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DETECTION OF BOILING LIQUID LEVELS BY DECOMPOSITION OF ACOUSTIC WAVES MEASURED USING A WAVEGUIDE

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In industrial systems at many instances processes require knowledge of the level of hot bubbling liquid in a closed vessel. However, many a times conventional instrumentation is unsuitable since the temperatures are extremely high, and/or the chemical processes generate poisonous gases. This paper proposes a method to detect such boiling liquid levels by remotely monitoring the boiling/bubbling noise in the vessel using a waveguide. The principle of this method is that the resonance of the air mass in the closed vessel would change with change in the liquid height, which would change the resonant frequencies of the waves generated in the vessel. These waves could be captured by decomposition in a waveguide coaxially attached to the vessel, and the resultant incident wave spectrum would show peaks at frequencies as a function of the liquid height. The proposed method was verified through finite element simulations and experiments of boiling water in a closed cylindrical vessel to which a waveguide was coaxially attached at the top. Results show that different boiling water levels resulted in different resonant frequencies of the decomposed incident wave corresponding to the water height. Thus, this waveguide system can remotely monitor boiling liquid levels based on the incident wave frequency and amplitude. Such a system has wide industrial applications, particularly in steel making industry where knowing the amount of molten steel during oxygen lancing faces many challenges.
Keywords: waveguide, wave decomposition, resonance, boiling liquid

1. Introduction

In industrial systems many processes require the knowledge of the level of hot bubbling liquids in closed vessels. For example, in steel plants, the process of oxygen lancing requires the knowledge of the molten steel volume. However, the temperature in and around the vessel is extremely high and poisonous gases are generated that render the conventional instrumentation unsuitable to measure the steel volume [1-2]. Similarly, in nuclear power plants, it is challenging to monitor volumes of boiling liquid metals in fast breeder reactors due to similar reasons [3-5]. This paper introduces a technique to detect such boiling liquid levels by remotely monitoring the bubbling noise of the boiling liquid through a waveguide connected to the vessel. The principle behind this technique is that a change in the boiling liquid heights will change the length of the air mass in the closed vessel, which in turn will change its resonant frequencies. Therefore, a noise source in the vessel such as a harmonic excitation or the bubbling noise of the boiling liquid will produce acoustic waves having resonant frequencies dependent on the liquid level. A waveguide attached along the axis of the vessel can capture these waves whose frequency and amplitude would depend upon the liquid level in the vessel. The following sections discuss the theory of this method and test it through finite element simulations and experiments.

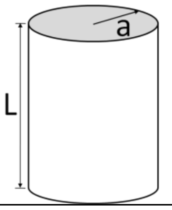
2. Analytical solution

The analytical model for boiling liquid height detection has a prerequisite that the liquid is measured during the phase of nucleate boiling when there is significant bubbling within the liquid. This bubbling/boiling noise acts as the sound source within the vessel. It is also assumed that at the time of measurement the boiling liquid and the gaseous vapours within the vessel have attained thermal equilibrium. The theoretical development of this method is discussed in details below:

2.1 Resonant frequencies of a cylindrical vessel containing hot bubbling liquid

Let ‘L’, ‘a’ be the length and radius of a cylindrical vessel containing boiling liquid at height h from the vessel base. Let c_L and c_G be the speed of sound in the liquid and the gas media of the vessel respectively, and ρ_L and ρ_G be their specific densities. Since, the liquid and the gaseous vapours within the vessel are at thermal equilibrium, they would be at approximately the saturation temperature of the boiling liquid. Therefore, c_L and c_G are the speed of sound for the gas and liquid column at the liquid’s saturation temperature. The closed cylindrical vessel when empty or completely filled with the liquid is acoustically equivalent to a rigid-walled both end closed cylindrical duct with speed of sound as c_G or c_L respectively. When the vessel is partially filled, the vessel’s liquid column is acoustically equivalent to a rigid-walled one end open and one end closed cylindrical duct with speed of sound as c_L. Now, considering that $\rho_L >> \rho_G$, the vessel’s gas column is acoustically equivalent to a rigid-walled both end closed cylindrical duct having speed of sound as c_G. A cylindrical duct has l,m,n resonant modes corresponding to the axial, radial, and circumferential directions. In case of the vessel, as the length of the liquid column changes, the axial modes corresponding to the liquid and gas columns change but the radial and circumferential modes remains constant. Therefore, only axial modes are indicators of the changing liquid height and only these modes are discussed. The frequencies of the axial modes of a cylindrical duct are given in Table 1 [6-7].

