NONLINEAR DYNAMICS FOR AN AUTOMOTIVE WIPER SYSTEM

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This paper shows the complex dynamic behaviors of an automotive wiper system. The rich dynamic behaviors are numerically studied by means of time responses, Poincare maps and frequency spectra. Next, the largest Lyapunov exponent is estimated to verify chaotic motions. Therefore, understanding the dynamics behaviours of a wiper system provides theoretical and practical ideals for engineers in designing and controlling the automotive wiper system in the future.
Keywords: wiper, Lyapunov exponent, chaotic motion

1. Introduction

Many vibrations that may be harmful to the driver can be observed when a wiper, driven by on an automotive windshield wiper system, is operational. These vibrations reduce the comfort of driving. To find an effective way to control vibrations, we attempt to study the dynamic behaviors of the wiper system. Several studies have been carried out to investigate the chatter vibrations in an automotive wiper system [1-3]. The analysis of chatter vibrations performed by Suzuki and Yasuda [3], leads to an important conclusion: the chattering behavior is a self-excited vibration based on a stick-slip phenomenon and exits only in a certain range of wiping speed. Beyond this range, the chatter vibrations no longer occur. This property is a feature of the stick-slip phenomenon that can be observed in other physical systems [4-6].

Various numerical analyses including a bifurcation diagram, phase portraits, a Poincare map, frequency spectra and Lyapunov exponents are presented to observe periodic and chaotic motions. For a broad range of parameters, the Lyapunov exponent provides the most powerful method for measuring the sensitivity of the dynamical system to change in initial conditions. It can help us examine whether the system is in chaotic motion or not. The algorithms for computing Lyapunov exponents of smooth dynamical systems are well developed [7-10]. Nevertheless, some non-smooth dynamical systems have discontinuities where this algorithm cannot be directly applied, such as those associated with dry friction, backlash, or impact. Many works have presented methods for calculating the Lyapunov exponents of non-smooth dynamical systems [11-13]. The method proposed by Stefanski [13] for estimating the largest Lyapunov exponent for a wiper system is employed in this study..

2. Model description

A front wiper system has two blades. They are attached to the windshield at the driver's side and the passenger's side. Each blade is supported by an arm, which moves to and fro around the pivot. This motion is given by the rotation of a DC motor via the two connected four-bar linkages. The schematic diagram of an automotive wiper system is shown in Fig. 1. In this figure, the symbols with subscripts D and P are referred to as driver's and passenger's side, respectively. The lines denoted L_i represent no deflection positions. The terms θ_i (i=D, P) are the angular deflections with respect to the position L_i while the notations $\dot{\psi}_i$ are the angular velocity of the arms. The symbols l_i represent the length of the wiper arm between the pivot center and the top. The terms $\dot{z}_i$ represent the absolute velocities of the blades. Then,

$$\dot{z}_i = (\dot{\theta}_i + \dot{\psi}_i) l_i \qquad (i = D,\ P)\,. \tag{1}$$

According to Newton's second law, the governing equations for a wiper on the i's (i = D, P) side can be expressed as follows [3]

when $\dot{z}_i \neq 0$, slip,

$$I_i \ddot{\theta}_i = -R_i - D_i - M_i(\dot{z}_i)\,,$$

when $\dot{z}_i = 0,\ |R_i| \geq N_i l_i \mu_0$, stick to slip transition,

$$I_i \ddot{\theta}_i = -R_i - D_i\,,$$

when $\dot{z}_i = 0,\ |R_i| < N_i l_i \mu_0$, stick,

$$I_i \ddot{\theta}_i = 0 \quad (\dot{\theta}_i = -\dot{\psi}_i) \tag{2}$$

where the symbols I_i are the moments of inertia and M_i are the moments induced by the friction force between the wiper blades and the windshield. R_i and D_i are the moments produced by the restoring force and the damping force, respectively, as follows

$$R_D = k_D \theta_D - k_{DP} \theta_P\,,\ R_P = k_P \theta_P - k_{PD} \theta_D\,, \tag{3}$$

