A ROBUST IDENTIFICATION AND LOCALIZATION METHOD OF NOISE SOURCE USING SECOND ORDER CONE PROGRAMMING (SOCP) AND NEARFIELD AMPLITUDE COMPENSATION

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In order to improve the robustness of the noise source identification and localization in high SNR environment, a new minimum variance distortion response algorithm (AC-SOCP-MVDR) is proposed in this paper. The proposed method combines second order cone programming (SOCP) with near-field amplitude compensation (AC). On the basis of previous MVDR focused beam-former, this new method imposes the inequality norm constraint on the steering vector, and then it acquires the optional steering vector by YALMIP toolbox and amplitude compensation. It can improve the robustness of beam-former and achieve the true source level of noise in near-field. Simulation and experimental studies prove that AC-SOCP-MVDR is a practical and feasible method, which can realize identification and localization of noise source, not only estimate the relative value but also the true signal of interest (SOI) power in high SNR environment.

Keywords:second-order cone programming; nearfield compensation; sample matrix inversion

1. Introduction

On the condition that the array steering vector corresponding to the SOI and sample matrix inverse (SMI) are accurately known, the MVDR beam-former has better resolution and interference rejection capability than the data-independent beam-former. However, whenever the knowledge of the SOI steering vector and sample matrix inverse are imprecise and inevitable. So the performance of the MVDR beam-former may become worse than that of the standard beam-formers (SCB)[1], especially in high SNR environment.

Li proposed a Robust Capon Beam-former (RCB) based on an ellipsoidal uncertainty set of the array steering vector, which has less sensitive to steering vector mismatches than the Standard Capon Beam-former. The RCB problem can be readily reformulated as a second order cone program and solved by SeDuMi[2].

Chen proposed a new method for underwater noise source identification in near-field location based on MVDR with normalized amplitude compensation (AC-MVDR) which greatly enhanced the spatial resolution rate and effectively reduced the side-lobe level at the same time[3]. AC-MVDR supply high resolution rate in noise source location and identification, but just estimated relative intensity of SOI, not the true value.

In this paper, a robust adaptive beam-former based on AC-MVDR and SOCP is proposed, the proposed algorithm can diminish the error of covariance matrix through inequality norm constraint on the steering vector, then locate the high SNR noise source and achieve the true SOI power in near-field. Simulation and experimental studies proved that AC-SOCP-MVDR is a practical and feasible method applying in practical engineering.

2. Nearfield observation model

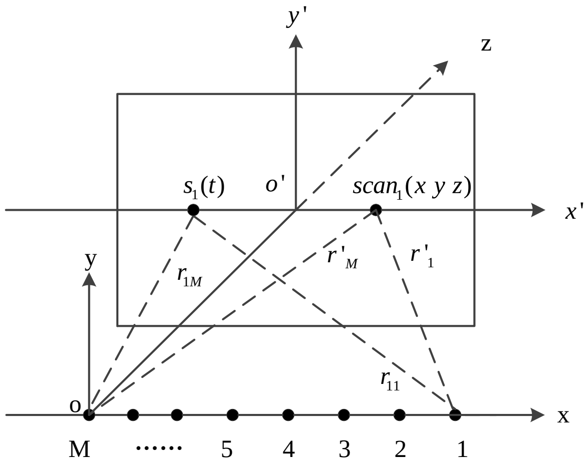


Figure 1: The near-field observation model of noise source

Consider the M-element linear sensor array to capture unknown signal in near-field environment as shown in Fig.1. The received signals are expressed as (1)

$$x(t) = (\alpha \otimes A)s(t) + n(t) \tag{1}$$

Where $x(t) = [x_1(t)\ x_2(t) \cdots x_M(t)]^H$ is the received matrix；

α is the parameter matrix of the spherical propagation loss from the source to sensor, A is the steering matrix, $\otimes$ stands for element product operational, and $n(t)$ denotes the white noise matrix. Note that $s(t) = [s_1(t)\ s_2(t) \cdots s_N(t)]^H$ is the unknown signal matrix and N is the number of signal source. We assume that there is a signal source $s_1(t)$ on the plane $x'o'y'$, and $\tau_1 = [\tau_{11}\ \tau_{12} \cdots \tau_{1M}\ ]$ is the propagation delay between $s_1(t)$ and each sensor element, $r_1 = [r_{11}\ r_{12} \cdots r_{1M}\ ]$ is the distance between $s_1(t)$ and each sensor element.