Table 1: Natural frequencies of a cylindrical duct

Configuration of cylindrical pipe/duct	Schematic	Mode index (l)	Natural frequencies of axial modes (f_{zl})
rigid-rigid (both ends rigid)		0, 1, 2, ...	$\frac{c}{2L} \times l$
open-rigid (one end open, one end rigid)			$\frac{c}{4L} \times (2l+1)$
open-open (both ends open)		1, 2, 3, ...	$\frac{c}{2L} \times l$

From Table 1, when the vessel is empty then the first axial mode is obtained at $c_G/(2L)$, and when vessel is completely filled with liquid then the first axial mode is obtained at $c_L/(2L)$. However, when the vessel is partially filled to a height h, the resonant frequencies of axial modes of the gas and the liquid column are $c_G/(2(L-h))$, and $c_L/(4h)$ respectively. Since $c_G<c_L$, the first axial mode of the vessel is due to the gas column until the liquid length becomes sufficiently long as given by following equation:

$$\frac{c_G}{2(L-h)} \geq \frac{c_L}{4h} \Longrightarrow h \geq \frac{c_L}{2c_G+c_L} L \quad (1)$$

Therefore, the first resonant frequencies for the axial pressure variations of this vessel are given by the following function:

$$f_{zl} = \begin{cases} \frac{c_G}{2(L-h)};\ h < \frac{c_L}{2c_G+c_L}L \\ \frac{c_L}{4h};\ \frac{c_L}{2c_G+c_L}L \le h < L \\ \frac{c_L}{2h};\ \mathrm{h = L} \end{cases} \tag{2}$$

2.2 Waveguide for capturing axial modes of the vessel

Waveguides have axial dimension much larger than the cross-sectional dimension. Therefore, in general, irrespective of the noise source at one end of the waveguide only plane waves propagate along the axis of the waveguide [8]. Thus, they have been widely used as an efficient means of transmitting and capturing plane waves [8]. This property of the waveguide is used for separating the axial modes of the cylindrical vessel from the other radial and circumferential modes. For this purpose, the waveguide is connected coaxially with a cylindrical vessel and it is expected that it will allow propagation only along the axial direction. Thus, travelling wave in the waveguide will have propagation vector 'k' along the axis of the vessel. Therefore, only axial pressure variations generated within the vessel will propagate, but not the orthogonal pressure variations. These travelling waves will have resonance at frequencies dependent on the liquid height and governed by equation (2). The total pressure at any point inside the waveguide will also contain reflected pressure waves that depend on the waveguide termination condition. The incident wave from vessel to waveguide termination is a more accurate indicator of liquid height, and this incident wave spectrum will show resonance at frequencies that are a function of the liquid height. This incident wave can be obtained by wave decomposition using two microphones along the axis of the waveguide.

2.3 Waveguide-based liquid height measurement system

Based on the above discussion, a waveguide-based measurement system is devised to monitor hot bubbling liquid levels in closed vessels. Figure **1** shows the schematic of the measurement system. Here, a waveguide is attached coaxially to the top of a boiling liquid vessel. The incident wave spectrum in the waveguide would show these resonant frequencies. The minimum range of the driving frequency, f_d corresponds to the range of the first axial modes of the vessel and are given as:

$$\frac{c_G}{2L} \le f_d \le \max\left(\frac{2c_G+c_L}{4L}, \frac{c_L}{2L}\right) \tag{3}$$