$$D_D = c_D \dot{\theta}_D - c_{DP} \dot{\theta}_P\,,\ D_P = c_P \dot{\theta}_P - c_{PD} \dot{\theta}_D\,, \tag{4}$$

where,

$$k_D = K_D(K_P + K_M)/(K_D + K_P + K_M)\,,\ k_{DP} = k_{PD} = K_D K_P/(K_D + K_P + K_M)\,,$$

$$k_P = K_P(K_D + K_M)/(K_D + K_P + K_M)\,,\ c_D = C_{DP},\ c_P = C_{DP} + C_P\,,$$

$$c_{DP} = c_{PD} = C_{DP}.$$

The moments M_i can be written as

$$M_i(\dot{z}_i) = N_i l_i \mu(\dot{z}_i)\,, \tag{5}$$

where N_i is the normal force. According to the experimental data [4], wiper friction can be approximated reasonably well by the combination of coulomb friction and viscous friction. Accordingly, the coefficient of wiper friction, μ, is given as

$$\mu(\dot{z}_i) = \mu_0 \operatorname{sgn}(\dot{z}_i) + \mu_1 \dot{z}_i + \mu_2 \dot{z}_i^{\,3} \quad (i = D,\ P)\,. \tag{6}$$

Let $x_1 = \theta_D$, $x_2 = \dot{\theta}_D$, $x_3 = \theta_P$ and $x_4 = \dot{\theta}_P$ be the state variables, the state equations of the wiper system (Eq. (2)) on the driver's side can be written as follows

when $\dot{z}_D \neq 0$, slip,

$$\dot{x}_1 = x_2\,,$$

$$\dot{x}_2 = (-R_D - D_D - M_D(\dot{z}_D))/I_D\,,$$

when $\dot{z}_D = 0,\ |R_D| \geq N_D l_D \mu_0$, stick to slip transition,

$$\dot{x}_1 = x_2\,,$$

$$\dot{x}_2 = (-R_D - D_D)/I_D\,,$$

when $\dot{z}_D = 0,\ |R_D| < N_D l_D \mu_0$, slip,

$$x_2 = -\dot{\psi}_D\,,$$

$$\dot{x}_1 = x_2\,,$$

$$\dot{x}_2 = 0. \tag{7a}$$

The state equations of the wiper system (Eq. (2)) on the passenger's side can be written as follows

when $\dot{z}_P \neq 0$, stick,

$\dot{x}_3 = x_4$,

$\dot{x}_4 = (-R_P - D_P - M_P(\dot{z}_P))/I_P$,

when $\dot{z}_P = 0,\ |R_P| \geq N_P l_P \mu_0$, stick to slip transition,

$\dot{x}_3 = x_4$,

$\dot{x}_4 = (-R_P - D_P)/I_P$,

when $\dot{z}_P = 0,\ |R_P| < N_P l_P \mu_0$, slip,

$x_4 = -\dot{\psi}_P$,

$\dot{x}_3 = x_4$,

$\dot{x}_4 = 0$. (7b)

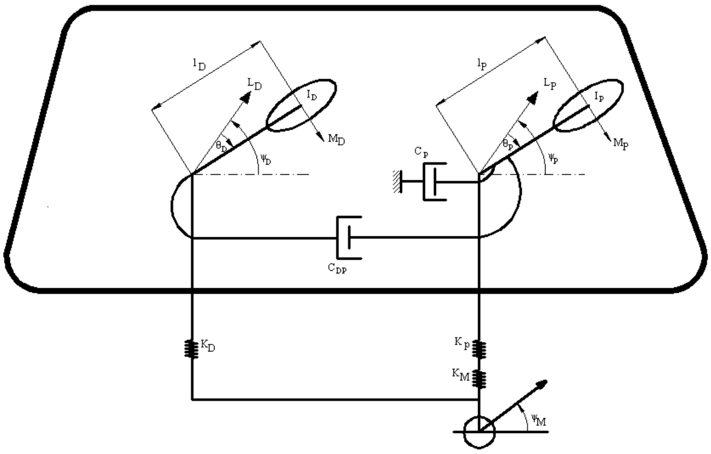


Figure 1: The analyzed automotive windshield wiper system.

