used to constrain each estimate at the object domain, satisfactory reconstruction is obtained except for the singularity in the centre portion of the image which is an artefact of the band limitation of the propagation filter.

6. REFERENCES

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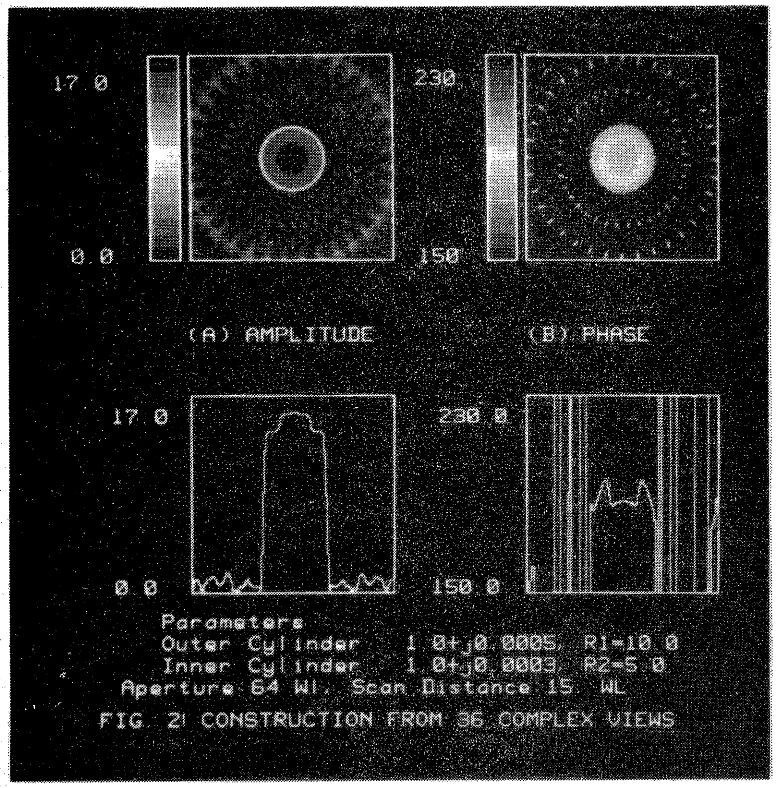

FIG. 2: CONSTRUCTION FROM 36 COMPLEX VIEWS

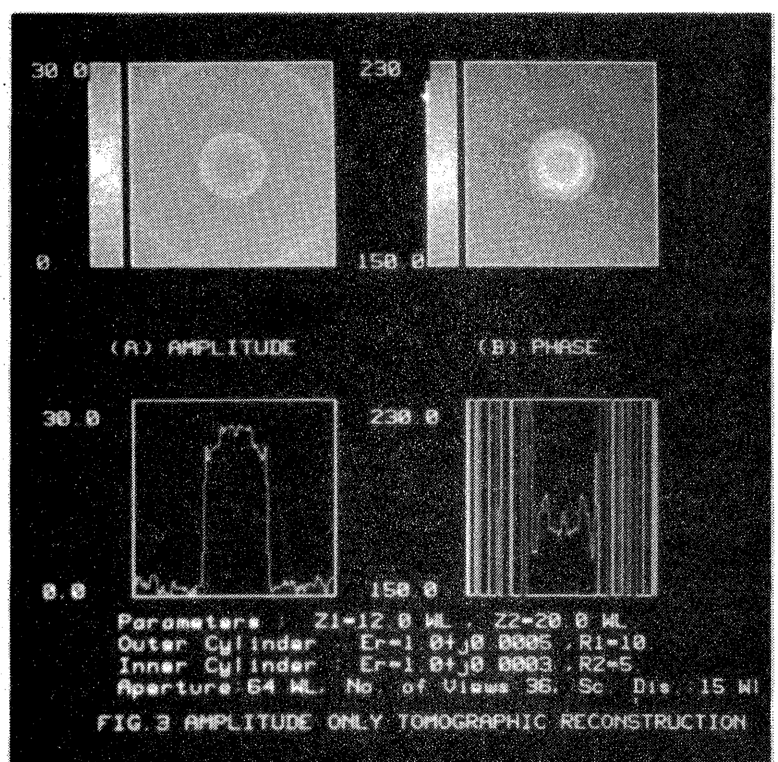

FIG.3 AMPLITUDE ONLY TOMOGRAPHIC RECONSTRUCTION

Iterations return to step 1 and continue until a satisfactory convergence is obtained. In equations above, H_{01}; H_{10} and H_{02};H_{20} are the forward and back-propagation filters for the associated distances between the object and first and second measurement planes respectively. They are given as :

$$H_{01} = \exp(jk|z_1|m)$$

$$H_{02} = \exp(jk|z_2|m)$$

$$m = \sqrt{1 - (\lambda sx)^2 - (\lambda sy)^2}$$

where λ = free space wavelength
k = free space propagation constant
$sx;sy$: fourier domain variables

A2. <u>A proof of convergence based on the error reduction</u>

The convergence of the algorithm may be monitored by computing the squared error. Based on the additional constraint provided by the object support it may be proved that the error energy during the successive iterations is reduced.

