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STATISTICAL ESTIMATION METHOD OF ROAD TRAFFIC NOISE BY USE OF MULTIPLICATION TYPE MODEL OF RANDOM PROCESS AND ITS EXPERIMENTAL CONFIRMATION

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INTRODUCTION

Up to now, many methods for estimating the statistics of road traffic noise have been proposed under the introduction of specific vehicle distribution models, such as an equally spaced model, an exponentially distributed model, or an Erlang distribution type model, especially in a simplified sound propagation environment such as a free sound field. In the above theoretical methods, however, the statistical information on headway distribution as well as the statistical property of number of vehicles flowing on the road per unit time interval are fundamentally necessary to estimate the statistics of road traffic noise fluctuation. But, it is, in general, difficult to grasp the headway distribution form in an actual observation.

From the above practical point of view, in this paper, a simplified method for estimating the evaluation indices of road traffic noise such as median, L_5 and L_{10} (in general, L_x: (100-X) percentage point of noise level distribution) from only the statistical information on number of vehicles flowing on the road per unit time interval is newly proposed. Concretely, a new explicit expression of the probability density function for the noise intensity fluctuation, from which every noise evaluation indices like L_{eq}, variance and L_x can be derived, is firstly found from the theoretical point of view, by newly introducing the multiplication type model for random process fluctuating only in positive region and the Mellin transform type characteristic function method.

For purpose of finding an effect of statistical properties of number of vehicles on the resultant intensity distribution form of road traffic noise, the noise intensity distribution expression is theoretically derived in a general expansion form taking the noise intensity distribution function in a typical case when the number of vehicles flowing on the road per unit time interval is constant into the first expansion term. Finally, the above theoretical estimation method is experimentally confirmed by applying it to the actual road traffic noise data.

THEORETICAL CONSIDERATION

Probability density expression for random noise intensity fluctuation based on multiplication type model

Let $Z(t)$ be the noise intensity fluctuation wave of arbitrary probability distribution type. And suppose that $Z(t)$ is constructed as the product of two different random component processes, $X(t)$ and $U(t)$, fluctuating over a positive semi-infinite region $(0,\infty)$, and that the choice of the probability density function form of $X(t)$ is quite unrestricted. At this time, we can describe $Z(t)$ by the multiplication type model as follows:

$$Z(t) = X(t)\times U(t) \quad (X(t),U(t)>0) \quad , \tag{1}$$

where the probability density function form of a component process, $X(t)$, is already known or is advantageously established in advance to our statistical noise analysis.

In order to derive an explicit expression of the probability density function, $P(Z)$, for an arbitrary noise intensity fluctuation $Z(t)$ especially based on the probability density function, $P_X(X)$, of random process $X(t)$, we newly introduce a Mellin transform type characteristic function, $M_Z(s)$, defined as [1] :

$$M_Z(s) \overset{\Delta}{=} m[P(Z)] = \int_0^\infty Z^{s-1}P(Z)dZ \quad . \tag{2}$$

By substituting Eq.(1) into Eq.(2), we can easily obtain

$$M_Z(s) = \int_0^\infty X^{s-1}P_X(X)[\int_0^\infty U^{s-1}P(U|X)dU]dX \quad , \tag{3}$$

and the above expression becomes

$$M_Z(s) = M_X(s)\times M_U(s) \quad , \tag{4}$$

in a special case when the random variables X and U are statistically independent of each other. Hereupon, $M_X(s)$ and $M_U(s)$ are Mellin transform type characteristic functions for random variables X and U respectively. The above relationship, Eq.(4), makes a striking contrast to the well-known Fourier transform type characteristic function suitable for a random process described especially by the addition type model [2].

Now, in Eq.(3), by carrying out Taylor's expansion theorem on the part in the square bracket, we can easily obtain

$$\begin{aligned} M_Z(s) &= \int_0^\infty X^{s-1}P_X(X)\sum_{i=0}^{\infty} b_i(X)(s-1)^i dX \\ &= \sum_{i=0}^{\infty} (s-1)^i m[P_X(X)b_i(X)] \end{aligned} \tag{5}$$

with

$$b_i(X) = \frac{1}{i!}\frac{d^i}{ds^i}\int_0^\infty U^{s-1}P(U|X)dU\Big|_{s=1} = \frac{1}{i!}\cdot\langle(\ln U)^i|X\rangle_U \quad . \tag{6}$$

After making use of the following formula obtained from the definition of the Mellin transform:

$$(-1)^n(s-1)^n m[f(X)] = m[(\frac{d}{dX}X)^n f(X)] \quad , \tag{7}$$

we can easily derive the following relationship (see Eqs.(2) and (5)):

$$\int_0^\infty Z^{s-1}P(Z)dZ = \int_0^\infty X^{s-1}\sum_{i=0}^{\infty}(-1)^i(\frac{d}{dX}X)^i\{b_i(X)P_X(X)\}dX. \tag{8}$$