3. Numerical simulation results

To clearly understand the characteristics of this system, we carry out a series of numerical simulations from Eqs. (7). The resulting bifurcation diagram is shown in Fig. 2. It is a widely used technique to describe a transition from periodic motion to chaotic motion for a dynamical system. It can be clearly seen from this figure that the chaotic motions appear approximately at regions II and IV. The dynamic behavior may be observed more completely over a range of parameter values by the bifurcation diagram. The details of the various responses exhibited by the system are presented in Fig. Here, each type of response is characterized by a phase portrait, Poincare map (Poincare velocity vs. phase angle) and frequency spectrum.

Fig. 3 shows the period-one solution. In other words, while wiping speed is high enough, equilibrium point of Eqs. (7) is stable. This means that no chatter vibrations will occur. This stable situation continues until the wiper speed decreases into the region I in Fig. 2, the stable period-*n* orbits, such as period-7 orbit (see Fig. 4) or a quasi-periodic motion (presented in Fig. 5), namely "torus motion" that produces by two incommensurate frequencies, appear to this system. As the wiper speed continues to decrease into the regions II and IV in Fig. 2, the chaotic vibrations take place. This means that the chatter vibration occurs. The particular features of two descriptors characterize the essence of the chaotic behavior: the Poincare map and the frequency spectrum. The Poincare map shows an infinite set of points referred to as a strange attractor. Simultaneously, the frequency spectrum of chaotic motion contains a board band. The two features that strange attractor and continuous type Fourier spectrum are strong indictors of chaos. Their phase portraits, Poincare maps, and frequency spectra are shown in Figs. 6. The period-three bifurcation occurs in region III in Fig. 2, which eventually results in a chaotic motion. To see this behavior in detail, phase portrait, Poincare map, and frequency spectrum are shown in Fig. 7.

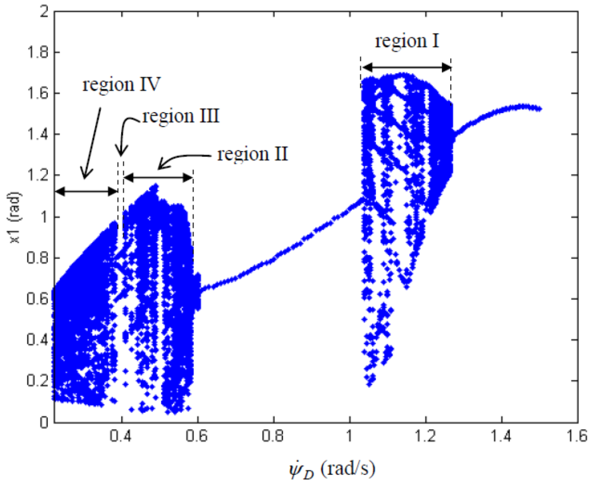


Figure 2: Bifurcation diagram of the angular velocity of the arm of driver's side $\dot{\psi}_D$ versus angular deflection x_1.

