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AN ACOUSTIC PROPAGATION MODEL ALLOWING A MULTIPLY LAYERED POROELASTIC GROUND SURFACE

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INTRODUCTION

The method presented here allows the prediction of acoustic propagation over a multiply layered porous elastic ground from a point source. The fast field method of prediction of acoustic propagation relies on solution of a depth separated wave equation for up and down going wavefield in each layer in terms of horizontal wavenumber and depth. Because this solution is depth dependent only, the environment can be depth dependent only, and hence range independent.

The range dependent solution is then obtained by the application of a fast fourier transform approximation to a Hankel transform of the depth dependent solution in terms of horizontal wavenumber.

In this paper the global matrix method of obtaining the depth dependent solution, used by Schmidt [7], is used. The set of simultaneous equations from which the solution is obtained are equations of continuity of parameters at the layer interfaces, using a modified Biot Stoll model, as set out in Sabatier et al [5].

Richards' [6] method is used for the reduction of oscillations in the range dependent solution to produce an accurate model of propagation over a multiply layered poro-elastic ground surface.

This method is then used to examine the effect of including elasticity of the solid frame in predictions of excess attenuation over snow. The ability to predict normal surface impedance over multiply layered ground surfaces is used to deduce ground structure from measured impedance.

BIOT STOLL MODEL

The multilayered poro-elastic model is an extension of Sabatier's poro-elastic single layered model [5] .This model is based on a modified Biot-Stoll theory [3][10][2] and assumes plane wave incidence. There are three different wavetypes; fast,slow and shear. These are derived from a solution to the Biot-Stoll equations for wave propagation. Inputs used to calculate propagation constants are; fast wave speed(v_p), shear wave speed(v_s), bulk modulus of solid material(k_b), porosity (Ω), angular frequency(ω), flow resistivity (σ), fluid density (ρ_f), material density (ρ), grain shape factor (n'), and pore shape factor ratio (s_p).

PLANE WAVE MODEL GIVING THE DEPTH DEPENDENT SOLUTION.

The depth dependent Greens function part of the integral can be found using the same boundary condition equations as for a plane wave incidence model [5]. At each boundary a set of boundary condition equations in terms of up and down going particle displacements due to each of the wave types is calculated. The number of boundary conditions depends on the type of interface.

Sabatiers model involves a single poroelastic layer over an elastic halfspace, leading to four boundary conditions at the air to layer interface, and five at the layer to half-space interface.

This model involves six boundary conditions for each poro-elastic to poro-elastic interfaces as follows;

1. Continuity of normal frame velocity.
2. Continuity of fluid pressure.
3. Continuity of total normal stress .
4. Continuity of tangential stress.
5. Continuity of normal fluid velocity.
6. Continuity of tangential frame velocity

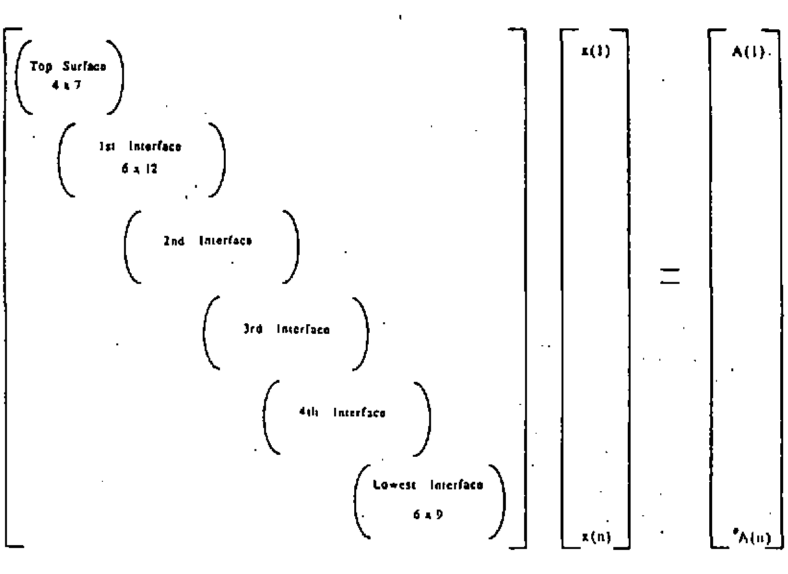
Of these boundary conditions numbers 1,2,3,and 4 are also boundary conditions for the air to poro-elastic interface and 2 and 5 are also boundary conditions for air-air interfaces. Each layer has its own boundaries as local origins. The amplitudes in the pore fluid and solid frame and the boundary condition equations are formulated in Sabatier et al [5] for five of the six conditions ,the sixth may easily be calculated.

