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"Dynamic Properties of Viscoelastic Composites"

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INTRODUCTION

High structural damping can be beneficial in reducing the transmission of noise and vibrations. However, high damping materials such as plastics have usually poor mechanical properties, whereas stiff materials such as metals have negligible damping. A construction possessing high stiffness and high damping simultaneously can only be produced by a combination of two materials with different stiffness and damping, either as a composite material or as a composite structure such as a multilayer sandwich or unconstrained twin layers. Even with sandwich structure, good damping over a wide frequency range can only be achieved by incorporating several high damping viscoelastic materials with very different stiffnesses [1], while with unconstrained layer structures optimum damping occurs when the elastic modulus of the damping material approaches that of the metal base layer [2]. In this work the effects of spherical or fibrous inclusions upon the storage and quadrature moduli of an already lossy matrix material have been computed.

THEORETICAL STUDY

When either a compressional or a shear stress wave travelling in an infinitely extended matrix impinges upon an elastic inclusion two reflected waves are set up in the matrix and two refracted waves in the inclusion. It has to be assumed that there is perfect adhesion between the inclusion and the matrix to give continuity of stresses and displacement at the interface. The theory has been developed for a viscoelastic matrix with an inclusion of either spherical or cylindrical form of infinite length (a fibre) with a diameter very small compared with the wave length. The stress and strain fields in and around the inclusion can be evaluated from stress field theory allowing for the inertia effect of the inclusion. The energy stored, W', and the energy dissipated, W'', in both the matrix and the inclusion can be calculated by integrating the product of stress, σ, and the appropriate strain vector, ϵ, respectively as:-

$$W' = \tfrac{1}{2}\int \sigma' \epsilon' dv \ ; \ W'' = \pi \int \sigma'' \epsilon'' dv$$

The total energies stored and dissipated in the composite are the summation of those for the matrix and the inclusions. The overall dynamic properties (a storage modulus and a loss factor, tan δ) can be derived from these energy relations. The method is valid for low filler concentrations only. The following relationships have been obtained, where a subscipt 1 or 2 refers to the matrix or the inclusion and no subscript to the composite material, v, is the filler volume concentration and p the density.

Longitudinal properties:

Shear moduli: $\frac{\mu}{\mu_1} = \left(\frac{\rho}{\rho_1}\right)^{1/3}\left[1+\left(\frac{\mu_2}{\mu_1}+1\right)v\right]^{2/3}$

Lamé constants: $\frac{\lambda+2\mu}{\lambda_1+2\mu_1} = \left[\left(\frac{\rho}{\rho_1}\right)^{1/3}\left(1+\left[\frac{\lambda_2+2\mu_2}{\lambda_1+2\mu_1}-1\right]v\right)\right]^{2/3}$

Loss moduli: $\frac{E''}{E''_1} = \left(\frac{\rho}{\rho_1}\right)^{1/2}\left[\frac{\lambda_1+2\mu_1}{\lambda+2\mu}\right]^{1/2}\left[1+\left(\frac{E''_2}{E''_1}-1\right)v\right]$

Transverse properties:

Shear moduli: $\frac{\mu}{\mu_1} = \left(\frac{\rho}{\rho_1}\right)^{4/3}\left[1+\left[\left(\frac{\rho_2}{\rho_1}\right)^2\frac{\mu_1}{\mu_2}-1\right]v^3\right]^{-1/3}$

Poisson's ratio: $\nu = 1 - q + \sqrt{q(q-1)}$, where

$$\frac{1}{q} = \frac{\mu}{\mu_1}(1-2\nu_1)\left(\frac{\rho}{\rho_1}\right)^2\left[1+\left\{\frac{\lambda_2+2\mu_2}{\lambda_1+2\mu_1}-1\right\}v\right]$$

Loss moduli: $\frac{E''}{E''_1} = \left(\frac{\rho_1}{\rho}\right)^2\left(\frac{\lambda+2\mu}{\lambda_1+2\mu_1}\right)^2\left[1+\left\{\frac{E''_2}{E''_1}\frac{\lambda_1+2\mu_1}{\lambda_2+2\mu_2}-1\right\}v\right]$

Expressions for randomly orientated fibres have also been obtained, though more complex and requiring numerical processing [3].

For spherical inclusion:

Shear moduli: $\frac{\mu}{\mu_1} = \frac{(\rho/\rho_1)^2}{1+\left[\frac{\mu_2}{\mu_1}-1\right]v^{2/3}}$

Poisson's ratio: $\nu = \tfrac{1}{4}\left[q-1+\sqrt{(q-1)(q-3)}\right]$

where $q = (1-2\nu_1)\frac{\mu}{\mu_1}\left(\frac{\rho}{\rho_1}\right)^2\left[1+\left(\frac{3\lambda_2+2\mu_2}{3\lambda_1+2\mu_1}-1\right)v\right]$

Loss moduli:

$$\frac{E''}{E''_1} = \left(\frac{\rho_1}{\rho}\right)^2\left(\frac{\lambda+2\mu}{\lambda_1+2\mu_1}\right)^2\left[1+\left\{\frac{E''_2}{E''_1}\left(\frac{3\lambda_1+2\mu_1}{3\lambda_2+2\mu_2}\right)^2-1\right\}v\right]$$

DISCUSSION AND CONCLUSION. Theoretical curves of the storage modulus, E', and the loss factor as a function of frequency and volume concentration by weight are shown in Figure 1 for a polypropylene matrix with either glass fibres or glass beads as filler. The computation was based on the measured properties of the unfilled polypropylene.