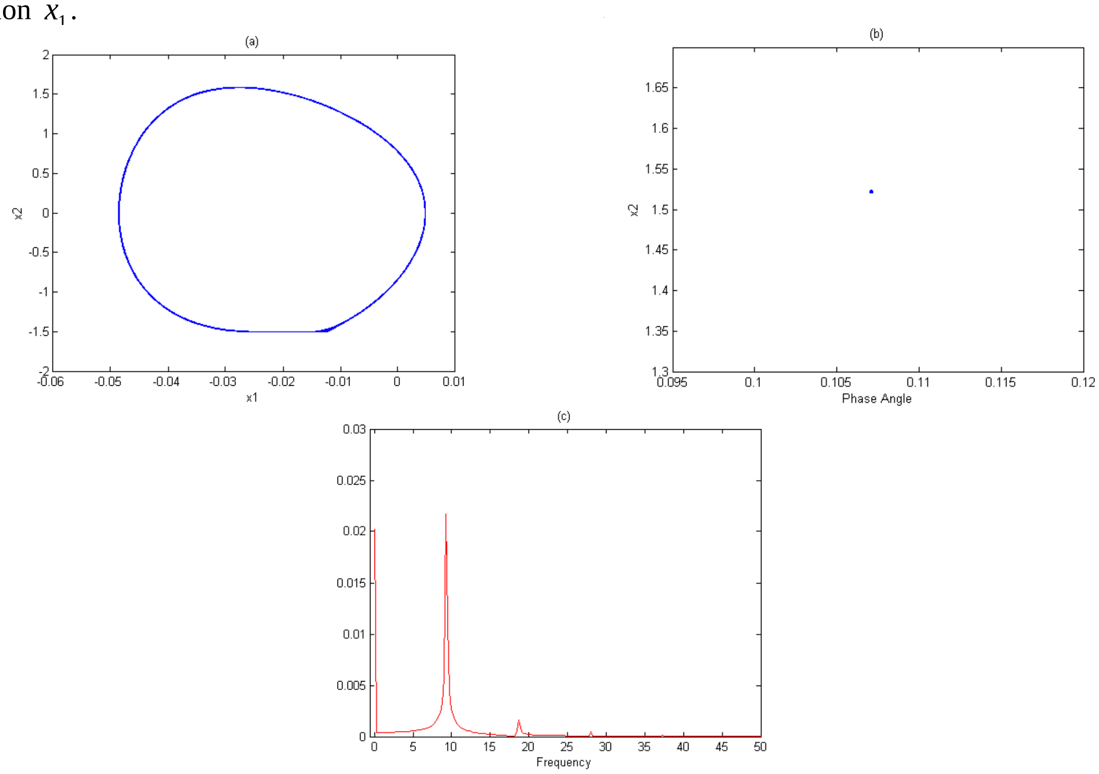
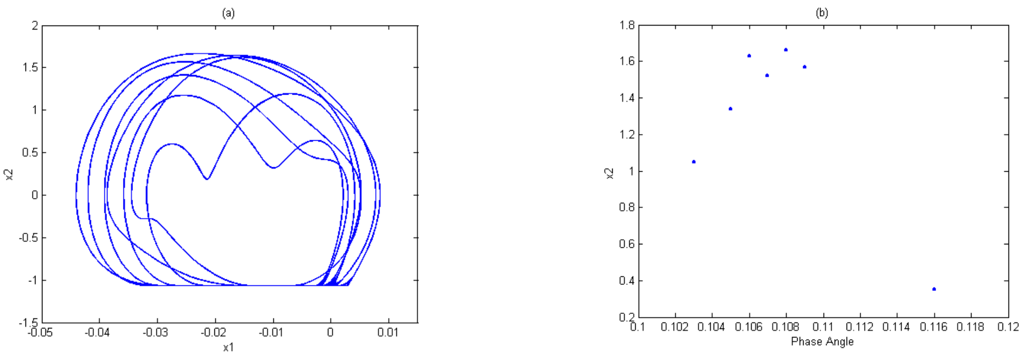


Figure 3: Period-one orbit for $\dot{\psi}_D = 1.5$ (rad/sec):
(a)phase portrait; (b)Poincare map; (c)frequency spectrum.



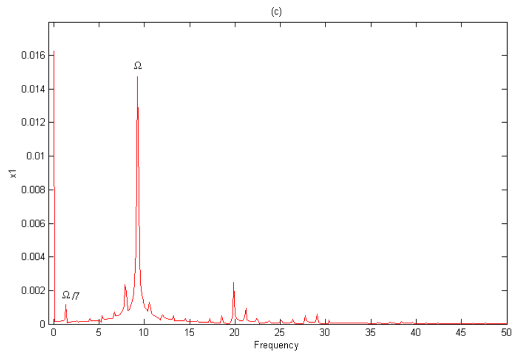


Figure 4: Period-seven orbit for $\dot{\psi}_D = 1.068$ (rad/sec):
(a)phase portrait; (b)Poincare map; (c)frequency spectrum.