The coefficients of the boundary condition equations for each interface are mapped onto a single global array G of side $p = 6n+4$ where n is the number of layers, as shown in figure 1, and then the equation;

$$Gx = A \tag{1}$$

is solved for the wave amplitudes $x(1)$to $x(p)$,where A is a matrix of the incident wave terms in the boundary condition equations. The acoustic pressure, for example, can then be calculated knowing the propagation constants, reflection coefficient and angle of incidence. The sound pressure level or particle displacement at any point in a many layered system can be modelled by this approach.

Figure 1. Mapping of interface boundary condition equations onto global matrix



RANGE DEPENDENT SOLUTION

THE DEPTH SEPARATED WAVE EQUATION

The basis of the FFP method of predicting pressure and wave amplitudes in a range independent environment from a point source is the depth separated wave equation [7]. From a linear wave equation in terms of wavefield potentials:

$$\left[\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\Psi(r,z,t) = H(r,z,t) \tag{2}$$

where $H(r,z,t)$ is the forcing term. By applying a Fourier transform in time one obtains a frequency domain wave equation, the Helmholtz equation:

$$\left(\nabla^2 - k_m^2\right)\Psi(r,z,\omega) = H(r,z,\omega) \tag{3}$$

where k_m is the propagation constant in the medium. Applying a forward Hankel transform in terms of range:

$$G_{(k,z)} = \int_{r=0}^{\infty} J_i(k_h.r).g(r,z).r.dr \tag{4}$$

to the Helmholtz equation one then obtains the depth separated wave equation:

$$\left(\frac{d^2}{dz^2} + \left(k^2 - k_m^2(z)\right)\right)\Psi(k,z) = H(k,z) \tag{5}$$

This equation is depth dependent only and is equivalent to the wave equation for continuous plane wave incidence, and hence has the solutions given by the above plane wave model. In order to obtain a range dependent solution one simply performs the inverse Hankel Transform on the solution to equation 5 , which is in terms of horizontal wavenumber. The transform (integration) over horizontal wavenumber is equivalent to an integration over all angles of incidence, where the horizontal wavenumber is the horizontal component of the wavenumber in the medium. So the sine and cosine of the propagation angle $\theta_i$in each medium is calculated as follows:

$$\sin\theta_i = \frac{k_h}{l_i} \qquad\qquad \cos\theta_i = \left(1 - \frac{k_h^2}{l_i^2}\right)^{\frac{1}{2}} \tag{6}$$

Where k_h is horizontal wavenumber, and l_i is the propagation constant in the relevant medium, of the relevant wavetype.

THE FFP METHOD OF INTEGRATION

The exact range dependent solution is in the form

$$F_{(x,d)} = \int_{k_h=0}^{\infty} J_i(k_h.x).\Gamma(k_h,d).dk_h \tag{7}$$

Where F is the output parameter (pressure, particle velocity, etc) and Γ is the corresponding depth dependent Greens function. For a receiver in the upper half space the depth dependent Greens function (Γ) for pressure is as follows [7]:

$$\Gamma = \rho.\omega^2\left[\frac{1}{\cos\theta_0.l_0}e^{i.|h_s-h_r|.l_0.\cos(\theta_0)} + x(1).e^{i.h_s+h_r.l_0.\cos(\theta_0)}\right] \tag{8}$$

The exact solution (equation 7)is the inverse Hankel Transform pair to equation 4. The order of the Bessel function J_ν is dependent on the output parameter required and is zero for pressure. The above equation can be approximated to the sum of two Fourier Transforms which are approximated by Fourier series.

