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EFFECTIVE PARAMETER IDENTIFICATION AND CONTROLLER DESIGN FOR ACTIVE MECHANICAL METAMATERIALS

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Recently there has been increasing interest in the design of active acoustic and mechanical metamaterials. Part of the interest lies in the ability of active material systems to provide effective properties which can be tuned to meet complex or changing performance demands. They provide an interesting framework through which some of the novel devices proposed for mechanical metamaterials can be implemented. However, the introduction of control hardware such as actuators and sensors complicates the design when compared to alternative passive approaches. In particular considerable inhomogeneity can be introduced, which can lead to an effective response that differs from that which is desired. In this paper a solution is proposed that uses the information from sensors embedded in the active system to identify its effective response, including any inhomogeneity. Under certain conditions, such an approach can also be extended to passive systems. The active system can then be subsequently modified to compensate for the identified inhomogeneity. This approach is demonstrated using a simulated example of an active mechanical metamaterial.

Keywords: Metamaterial, Active Control, Systems Identification, Effective Parameters

1. Introduction

Metamaterials are artificially created materials which derive their unique properties predominantly from their physical structure, as opposed to their chemical composition. The unique properties of mechanical metamaterials, in particular acoustic and elastic metamaterials, include the effective density and various moduli of the medium. Metamaterials have gained considerable interest from the research community as they are able to provide an effective material response for wavelengths much smaller than the spatial size of their sub-structure, which cannot be achieved with conventional materials. In particular, this includes negative effective constitutive values across a finite frequency (>0Hz) band. The ability to create materials with such a response has led to the possibility of realising designs for novel devices in a range of applications, such as acoustic imaging [1, 2].

Numerous designs for acoustic/elastic metamaterials and associated applications have been proposed. Some of these require a homogeneous effective response (the effective parameters do not change spatially throughout the metamaterial), while others inherently require a non-homogeneous (the effective parameters change spatially throughout the metamaterial) and/or anisotropic response to achieve the desired functionality [3, 4]. Progress in metamaterials research has inevitably led to a recent focus on manufacturing technologies and experimental validation. However, this leads to a fundamental problem. Existing methods to extract the effective properties of a metamaterial from experimental data assume that the metamaterial has a homogeneous effective parameter distribution [5, 6, 7]. While some functionality, such as the effectiveness of a lens to focus correctly, can be deduced from conventional experiments, the underlying properties of the metamaterial cannot be sufficiently determined in the case of a non-homogeneous design. The result is a incomplete set of analysis tools to completely understand the behaviour of metamaterials, validate new designs and to diagnose problems. As an example, inconsistencies in the manufacturing process could lead to an undesired non-homogeneity affecting the dynamic response of an otherwise homogeneous design and at the moment, the tools do not exist to be able to characterise the level of non-homogeneity.

In this paper a technique for homogeneous parameter estimation is extended to allow non-homogeneous parameters to be estimated from experimental data. The resulting method is validated using simulated data. The power of the technique to identify resulting non-homogeneity, which can subsequently be eliminated through appropriate design choices is also demonstrated. In this paper, the approach adopted to eliminate the non-homogeneity is to spatially tune the control parameters in an active elastic metamaterial. An active metamaterial is a system in which additional control forces generated by attached actuators and determined from sensor measurements and a control algorithm are applied to the system. Alternative approaches to this elimination include re-design and manufacture of the metamaterial or the incorporation of adaptive elements.

2. Theory - Effective Parameter Identification

The use of lumped element models is sufficient to represent a range of acoustic/elastic metamaterials due to the inherent requirement for a sub-wavelength structure. Figure 1 shows a basic infinite sequence mass-in-mass locally resonant metamaterial to which an active control force f_c can be applied to modify the response of the metamaterial [8, 9]. Such a metamaterial can be represented by the effective system of Eq. (1). For the passive case ($f_c = 0$) the effective parameters are described by the frequency response transfer functions in Eq. (2) and (3). For appropriately selected material parameters this class of metamaterial can posses a negative effective mass across a finite frequency band, i.e. a single negative metamaterial. The application of the feedback control force f_c can be used to further modulated both the effective mass and stiffness of such a metamaterial.