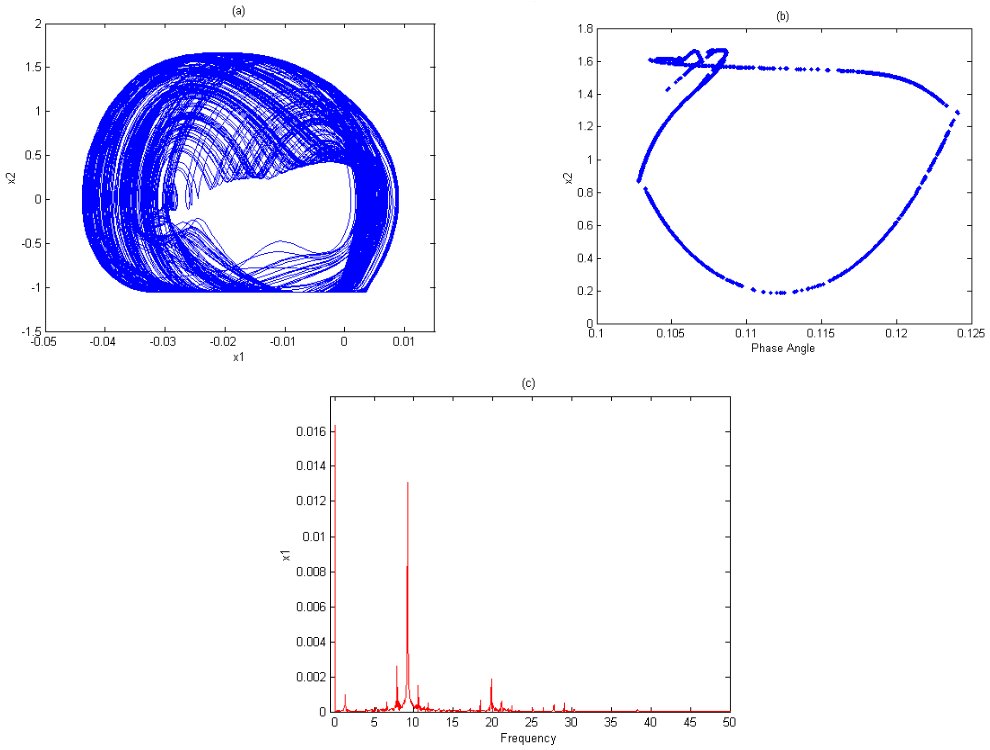
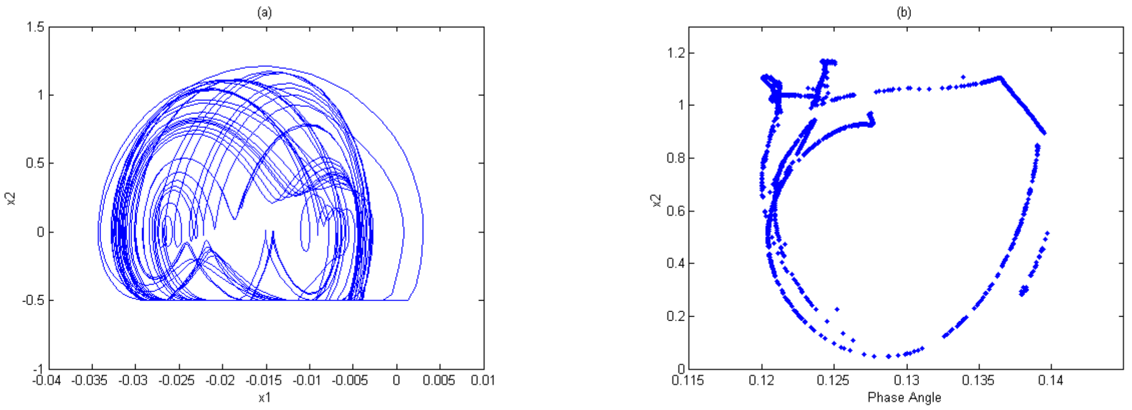


Figure 5: Quasi-periodic motion for $\dot{\psi}_D = 1.054$ (rad/sec):
(a)phase portrait; (b)Poincare map; (c)frequency spectrum.