Since the right-hand side of Eq.(8) is expressed in the form of a definite integral, it is possible to change the integral variable X to Z. Thus, we have

$$\int_0^\infty Z^{s-1}[P(Z)-\sum_{i=0}^{\infty}(-1)^i(\frac{d}{dZ}Z)^i\{b_i(Z)P_X(Z)\}]dZ = 0 \ . \tag{9}$$

Since s in Eq.(9) is quite arbitrary parameter, the following relation must be satisfied:

$$P(Z) = \sum_{i=0}^{\infty}(-1)^i(\frac{d}{dZ}Z)^i\{b_i(Z)P_X(Z)\}$$

$$= \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}(\frac{d}{dZ}Z)^i\{\langle(\ln U)^i|X=Z\rangle_U P_X(Z)\}. \tag{10}$$

Especially when X and U are statistically independent of each other, the above probability density expression can be rewritten as follows:

$$P(Z) = \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\langle(\ln U)^i\rangle_U(\frac{d}{dZ}Z)^i P_X(Z) \quad . \tag{11}$$

From the above theoretical result, we can find that the probability density expression, $P(Z)$, for a general random noise intensity fluctuation, $Z(t)$, described by the multiplication type model in Eq.(1), can be expressed in a universal expansion form. In this form of probability density expression, the probability density function, $P_X(X)$, of an arbitrarily introduced component random process $X(t)$ is taken into the first expansion term. The effect of statistical property of $U(t)$ on the resultant probability distribution form is reflected in each expansion term.

Application to the statistical estimation problem of road traffic noise

Let us consider the road traffic noise shown in Fig.1. The noise intensity, I, originated from the traffic flow, at an observation point, can be given as follows:

$$I=\sum_{\lambda=1}^{\Lambda}\sum_{i=1}^{n_\lambda} W_\lambda \cdot f(x_i). \qquad (12)$$

In the above equation, Λ is the number of different types of vehicles, n_λ denotes the number of the λ-th type vehicles located in the arbitrary line section $[-L,L]$ of the road, W_λ is the random acoustic power of the λ-th type vehicle and $f(x_i)$ denotes an acoustic characteristic of noise propagation environment. At this time, the mean value of noise intensity, $\langle I\rangle$, can be easily expressed as

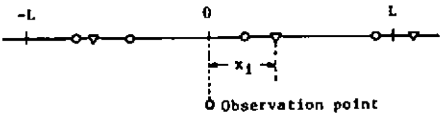

Fig.1. Model of traffic flow on straight road. (O , ∇): vehicle types.

$$\langle I\rangle = N\cdot \sum_{\lambda=1}^{\Lambda} \theta_\lambda \langle W_\lambda\rangle\langle f(x_i)\rangle \quad , \qquad (13)$$

where N is the number of vehicles located in the line section $[-L,L]$ and θ_λ denotes the ratio of the intermixture on the λ-th type of vehicles [3]. Since Eq.(13) can be rewritten as

$$\langle I\rangle = \frac{N}{N_0} N_0 \sum_{\lambda=1}^{\Lambda} \theta_\lambda \langle W_\lambda\rangle\langle f(x_i)\rangle \qquad (N_0\text{: arbitrary constant value}), (14)$$

the noise intensity at an observation point can be described by multiplication type model, Eq.(1), by use of replacement:

$$\langle I\rangle \to Z, \quad \frac{N}{N_0} \to U, \quad N_0 \sum_{\lambda=1}^{\Lambda} \theta_\lambda \langle W_\lambda\rangle\langle f(x_i)\rangle \to X \quad . \qquad (15)$$

EXPERIMENTAL CONSIDERATION

As an application of the above theory, we have first classifyed vehicles flowing on the road into two groups of light and heavy vehicles (i.e., $\Lambda=2$). Figure 1 shows the comparison between experimentally sampled values and theoretically estimated curves by use of Eq.(11), for the cumulative noise level distribution.

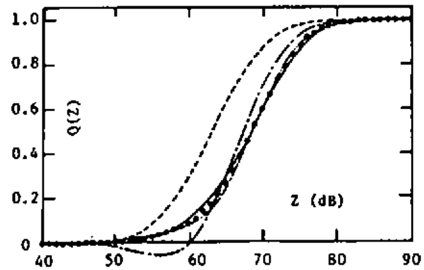

Fig.2. A comparison between theory and experiment (● experimental value, --- 1st approx., —·— 2nd approx., —··— 3rd approx. and ——— 4th approx.).

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