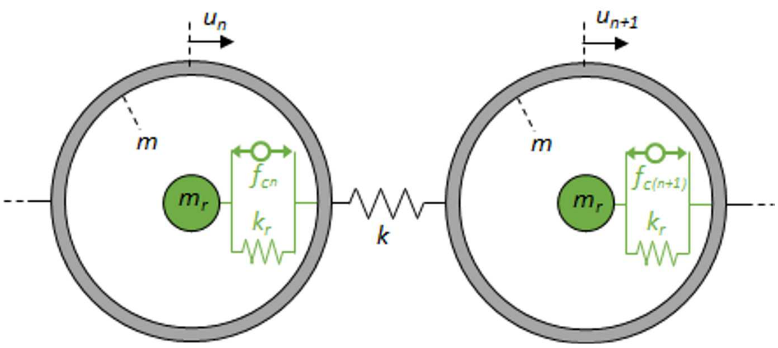

Figure 1: Two units of a mass-in-mass locally resonant active metamaterial

$$-m_e(i\omega)\,\omega^2 U_n = k_e(i\omega)\left(U_{n-1} + U_{n+1} - 2U_n\right) + F_n \tag{1}$$

$$m_e(i\omega) = m + \frac{m_r k_r}{\left(-m_r\omega^2 + k_r\right)} \tag{2}$$

$$k_e(i\omega) = k \tag{3}$$

2.1 Homogeneous Effective Parameter Identification

Equation (1) represents the situation where the effective response of the system has a homogeneous nature. The effective system can be written in terms of the frequency response functions (FRFs) between the accelerations and exogenous applied forces in the system, which leads to Eq. (4).

$$T\left(i\omega\right)_{F_n,A_n} = m_e\left(i\omega\right) - k_e\left(i\omega\right)\left(2 - T\left(i\omega\right)_{A_{n-1},A_n} - T\left(i\omega\right)_{A_{n+1},A_n}\right)\omega^{-2} \tag{4}$$

This sequence of mass-spring units can be represented in matrix form $T_F = T_A\beta$, with T_F, T_A and β given by Eq. (5)-(7) respectively. On the assumption that the FRFs can be determined and the sequence of units is finite, this system of equations can be solved using the standard least squares solution to give the effective parameter vector at each frequency, i.e. $\beta = \left(T_A^T T_A\right)^{-1} T_A^T T_F$ [9].

$$T_F = \begin{bmatrix} \vdots \\ T\left(i\omega\right)_{F_n,A_n} \\ T\left(i\omega\right)_{F_{n+1},A_{n+1}} \\ \vdots \end{bmatrix} \tag{5}$$

$$T_A = \begin{bmatrix} \vdots & \vdots \\ 1 & \left(T\left(i\omega\right)_{A_{n-1},A_n} + T\left(i\omega\right)_{A_{n+1},A_n} - 2\right)\omega^{-2} \\ 1 & \left(T\left(i\omega\right)_{A_n,A_{n+1}} + T\left(i\omega\right)_{A_{n+2},A_{n+1}} - 2\right)\omega^{-2} \\ \vdots & \vdots \end{bmatrix} \tag{6}$$

$$\beta = \begin{bmatrix} m_e\left(i\omega\right) \\ k_e\left(i\omega\right) \end{bmatrix} \tag{7}$$