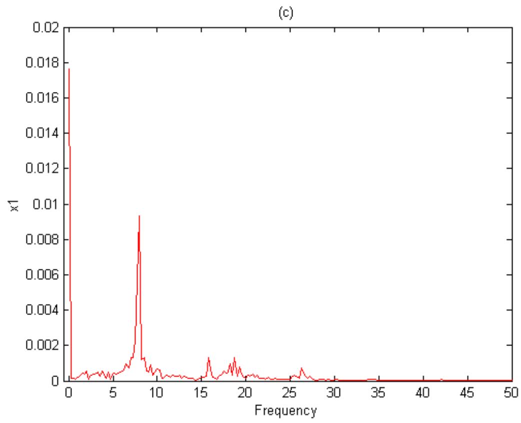


Figure 6: Chaotic motion for $\dot{\psi}_D = 0.5$ (rad/sec):
(a)phase portrait; (b)Poincare map; (c)frequency spectrum.

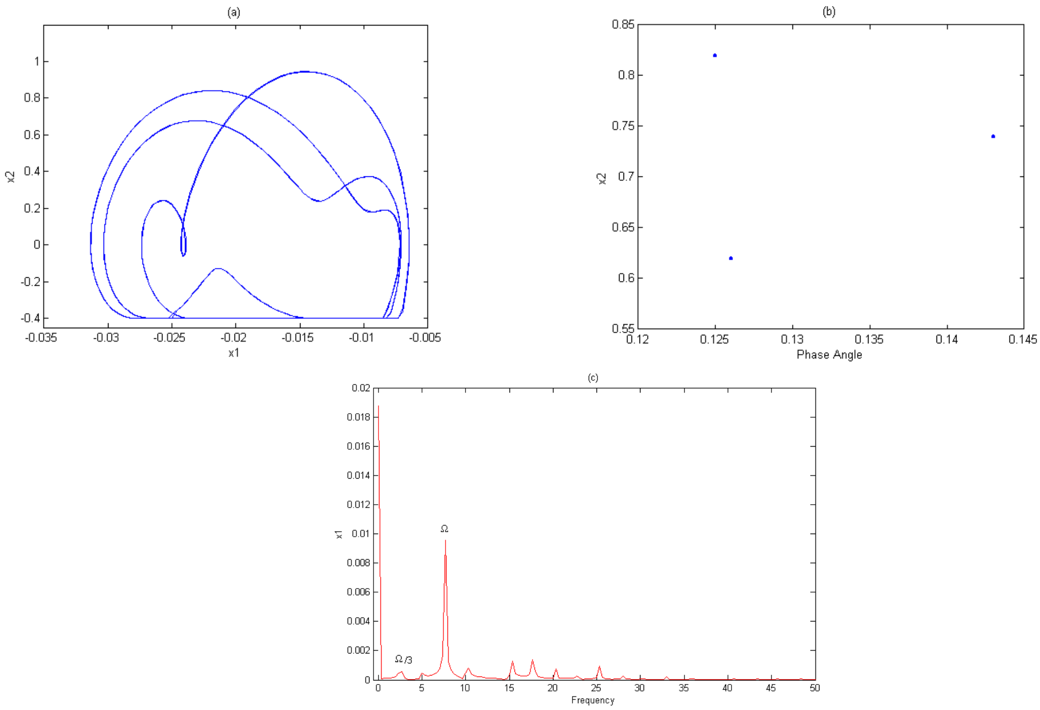


Figure 7: Period-three orbit for $\dot{\psi}_D = 0.398$ (rad/sec):
(a)phase portrait; (b)Poincare map; (c)frequency spectrum.