$$A = \begin{bmatrix} e^{-j2\pi f_0 \tau_{11}} & e^{-j2\pi f_0 \tau_{21}} & \cdots & e^{-j2\pi f_0 \tau_{N1}} \\ e^{-j2\pi f_0 \tau_{12}} & e^{-j2\pi f_0 \tau_{22}} & \cdots & e^{-j2\pi f_0 \tau_{N2}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2\pi f_0 \tau_{1M}} & e^{-j2\pi f_0 \tau_{2M}} & \cdots & e^{-j2\pi f_0 \tau_{NM}} \end{bmatrix} \tag{2}$$

$$\alpha = \begin{bmatrix} 1/r_{11} & 1/r_{21} & \cdots & 1/r_{N1} \\ 1/r_{12} & 1/r_{22} & \cdots & 1/r_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ 1/r_{1M} & 1/r_{2M} & \cdots & 1/r_{NM} \end{bmatrix} \tag{3}$$

3. MVDR beam-former based on near-field amplitude compensation (AC-MVDR)

The AC-MVDR meets two conditions, the first is to minimize the variance of the noise component of $w^H x$ and the second is to subject a constraint gain 1 in the look direction of the desired signal. The corresponding steering vector w is the solution to the following optimization problem：

$$\begin{cases} \min \ w^H R_x w \\ s.t. \ \ w^H \dfrac{a(x\ y\ z)}{r'(x\ y\ z)} = 1 \end{cases} \tag{4}$$

$$r'(x\ y\ z) = [r'_1\ r'_2\ ...r'_M]^T \tag{5}$$

Where:

$R_x = E\left\{xx^H\right\}$ is the spatial autocorrelation matrix of sensor output;

$r'(x\ y\ z) = [r'_1\ r'_2\ ...r'_M]^T$ is the distance between the desired signal and each sensor element. In practical applications, the true matrix R_x is unknowable but can be estimated from the receiving data.

The steering vector is given by：

$$a(x\ y\ z) = [0, e^{-j2\pi f_0(\tau_2-\tau_1)}, e^{-j2\pi f_0(\tau_3-\tau_1)}, \cdots, e^{-j2\pi f_0(\tau_M-\tau_1)}]^T \tag{6}$$

Where:

τ_i stands for propagation delay between the desired signal and sensor element i.

This problem was solved from well-known method of Lagrange multipliers, the following solution was derived:

$$w_{ac-MVDR}(x\ y\ z) = \frac{R_x^{-1} \dfrac{a(x\ y\ z)}{r'(x\ y\ z)}}{[\dfrac{a(x\ y\ z)}{r'(x\ y\ z)}]^H R_x^{-1} \dfrac{a(x\ y\ z)}{r'(x\ y\ z)}} \tag{7}$$

The SOI power of AC-MVDR beam-former is：

$$p_{ac-MVDR} = w(x\ y\ z)^H R_x w(x\ y\ z) \tag{8}$$

Similarly, the optimal steering vector for amplitude compensation common beamformer (AC-CBF) is：

$$w_{ac-CBF}(x\ y\ z) = \frac{a(x\ y\ z)\cdot r_0(x\ y\ z)}{M} \tag{9}$$

However, the result of MVDR beam-former depends on its steering vector mismatch and the error of covariance matrix inverse. Particularly, high SNR signal results in a significant degradation in the consequence of MVDR beam-former. While the Inequality norm constraint on the steering vector can improve MVDR beam-former performance. As is shown in Section.4.