2.2 Non-Homogeneous Effective Parameter Identification

The assumption of homogeneity is relatively strong and not valid when manufacturing variance can lead to different parameter values, or for applications which require inherent homogeneity. In such a situation the least squares parameter estimation approach can be modified. For the example presented here, it is assumed that the non-homogeneous system is represented by Eq. (8). However, the set of individual equations formed from Eq. (8) are underspecified and cannot be solved uniquely using least squares. An alternative effective system is described by Eq. (9). In this system the transmission parameters (outer mass and connecting springs) are separated from the local resonators and are assumed to be homogeneous. The non-homogeneity is then introduced into the local resonators.

$$F_n = -m_{e_n}\left(i\omega\right)\omega^2 U_n + k_{e_n}\left(i\omega\right)\left(2U_n - U_{n-1} - U_{n+1}\right) \tag{8}$$

$$F_n = -\left(m_t\left(i\omega\right) + m_{e_n}\left(i\omega\right)\right)\omega^2 U_n + \left(k_t\left(i\omega\right) + k_{e_n}\left(i\omega\right)\right)\left(2U_n - U_{n-1} - U_{n+1}\right) \tag{9}$$

The parameters in Eq. (9) can be solved in a two step process and requires some pre-requisite knowledge of the system. The first step is to use the homogeneous parameter identification method described in section 2.1 to estimate the transmission parameters. This requires the transmission system to be measured in isolation without the local resonators attached. The second step is to then estimate the parameters of the non-homogeneous locally resonant system. In the case of the passive system Eq. (1)-(3) the metamaterial is of the single negative type and the contribution to the effective stiffness $k_{e_n} = 0$. By combining the contribution from the transmission system determined in step one with the force-acceleration transfer functions in T_F, the exactly specified set of equations described by Eq. (10) - (13) can be solved to give the contribution m_{e_n} from the local resonators to each of the effective masses. Any further contribution from the active control system $f_c \neq 0$ can then be determined in a similar manner using a third step.

$$T_F = \begin{bmatrix} \vdots \\ T(i\omega)_{F_n, A_n} - T_{t_n} \\ T(i\omega)_{F_{n+1}, A_{n+1}} - T_{t_{n+1}} \\ \vdots \end{bmatrix} \quad (10)$$

$$T_{t_n} = m_t(i\omega) + k_t(i\omega)\left(T(i\omega)_{A_{n-1}, A_n} + T(i\omega)_{A_{n+1}, A_n} - 2\right)\omega^{-2} \quad (11)$$

$$T_A = \begin{bmatrix} \ddots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \ddots \end{bmatrix} = I \quad (12)$$

$$\beta = \begin{bmatrix} \vdots \\ m_{e_n}(i\omega) \\ m_{e_{n+1}}(i\omega) \\ \vdots \end{bmatrix} \quad (13)$$

3. Simulation Results - Effective Parameter Identification

As an example of application of the non-homogeneous parameter estimation method it is applied to a simulated mass-in-mass system of the type shown in Fig. 1. The simulated system consists of three mass-in-mass units. Mass 1 is subjected to a free boundary condition and mass 3 is connected to a fixed termination through a spring of stiffness k. The values for the mass and spring elements for the homogeneous case are given in Table 1. The transmission values have been normalised in terms of the transmission mass m and the local resonator values selected to give a resonance at 10Hz and a corresponding negative mass of approximately -1Kg in the region immediately above the resonant frequency. The damping in the system is represented as complex values for the springs, which is equivalent to the Kelvin-Voigt model of damping. Non-homogeneity is introduced into the system by changing the resonator stiffness, which allows the frequency and associated peak negative values associated with each of the resonators to be changed. All simulations have been conducted in MatLab-Simulink. The FRF's are calculated using a 60 second window of data taken at a sample rate of 2kHz.