A large argument approximation to the Bessel function [1] is:

$$J_\nu(z) \simeq \frac{1}{\sqrt{2\pi z}}\left[e^{i(z-\pi\nu/2-\pi/4)} + e^{-i(z-\pi\nu/2-\pi/4)}\right] \qquad (9)$$

This approximation together with the replacement of the integration by a finite sum gives the approximate equation for $F_{(x,d)}$:

$$F_{(x_m,d)} \simeq \frac{\delta k N^{1/2}}{2\pi m^{1/2}}\left[e^{-i\pi/4}\sum_{n=0}^{N-1}\Gamma_{(k_n,d)} n^{\frac{-1}{2}} e^{\frac{2i\pi mn}{N}} + e^{i\pi/4}\sum_{n=0}^{N-1}\Gamma_{(k_n,d)} n^{\frac{-1}{2}} e^{\frac{-2i\pi mn}{N}}\right] \qquad (10)$$

So Γ is calculated for a set of values of horizontal wavenumber $k_h = k_n$ corresponding to values of n from 0 to $N-1$ where:

$$k_n = k_{h(min)} + n.\delta k \qquad (11)$$

This Fourier series approximation can then be improved by corrections to allow for the truncation of the integral to infinity to a finite wavenumber, the discretization of the integral and the avoidance of pole(s) on the real axis, which together lead to inaccuracies and oscillations in the result. The method used to achieve this is that put forward by Richards [6] .This involves the choice of two variable parameters α and Δ , and the modification of the depth dependent integrand. The final form for the Hankel transform is:

$$F_{(x_m,d)} \simeq \frac{\delta k N^{\frac{1}{2}}}{2\pi m^{\frac{1}{2}}}\left[e^{-i\pi/4}\sum_{n=0}^{N-1} C_{(k_n,d)} e^{\frac{2i\pi mn}{N}} e^{\frac{2\pi m\alpha}{N}} + e^{-i\pi/4}\sum_{n=0}^{N-1} C_{(k_{N-n},d)} e^{\frac{2i\pi(N-m)n}{N}} e^{\frac{-2\pi m\alpha}{N}}\right] \qquad (12)$$

where:

$$C_{(k_n,d)} = G_{(k_n,d)}.(n-i\alpha)^{-1/2} + G_{(k_{N-1},d)} N^{-1/2} S^* \qquad (13)$$

with:

$$G_{(k_n,d)} = \Gamma_{(k_n,d)} + \frac{iN\Gamma_{(k_0,d)}}{\alpha\Delta}\left[1 - e^{(\Delta(i\alpha-n)/N)}\right] \qquad (14)$$

and S^* is an approximation to the sun of the series S where:

$$S = \sum_{j=1}^{\infty}(j + [(n-i\alpha)/N])^{-1/2} \qquad (15)$$

OUTPUT

The output from so called ffp programs is range dependent. From this model the output could be in terms of pressure, or any other parameter in any of the solid or fluid layers, here only the excess attenuation in pressure predictions are presented.

Using the depth dependent solution normal surface impedance at some angle of incidence can be predicted.

WET SOIL

Crammond and Don [4] have conducted measurements of soil normal surface impedance using an impulse technique. They found that when they examined the normal surface impedance of a soil which had been artificially wetted, both sharp and rounded peaks in impedance at certain frequencies resulted(see figure 2). Furthermore they found that even a change in source or receiver position of a few centimetres resulted in a large change the frequencies and amplitudes of the peaks in impedance. On examination of the soil it was found that a thin wetted layer one to two centimetres deep existed at the surface. Using soil parameters derived from analysis of the surface impedance of the dry soil given by Crammond and Don [4] and assuming an increase in flow resistivity and a decrease in porosity in the wetted layer, it was attempted to produce a predicted normal surface impedance similar to those measured. This was done by the use of the depth dependent code (see figure 3). It

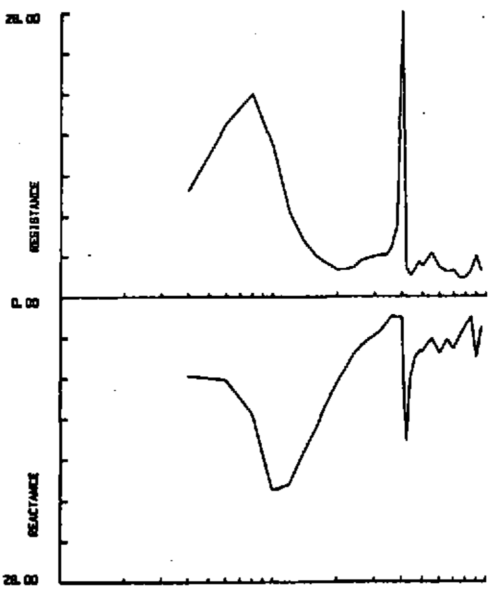


Figure 2. Impedance measured by Crammond and Don over artificially wetted grassland.