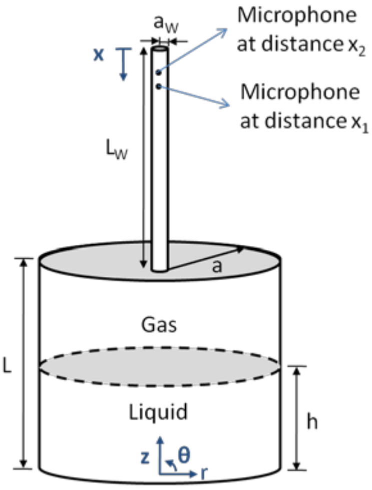


Figure 1: Schematic of the vessel attached to a waveguide

To obtain incident wave spectrum the wave decomposition algorithm by Seybert and Ross (1977) and Seybert (1988) [9-10] is used. This method requires two microphones that are placed inside and along the axis of the waveguide, near its termination. If the spectrum of sound pressure measured in the waveguide at microphone locations, x_1 and x_2 shown in Figure **1** are denoted as S_{11} and S_{22}, then with the assumption of no flow condition in the waveguide, the decomposition equation gives the incident wave spectrum, S_{II} as follows:

$$S_{II} = S_{11} + S_{22} - 2C_{12}\cos(ks) + 2Q_{12}\sin(ks); k = \frac{2\pi f_d}{c}; s = x_1 - x_2 \quad (4)$$

Where, C_{12} and Q_{12} are real and imaginary parts of the cross spectrum between the pressures at x_1 and x_2, respectively. These can be obtained in terms of the transfer function H_{12} and the phase difference θ_{12} between microphones at x_1 and x_2 as follows:

$$C_{12} = |H_{12}|S_{11}\cos(\theta_{12}) \quad (5)$$

$$Q_{12} = |H_{12}|S_{11}\sin(\theta_{12}) \quad (6)$$

Thus, this waveguide measurement system requires two microphones with known spacing, and the signal data acquired are the spectrum at each microphone location, and transfer function and phase difference between the two microphone locations.

3. Simulation

3.1 Procedure

The above derived theory was tested through finite element simulation in ANSYS Workbench 15. A cylindrical vessel with a waveguide coaxially attached to its top as proposed in Figure **1** was modelled. The vessel had dimensions of L = .195 m, a =.1325 m, and the dimensions of the waveguide were L_w = 1.25 m, a_w =.035 m. A harmonic normal surface velocity excitation of 1 mm/s was given to the vessel base along the z direction, to simulate the phenomenon of boiling liquid. The speed of sound and density of the acoustic media within the vessel were taken as c_G = 391 ms^{-1}, ρ_G = 0.928 Kgm^{-3}, c_L= 1543 ms^{-1} and ρ_L = 958.4 Kgm^{-3} for the air and water column respectively at a temperature of 100°C. The air column in the waveguide was given a linear temperature gradient from 100°C at the vessel-waveguide connection to 20°C at the waveguide termination. The speed of sound and density of air column in the waveguide varied based on the temperature. The vessel was excited between frequencies 950 to 4000 Hz to cover the resonant frequency range of the axial mode of the vessel as calculated from equation (3). In the FEM model, harmonic response was obtained at two microphone locations situated at a distance of x_1 = 0.085 m and x_2 = 0.050 m from the waveguide termination so the microphone spacing, x_1-x_2 = s = 0.035 m. The harmonic response was obtained at these microphones for water heights of h=0.031 m and h=0.062 m.

3.2 Results and Discussions

3.2.1 Resonant frequencies of the model

Table 2 shows the analytical solutions to the resonant frequencies of the vessel.

Table 2: Results for resonant frequencies of the vessel (L=0.195 m, c_G = 391 ms^{-1}, c_L= 1543 ms^{-1})

Water height	**Natural frequency (f_{zl}) between (950 Hz to 4000 Hz)**
0.031 m	1192
	2384
	3576
0.062 m	1470
	2940

3.2.2 Harmonic Response

Figure 2 show the harmonic response of the decomposed incident wave amplitude and the individual pressure amplitude at x_1 and x_2 for the FEM model. It is found that the incident wave and the individual pressures at x_1 and x_2 achieve their first resonance at the first axial mode of the vessel that corresponds to the liquid height as shown in Table 2. Additionally, the spectrum shows peaks at the successive axial modes of the vessel. All these resonance frequencies are a function of the water height as predicted by equation (2). However, simulations show few other peaks which may be due to the axial modes of the waveguide. Taking an average speed of sound for the waveguide column as 367 ms^{-1}, the results shows resonance at the axial waveguide modes namely, 1630 Hz (≈367/(2X1.25)X11), and 2210 Hz (≈367/(2X1.25)X15).