4. Robust MVDR beam-former based on near-field amplitude compensation and SCOP(AC-SOCP-MVDR)

Based on the observation model and AC-MVDR, the eq.(4) add the constraints in order to get the most appropriate results. Mathematically, the steering vector can be found by solving the following convex optimization:

$$\begin{cases} \min \ w^H R_x w \\ s.t. \ \ w^H \dfrac{a(x\ y\ z)}{r'(x\ y\ z)} = 1, \left\|w\right\|^2 \le \zeta \ (\zeta \ge 1/\displaystyle\sum_{m=1}^{M} \frac{1}{{r_m}^2}) \end{cases} \tag{10}$$

Where:

ζ is the restricted factor. The value of ζ is less, the robustness of MVDR beam-former is higher. However eq.(11-12) proves that ζ should be satisfied with $\zeta \geq 1/\sum_{m=1}^{M}\frac{1}{r_m^{\ 2}}$. The design problems can be reformulated as convex optimization form And then it solved efficiently via the well-established interior point methods, for example, by SeDumi [2][4].

$$1=\left|w^H\frac{a}{r'}\right|^2 \leq \|w\|^2\left\|\frac{a}{r'}\right\|^2=\|w\|^2\sum_{m=1}^{M}\frac{1}{r_m^{\ 2}} \tag{11}$$

$$\|w\|^2 \geq 1/\sum_{m=1}^{M}\frac{1}{r_m^{\ 2}} \tag{12}$$

Note that solving this problem by SeDuMi is complex, which need allocating many parameters, then formulate eq.(10) in the dual standard form[5]. In addition, compensating amplitude and phase at the same time leads to the optimal solution not unique. So we can rewrite the eq.(10) in eq.(13-14). In other words, compensating amplitude and phase individually can improve the accuracy of optimal solution.

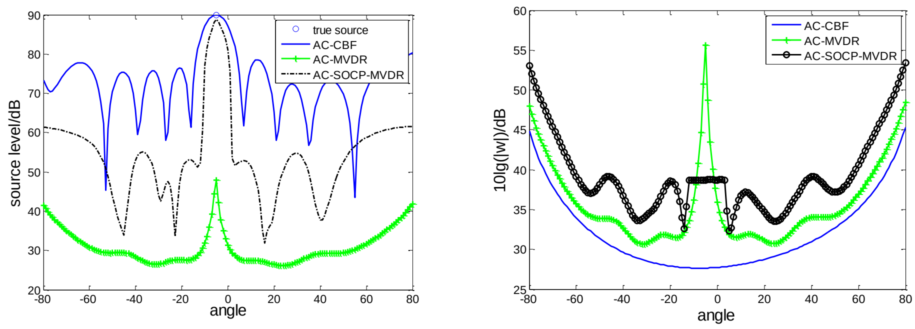
After that we propose using YALMIP[6] to deal with the SOCP problem, which need input directly objective function and constraint parameters directly, without being formulated into the dual standard form.

$$\begin{cases}\min\ \tilde{w}^H R_x \tilde{w} \\ s.t.\ \ \tilde{w}^H a(x\ y\ z)=1, \|\tilde{w}\|^2 \leq \zeta\ (\zeta \geq 1/M)\end{cases} \tag{13}$$

$$w_{ac-SOCP-MVDR}(x\ y\ z)=\tilde{w}(x\ y\ z)\cdot r'(x\ y\ z) \tag{14}$$

5. simulation and experimental results

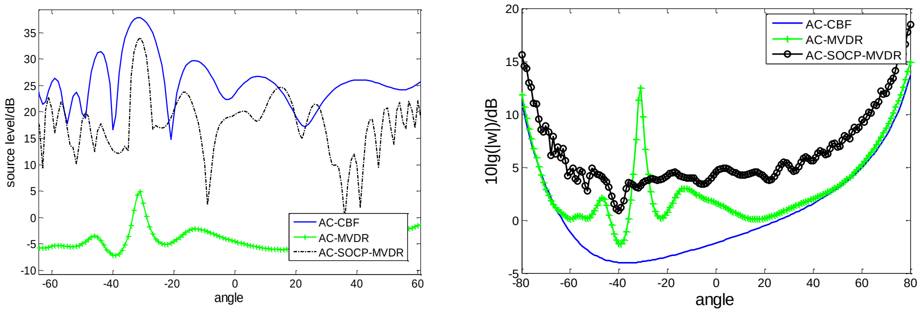
White noise gain(WNG) is defined as the gain applied by the adaptive beam-former to a spatially white input noise process[7], and is represented by $WNG = w^H * w$. The result of applying the various beam-formers to the simulated data are shown below. The AC-SCOP-MVDR was calculated with the factor $\zeta = 0.9$. Fig.2 is the result of applying different beam-formers to simulate. Beam-formers use 11-elements arrays and the true SOI power is 90dB, signal frequency is 500Hz, sound velocity is 1500m/s. The horizon range between sensor array and signal are 50m and spherical propagation loss is also considered. It can be seen from Fig.2(a), AC-SOCP-MVDR achieve the true source level (90dB) and have low sidelobe. The Fig.2 compares the WNG for three different beam-formers. Obviously，the WNG of AC-SOCP-MVDR is constrained and decreased in the looking direction(-5 °).