Table 1: Parameter values for the mass-in-mass locally resonant metamaterial

Parameter	Value
m	1 Kg
k	1000+50$i\omega$ N/m
m_r	0.5 Kg
k_r	19739+3$i\omega$ N/m

3.1 Non-Homogeneous Effective Mass Identification

To show the comparable performance of the two parameter estimation mechanisms they are applied to data from the simulated system. The non-homogeneity is in the form of different resonant frequencies for each of the mass-in-mass units. These frequencies are 8, 10 and 12Hz for units 1, 2 and 3 respectively. The imaginary part of the resonator stiffness is adjusted to ensure that the damping ratio for the resonators match. Figure 2 shows the results from the homogeneous parameter estimation method, together with the model values (i.e. Eq. (2) and (2)) for each of the three mass-in-mass units. It is apparent from panel (a) that the estimated effective mass (cyan) does not capture all the system dynamics introduced by the non-homogeneity. It is evident that the estimated effective mass is dominated by the response of the first unit. In contrast, the effective stiffness is estimated to a much higher level of accuracy since it is homogeneous (the model parameters for the three units overlay exactly). The poor estimation of the parameters of the non-homogeneous system is supported by panels (c) and (d), which show the estimation residuals (the rows of $T_F - T_A\beta$) for each of the three units. For all units the residuals are relatively high, in particular at frequencies away from their local resonances.

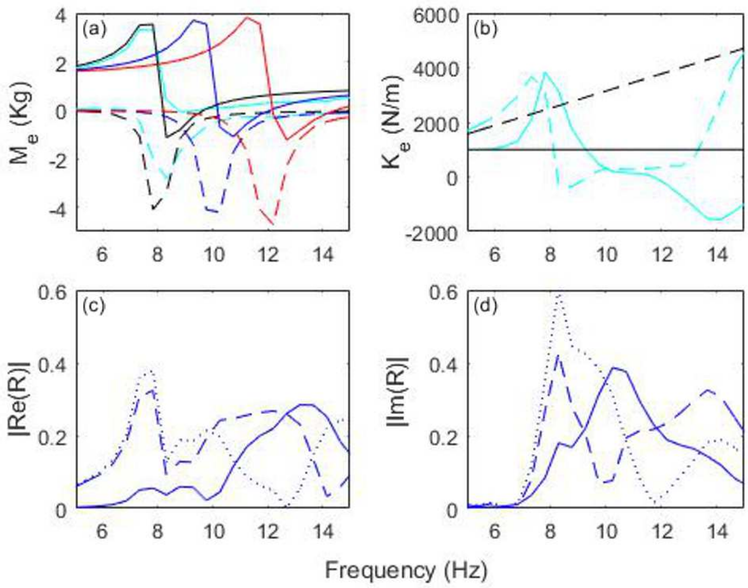

Figure 2: Panels (a) and (b) are the effective mass and stiffness of the non-homogeneous metamaterial (solid - real part; dashed - imaginary part). The cyan line is the estimated homogeneous effective parameter and the black, blue and red lines are the model response of the first, second and third mass-in-mass units respectively. Panels (c) and (d) are the real and imaginary parts of the residuals of the estimated system separated for each of the three units (solid - unit 1; dashed - unit 2; dotted - unit 3).

Figure 3 shows the results from applying the non-homogeneous parameters estimation method to the simulated data. The estimated parameters (solid and dashed lines) for each of the three units much better match the model parameters (crosses and circles), when compared to the results in Figure 2. The residuals (not shown) for the non-homogeneous method are zero since Eq. (12) is the identity matrix and therefore each value for Eq. (13) is solved exactly. However, while the overall dynamics, in-particular the location of the resonances are estimated well, there are errors in the estimated magnitudes of the resonances. The reason is that the error lies in the calculation of the FRF's. This is likely to be improved by a combination of a higher sample rate and longer data set. The sample rate used is chosen to match the sample rate of the experimental data used in Section **??**.