The effect of the waveguide could be compensated for theoretically if the temperature dependent convective heat transfer coefficients and thermal conductivities are known at every point in the waveguide to calculate accurate speed of sound at every point. This will be done in future studies.

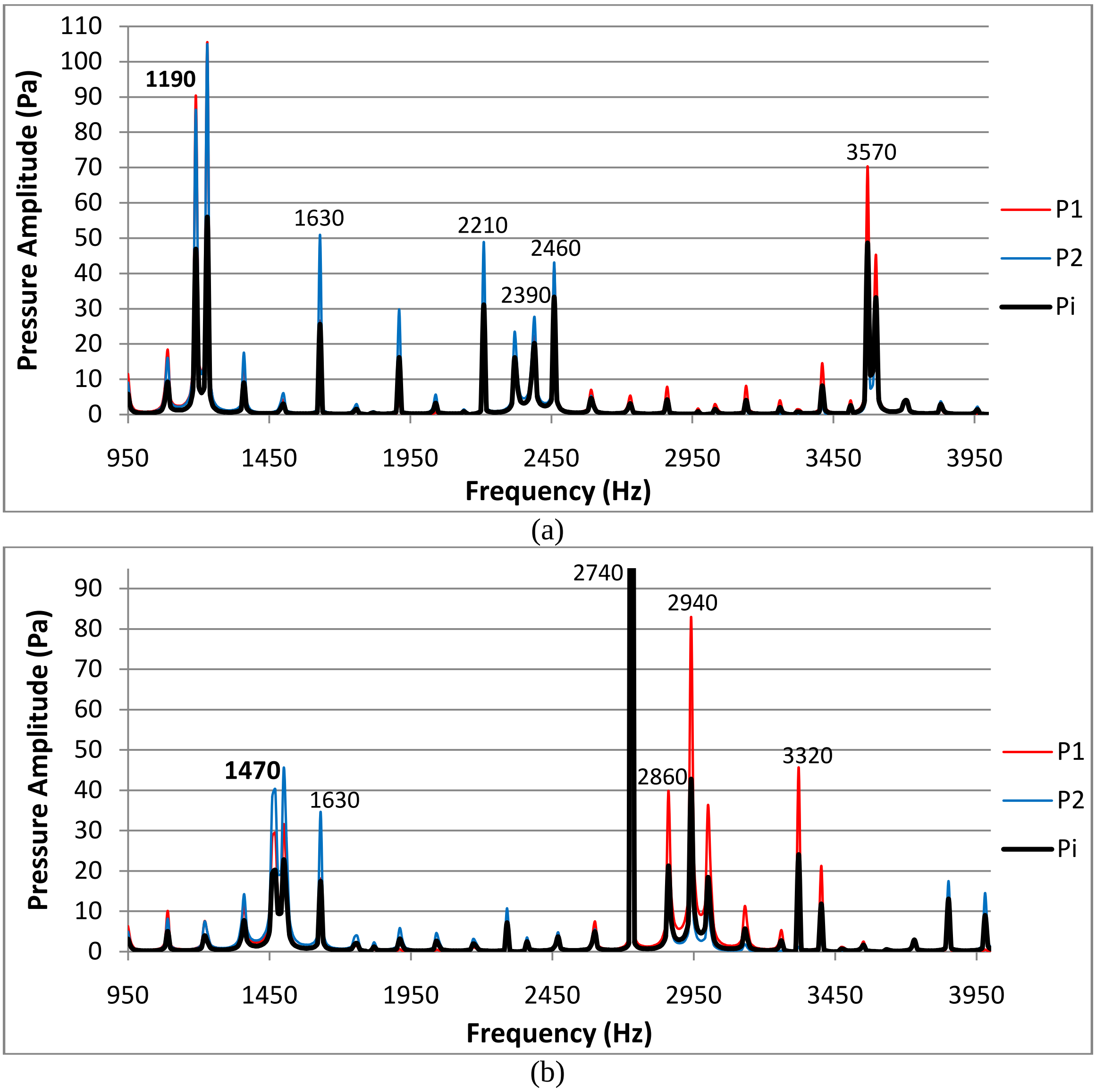


Figure 2: Harmonic response of the vessel-waveguide system, L=0.195 m, a=0.1325 m, L_W= 1.25 m at water heights of (a) 0.031 m, (b) 0.062 m.