The dynamic properties of a number of samples with different filler concentrations, made with the assistance of the Plastics Division of I.C.I. Limited, were measured at different frequencies with a B. and K. complex modulus apparatus and some of the results are shown in Figure 1. Agreement was in most cases better than 10%. The properties of the basic matrix are frequency dependent, but the inclusions do not affect further this frequency dependence, when the wave length is long compared with the size of the inclusion. However, the density ratio is important for the dynamic properties although it has no effect on the static stiffness.

With unidirectionally placed fibre inclusions the direction of the wave is important. For the longitudinal direction the storage modulus increases rapidly with an increase in volume concentration whereas the loss factor decreases. For the transverse direction there is only a slight increase in storage modulus but an appreciable increase in the loss factor. There is hardly any effect of densities on the longitudinal properties but the transverse properties increase rapidly with an increase in filler density. In both cases an increase in fibre length up to a critical value increases the storage modulus and decreases the loss factor. For randomly orientated fibres (see figure 1.a.) the values of the dynamic properties are inbetween those for the two cases of longitudinal and transverse directions and in general the storage modulus increases and the loss factor decreases with an increase in filler concentration.

With spherical particle inclusions the properties vary with filler concentration in a manner similar to the transversely reinforced composite (see figure 1.b.) and a high particle density is an advantage.

REFERENCES

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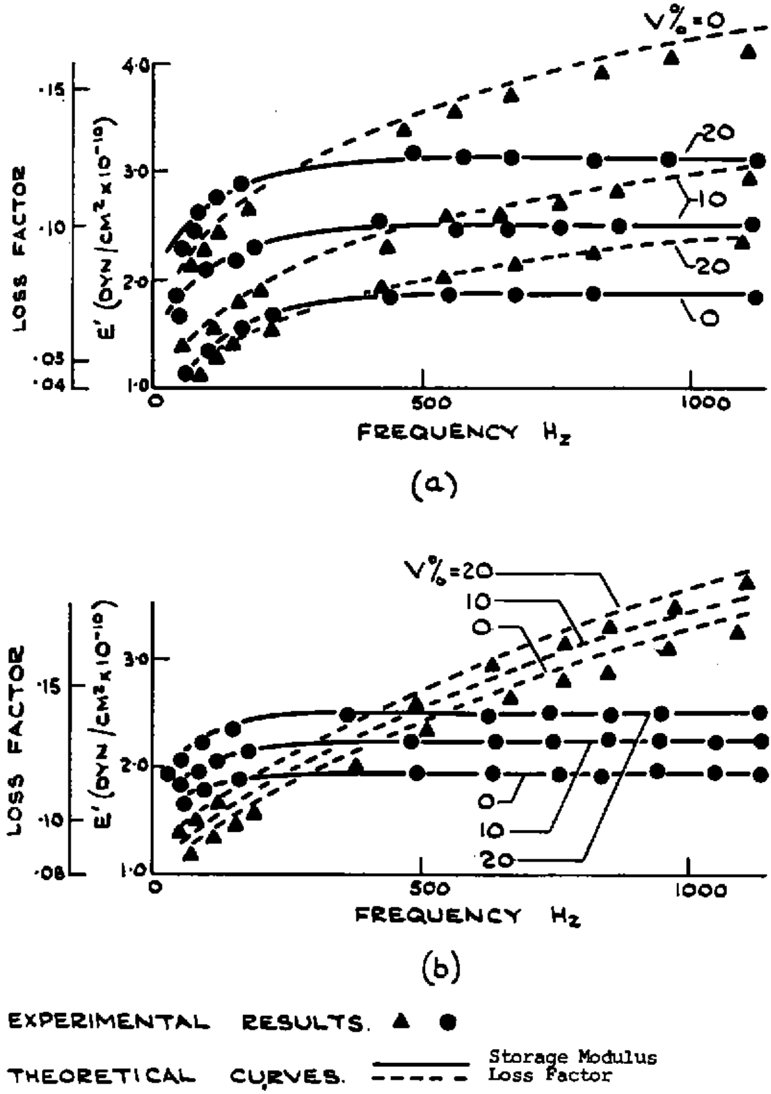

Fig. 1. Variation of the dynamic properties of some glass - polypropylene composites with filler concentration by weight and with frequency.

(a) randomly orientated glass fibres,

(b) spherical glass particles.