3.2 Application to an active metamaterial

One of the benefits of active metamaterials is that the control system can be designed to cancel out or introduce additional non-homogeneity into a passive structure. This is demonstrated in Fig. 4. The active metamaterial design follows that of previous work [9] and the control force f_c applied

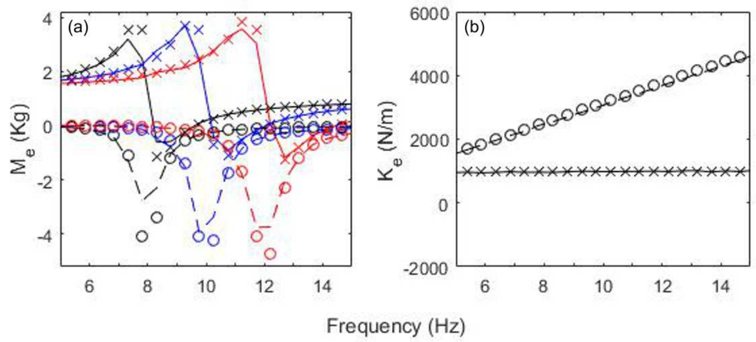

Figure 3: Panels (a) and (b) are the effective mass and stiffness of the non-homogeneous metamaterial. The solid and dashed lines are the real and imaginary parts of the estimated parameters and the crosses and circles are the real and imaginary parts of the model parameters respectively. The black, blue and red lines are the response of the first, second and third mass-in-mass units respectively.

to the passive metamaterial described by Eq. (1) - (3) is $f_{c_n} = -m_{c_n}\omega^2 U_n$. The control force is proportional to the acceleration of the transmission masses and by changing the value of the feedback gain m_c the magnitude of the effective mass near the resonant frequency can be tuned [9]. The resulting effective mass is now described by Eq. (14). In this simulation the resonant frequency of the passive local resonators for each units is the same and the non-homogeneity is in the damping ratio of the resonators. Specifically, the damping ratios of the resonators for units 1-3 are 100%, 150% and 50% of the nominal values resulting from the parameters in Table 1.

$$m_e\left(i\omega\right) = m + \frac{m_r\left(-m_c\omega^2 + k_r\right)}{\left(-m_r\omega^2 + k_r\right)} \tag{14}$$

If the effective response of the active system can be described by Eq. (15), the effect of the feedback gains m_{c_n} for each of the units can be determined using a three step process. The first two steps are identical to that used in Section to identify the effective parameters of the passive metamaterial. Once these have been identified, a third step, which mirrors that of step two, is used to identify the effect of the active force which is added to the passive metamaterial, i.e. $m_{a_n}\left(i\omega\right)$.

$$F_n = -\left(m_t\left(i\omega\right) + m_{e_n}\left(i\omega\right) + m_{a_n}\left(i\omega\right)\right)\omega^2 U_n + \left(k_t\left(i\omega\right) + k_{e_n}\left(i\omega\right)\right)\left(2U_n - U_{n-1} - U_{n+1}\right) \tag{15}$$

Once the effect of the control parameter m_{c_n} for each of the units has been identified a desired response can be achieved by appropriate selection of the control parameters. For this example, the desired response is to use the control system to remove the non-homogeneity in the passive system and create a metamaterial with a peak negative value in the real part of the effective mass of -2Kg. The control parameters required to achieve this are identified using this three step estimation process as $m_{c_1} = -0.32$, $m_{c_2} = -0.3$ and $mc_3 = 0$. The result of using these control parameters are shown by the blue lines in Fig. 4. These are the estimated response of the active metamaterial subjected to the control parameters, while the black lines are the estimated response of the passive non-homogeneous metamaterial. The control system creates the desired non-homogeneity in the real part of the effective mass, but with a detrimental effect on the imaginary part. The control system could be further extended to include an additional derivative term which would used to control the imaginary part of the effective mass.