4. Experimental verification

4.1 Procedure

An experiment was performed on an aluminium alloy pressure cooker with L= 0.195 m, a= 0.1325 m, and L_W= 1.25 m. This vessel was put on a noiseless 1000 W electric heater and the waveguide was attached to the top as shown in Figure 3. Water was put in this vessel at heights of 0.031 m, and 0.062 m from the cooker base and heated to its boiling point. The signal was measured by the two microphones during the bubbling phenomenon. Here, the bubbling noise of the boiling water was the excitation. Figure 3 shows the experimental setup. Two B&K 4136, ¼" pressure field microphones, phase calibrated as per ISO 10534 standard were used to measure the sound pressures at locations 1 and 2 on the walls of the waveguide, which were kept at a distance of 35 mm. The microphone signals and the transfer function between these signals were measured using an OROS 8 channel OR25 FFT analyzer. The signals obtained at the microphones were decomposed using equations (4) to (6).

Figure 3: Experimental set-up inside the semi-anechoic enclosure

4.2 Results and Discussions

Figure 4 shows the spectrum of the amplitude of the pressure waves at point x_1 and x_2, and the amplitude of the decomposed incident wave. Within the analysis frequency range of 950 to 4000 Hz, the incident wave spectrum always shows the first resonance at the first axial mode of the vessel corresponding to the liquid height as shown in Table 2. The spectrum of pressure at x_1 and x_2 also show resonance at the first axial mode of the vessel corresponding to the liquid height, but here other modes are also more prominent. Therefore, decomposed wave could be a more reliable indicator of liquid height. It is observed that decomposing to get incident wave attenuates other modes. These observations highlight the importance of using two microphones. However, more experimental data is required to confirm the necessity of using two microphones instead of one microphone.