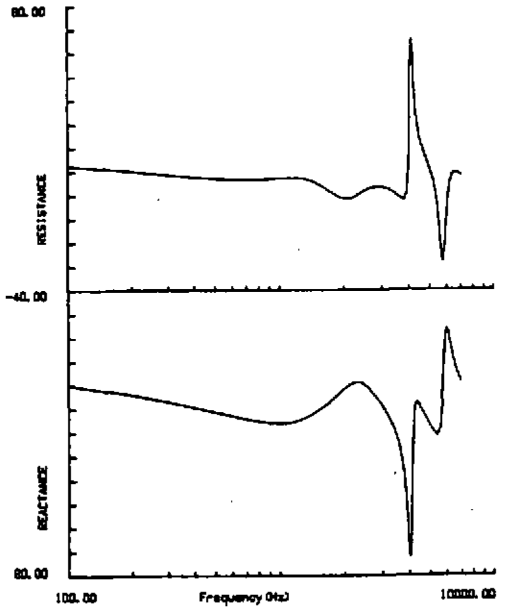


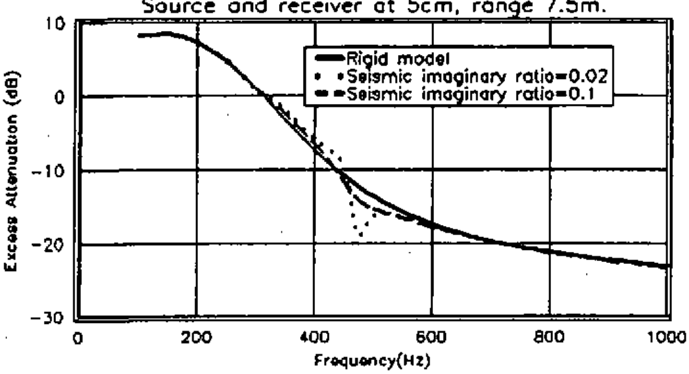
Figure 3. Theoretical normal surface impedance for high flow resistivity thin elastic layer over low flow resistivity elastic halfspace. Parameters taken from Crammond and Don wetted soil data as far as possible.

was found that in the parameter range studied the impedance was extremely sensitive to changes in some of these parameters; corresponding to the large changes in measured impedance with position. The parameters used to produce the results are not entirely realistic; in particular the tortuosity (given by $\Omega^{-n'}$) in the wetted layer was unusually high, and the seismic wave speed contrast is not that which would be expected. However, the results do show that with a combination of acoustic (slow wave) and seismic (fast wave) resonances, both the smooth and the sharp peaks in Crammond and Don's data can be predicted.

SNOW

Seismic wave speeds as a function of density are given by Sommerfeld [8] from a variety of sources and these have been extrapolated to low densities for this paper. The lack of literature makes the choice of attenuation factor in this study a little arbitrary. The four pore structure dependent parameters in the modified Biot Stoll model(flow resistivity σ,porosity Ω,grain shape factor n' and pore shape factor ratio s_p) have been derived for snow from Attenborough and Buser's work[9]. For a semi-infinite snow layer the effect of elasticity is small, but for a thin snow layer on a rigid ground surface, there are some frequencies and geometries where a difference of up to six decibels in excess attenuation is predicted due to snow elasticity (see figure 4).

Figure 4 Predicted excess attenuation over 8cm snow layer
Source and receiver at 5cm, range 7.5m.



DISCUSSION

It has been shown here that acoustic-seismic coupling effects can be important in the normal surface impedance of wetted ground surfaces, although the exact nature of the seismic discontinuity in this situation is unclear.

When the effect of frame elasticity in snow is examined the effect on a half-space is negligible. Near to the ground surface, on the other hand, over thin snow layers, there seem to be some frequencies and ranges where a significant change in predicted excess attenuation may be found, due to frame elasticity effects, although the size of this difference is very sensitive to the seismic attenuation in the snow and the actual magnitude of this attenuation is not well known.

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