(a) SOI Power estimates (b) SOI WNG estimates

Figure 2:simulated data of different beam-formers

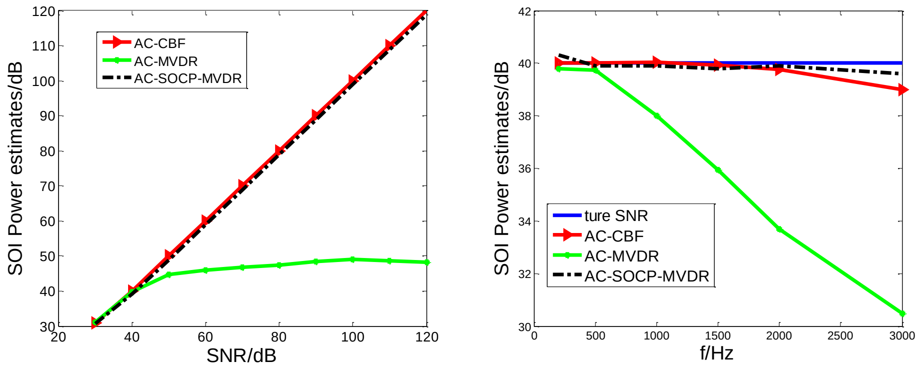
Fig.3 is experimental results of applying three different beam-formers in the anechoic chamber. The true SOI power is 36dB, signal frequency is 500Hz, sound velocity is 340m/s. The sensor array consists of 11 Microphones. The horizon range between sensor array and signal are 2m and spherical propagation loss is also considered. Fig.3(a) illustrates that AC-SOCP-MVDR achieves the true source level and has low sidelobe but has a error(less 3dB) compared with AC-CBF. The Fig.3(b) compares the WNG for three different beam-formers. Obviously, the WNG of AC-SOCP-MVDR is constrained and decreased in the looking direction(-31 °).



(a) SOI Power estimates (b) SOI WNG estimates

Figure 3:Experimental data of different beam-formers

Fig.4(a) illustrates that AC-SOCP-MVDR is better than AC-MVDR, which can achieve the true source level of noise (30~120dB) accurately. Furthermore, we investigate the algorithm’s performance under different frequencies which are shown in Fig.4 (b). The results demonstrate that AC-SOCP-MVDR always outperforms AC-MVDR under high frequency.



(a) SOI Power estimates under different SNR (b) SOI Power estimates under different frequency

Figure 4:The simulation data of different beam-formers

6. Conclusions

It is well-known that traditional MVDR beam-former identifies effectively and locates the near-field noise source, whereas the robustness of MVDR beam-former to SMI errors is sensitive especially in high SNR environment. The paper proposed a robust approach(AC-SCOP-MVDR) to identify and locate the near-field noise source. The algorithm has a good tolerance to the SMI errors by imposing the inequality norm constraint and compensating near-field transmission loss on the steering vector. Then it uses YALMIP to deal with the SOCP problem, which need input directly objective function and constraint parameters directly, without being formulated into the dual standard form.

The simulation and experimental result show that, compared with AC-CBF and AC-MVDR, this proposed method can achieve the expected designs of low sidelobe and high robustness, which realize identifying and locating the noise source in high SNR environment, and achieve the true source level rather than the relative value.

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