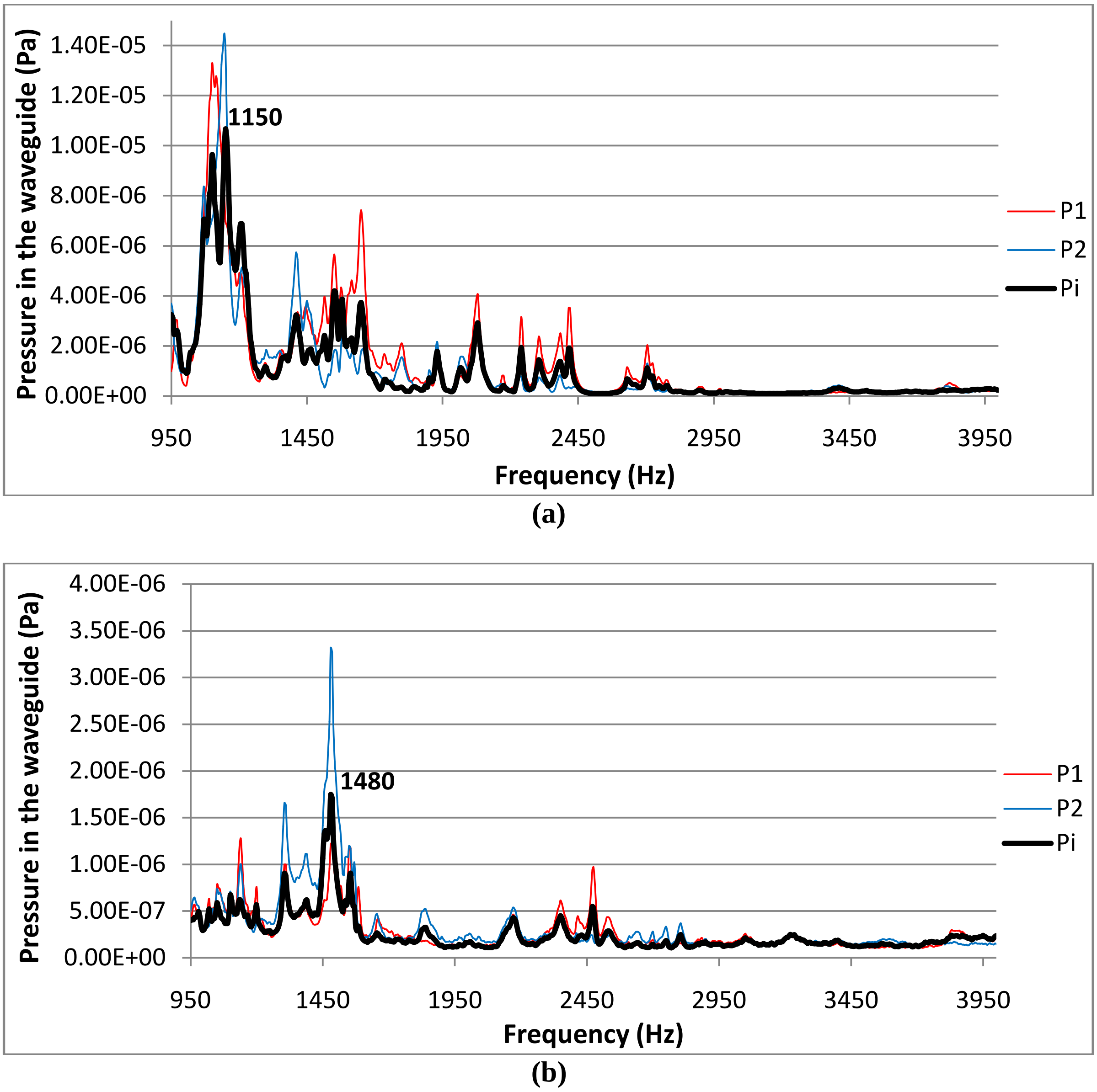


Figure 4: Wave decomposition of spectrum obtained from the experiment, L=0.195 m, a=0.1325 m, L_W= 1.25 m at water heights of (a) 0.031 m (b) 0.062 m.

4.3 General Discussions

It is observed that for simulations, the pressure measured at any point near the waveguide termination gives first resonance at the first axial mode of the vessel; here the total pressure in the waveguide is as good an indicator of liquid height as the incident wave spectrum. This is because in simulations, the waveguide termination condition is anechoic and hence it does not much affect the pressures in the waveguide. Decomposition may not be necessary during such ideal measurement conditions with no external noise and no acoustic barrier near the waveguide termination. However, experimental results suggest that total pressure in the waveguide may have other resonant peaks in addition to the peak corresponding to the liquid height. This problem will further increase in industrial applications where the environmental noise at the waveguide termination may contaminate the total pressure at any point in the waveguide. Here, decomposition will help attenuate those unnecessary modes.

Overall, simulations and experiments confirm that if the incident wave spectrum measured in the waveguide is analysed within the range of frequencies predetermined by the vessel length and the type of boiling liquid, the first resonant frequency gives the liquid height using equation (2). Thus, this waveguide system, when acoustically connected to the vessel, can be used to remotely monitor liquid levels based on the incident wave frequency and amplitude.

5. Conclusions

This paper aimed at devising a simple technique to detect boiling liquid levels in a closed vessel by monitoring the noise in the vessel using a waveguide. The theoretical solution showed that resonant frequencies of the axial modes of the vessel are a function of the liquid height. It is proposed to attach a waveguide coaxially on the top of such a vessel and measure the boiling noise generated in the vessel. The noise should be measured with two microphones attached to the waveguide along the vessel axial direction, and the signals decomposed to obtain the incident wave in the waveguide. The incident wave will show resonance at frequencies corresponding to the axial modes of the vessel, which in turn will depend on the liquid height. This method was tested and validated through finite element simulations and experiments. Results show that spectral content of the incident wave in the waveguide shows a sharp peak at the frequency that is a function of the liquid height. Overall, the proposed waveguide system has huge potential to determine boiling liquid levels remotely. Future experiments will collect and analyse more data to confirm this phenomenon. When fully developed, such a system would have wide industrial applications, particularly in steel plants, where the steel making process faces many challenges in knowing the amount of molten steel during the oxygen lancing in the vessel.

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