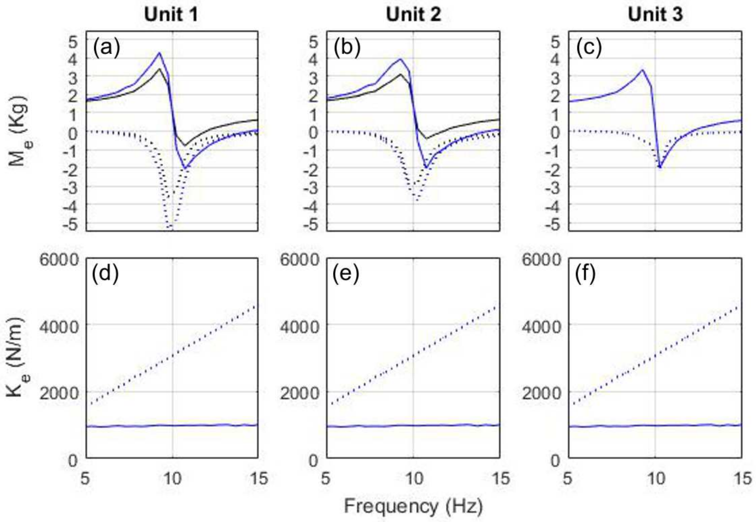

Figure 4: (a) - (c) are the estimated effective mass and (d) - (f) are the estimated effective stiffness for the three units of the metamaterial (solid - real part; dotted - imaginary part). The black lines represent the non-homogeneous passive metamaterial, while the blue lines represent the active metamaterial designed to compensate for the non-homogeneity.

4. Conclusion

The presence of spatial non-homogeneity in the effective parameters of metamaterials arise either through design (required to meet a particular application specification) or defects/inconsistency during manufacture (can lead to reduced performance if the non-homogeneity is significant). It is therefore important to be able to characterise any potential non-homogeneity to ensure that the requirements have been achieved. Previous methods to extract the effective properties of a metamaterial from experimental data rely on the assumption of a homogeneous response. In this paper, a "grey-box" method to estimate the effective parameters of non-homogeneous metamaterials is proposed and then validated using simulated data. An example, whereby the identified non-homogeneity can be removed through use of an active metamaterial design is subsequently demonstrated.

REFERENCES

1. Guenneau, S., Movchan, A., Petursson, G. and Ramakrishna, S. A. Acoustic metamaterials for sound focusing and confinement, *New J. Phys.*, **9**, 399, (2007).

2. Wen, J., Shen, H., Yu, D. and Wen, X. Exploration of amphoteric and negative refraction imaging of acoustic sources via active metamaterials, *Phys. Lett. A*, **377** (2199-2206), 167–170, (2013).

3. Cummer, S. A. and Schurig, D. One path to acoustic cloaking, *New Journal of Physics*, **9** (3), 45–45, (2007).

4. Popa, B.-I. and Cummer, S. A. Design and characterization of broadband acoustic composite metamaterials, *Physical Review B*, **80** (17), (2009).

5. Fokin, V., Ambati, M., Sun, C. and Zhang, X. Method for retrieving effective properties of locally resonant acoustic metamaterials, *Phys. Rev. B*, **76**, 144302, (2007).

6. Chen, H., Zeng, H., Ding, C., Luo, C. and Zhao, X. Double-negative acoustic metamaterial based on hollow steel tube meta-atom, *J. Appl. Phys.*, **113** (10), 104902, (2013).

7. Xie, Y., Popa, B.-I., Zigoneanu, L. and Cummer, S. A. Measurement of a broadband negative index with space-coiling acoustic metamaterials, *Phys. Rev. Lett.*, **110**, 175501, (2013).

8. Pope, S. A. and Daley, S. Viscoelastic locally resonant double negative metamaterials with controllable effective density and elasticity, *Phys. Lett. A*, **374** (41), 4250–4255, (2010).

9. Pope, S. A. and Laalej, H. A multi-layer active elastic metamaterial with tuneable and simultaneously negative mass and stiffness, *Smart Mater. Struct.*, **23** (7), 075020, wOS:000339657900036, (2014).