4. Chaotic Behavior

In this section, we wish to demonstrate that the automotive wiper system has chaotic behavior, by computing the maximal Lyapunov exponent. It should be noted that indicator such as the largest Lyapunov exponent is one of the most useful diagnostics for chaotic system. For every dynamic system, there is a spectrum of Lyapunov exponents (λ) that tells how length, area and volumes change in phase space. The existence of chaos can be established merely by calculating the largest Lyapunov exponent, to determine whether nearby trajectories diverge ($\lambda > 0$) or converge ($\lambda < 0$) on an average. Any bounded motion in a system containing at least one positive Lyapunov exponent is defined as chaotic, while non-positive Lyapunov exponents indicate periodic motion.

In this work the chaotic behavior of an automotive wiper system is demonstrated by computing the largest Lyapunov exponent. Any system with at least one positive Lyapunov exponent is defined as chaotic. Lyapunov exponents measure the rate of divergence (or convergence) of two initial nearby orbits. Algorithms for computing the Lyapunov spectrum of "smooth" dynamical systems are well established [8-10]. However, "non-smooth" dynamical systems with discontinuities such as the dry friction, backlash, or stick-slip prevent the direct application of this algorithm. Recently, Stefanski [13] suggested a simple and effective method to estimate the largest Lyapunov exponent, which utilizes the properties of synchronization. This method can be explained briefly: the dynamical system is decomposed into the following two subsystems:

drive system

$$\dot{x} = f(x), \tag{8}$$

response system

$$\dot{y} = f(y). \tag{9}$$

Consider a dynamical system, which is composed of two identical *n*-dimensional subsystems, where only the response system (8) is combined with a coupling coefficient *d*, while the equation of drive remain the same. The first order differential equations describing such a system can be written as

$$\dot{x} = f(x),$$
$$\dot{y} = f(y) + d(x - y). \tag{10}$$

Now the condition of synchronization (Eq. (10)) is given by the inequality

$$d > \lambda_{\max}. \tag{11}$$

In the synchronization, d_s, the smallest value of the coupling coefficient *d*, is assumed to be equal to the maximum Lyapunov exponent

$$d_s = \lambda_{\max}. \tag{12}$$

Fig. 8 presents the results of the numerical calculations which show the largest Lyapunov exponents that have been obtained using the described synchronization method. The system exhibits the chaotic motion because of all the largest Lyapunov exponents are positive for $\dot{\psi}_D < 0.425$ rad/s and $0.445\,\text{rad/s} < \dot{\psi}_D < 0.585\,\text{rad/s}$.

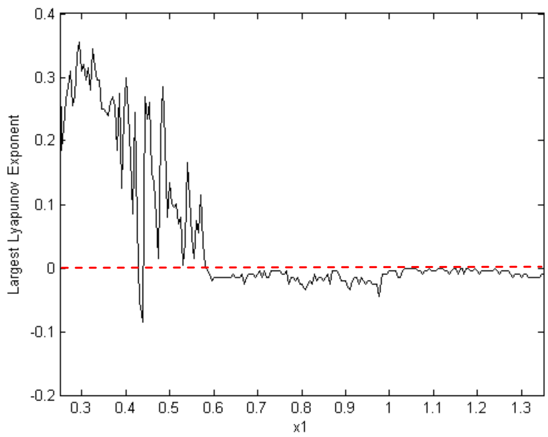


Figure 8: The evolutions of the largest Lyapunov exponent.

5. Conclusions

Our main purpose in this paper is to study chaotic attitude behavior and the problem of chaos control on an automotive wiper system. Numerical methods including time responses, Poincare maps, frequency spectra and the largest Lyapunov exponent are employed to obtain the characteristics of the nonlinear wiper system. Many nonlinear and chaotic phenomena have been displayed in bifurcation diagrams. From this diagram, we can find that the chaotic motion appears a lot in lower wiping speed for wiper system. In order to examine whether the system is in chaotic motion or not, the Lyapunov exponent will be the most useful method to diagnostics for chaotic system. The method of estimation of the largest Lyapunov exponent for wiper system uses the properties of synchronization phenomenon.

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