For the k'th iteration the error energy at the object domain is given by :

$$E^2_{k_0} = \sum_{z_0} |g'_k(z_0)|^2$$

From step i the error energy at the first measurement plane for the two successive iterations are

$$E^2_{k_1} = \sum_{z_1} |g_k(z_1) - g'_k(z_1)|^2$$

$$E^2_{(k+1)1} = \sum_{z_1} |g_{k+1}(z_1) - g'_k(z_1)|^2$$

By definition $g_{k+1}(z_1)$ is nearer to the complex function whose modulus is measured as it represents the propagation of the constrained and improved estimate of the complex object function. Thus

$$E^2_{(k+1)1} \leqq E^2_{k_1}$$

From step ii the error energy at the object domain

$$E^2_{(k+1)0} = \sum_{z_0} |g_{k+1}(z_0)|^2$$

It is clear that

$$E^2_{(k+1)0} \leqq E^2_{k_0}$$

From step iii the error energy at the second measurement planes for the same iterations are

APPENDIX

A1. Steps of the Algorithm

The algorithm consists of the following steps for the k'th iteration.

Step i : propagate the k'th estimate to first measurement plane at $z=z_1$

$$g'_k(z_1) = FT^{-1}\{FT\{g'_k(z_o)\}\ H_{01}\} = |g'_k(z_1)|\exp(j\eta(z_1))$$

and constrain the new estimate with measured modulus $|F(z_1)|$ in the domain B of the first measurement plane.

$$g_k(z_1) = |F(z_1)|\exp(j\eta(z_1)) \qquad \text{if } x_1;y_1\varepsilon B$$

$$g_k(z_1) = 0 \qquad \text{if } x_1;y_1\notin B$$

Step ii : back propagate the new estimate of the first measurement plane to the object domain

$$g'_k(z_o) = FT^{-1}\{FT\{g_k(z_1)\}\ H_{10}\}$$

and constrain to the object domain A

$$g_k(z_o) = g_k'(z_o) \qquad \text{if } x_o;y_o\varepsilon A$$

$$g_k(z_o) = 0 \qquad \text{if } x_o;y_o\notin A$$

Step iii : propagate the new object estimate to second measurement plane at $z=z_2$

$$g'_k(z_2) = FT^{-1}\{FT\{g_k(z_o)\}.H_{02}\} = |g'_k(z_2)|\exp(j\eta(z_2))$$

and constrain the new estimate

$$g_k(z_2) = |F(z_2)|\exp(j\eta(z_2)) \qquad \text{if } x_2;y_2\varepsilon B$$

$$g_k(z_2) = 0 \qquad \text{if } x_2;y_2\notin B$$

Step iv : back propagate the new estimate from second measurement plane to object domain

$$g'_{k+1}(z_o) = FT^{-1}\{FT\{g_k(z_2)\}.H_{02}\}$$

and constrain

$$g_{k+1}(z_o) = g'_{k+1}(z_o) \qquad \text{if } x_o;y_o;\varepsilon A$$

$$g_{k+1}(z_o) = 0 \qquad \text{if } x_o;y_o;\notin A$$

$$E^2_{k_2} = \sum_{z_2} |g_k(z_2) - g'_k(z_2)|^2$$

$$E^2_{(k+1)2} = \sum_{z_2} |g_{k+1}(z_2) - g'_k(z_2)|^2$$

As in the first plane, by definition, $g_{k+1}(z_2)$ is nearer to complex scattered field whose modulus is measured. Thus

$$E^2_{(k+1)2} \leqq E^2_{k_2}$$

It may be proved that

$$E^2_{(k+1)2} \leqq E^2_{(k+1)1} \leqq E^2_{k_2} \leqq E^2_{k_1}$$

which is the consequence of the fact that the error energy is reduced from one estimate to the next as propagation from first measurement plane to the second measurement plane is via the object functions for which the successive estimates are improved with proper object domain constraints. After (k+1)st iteration the error energy at the object domain

$$E^2_{(k+2)0} = \sum_{z_o} |g_{k+1}(z_o)|^2$$

which is nearer to the complex object function. Thus

$$E^2_{k_o} \geqq E^2_{(k+1)0} \geqq E^2_{(k+2)0}$$

which means the error energy can only decrease or remains the same at each iteration.