ITERATIONAL METHOD IN SOUND SCATTERING PROBLEM ON ELASTIC SHELLS

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1. INTRODUCTION

A method consecutive iterations is used by definition of boundary conditions on bodies in the scattering problem. If on the surface of the shell the oscillating velocity is given, it is necessary to know the preassure too for solving the integral Helmgoltz's equation. The method is effective when the shell may be split on pieces which are parts of canonical bodies. The angle dependence of the scattering amplitude for garmonical wave is obtained. The method has features like Schwarzshild's method. The angle dependence of the input of the butt-ends of the shell is calculated. Minimums of backscattering indicatris and small incident wave angles are analysed. The method permits to obtaine information of geometrical and elastic parameters of the scatterer.

2. ITERATIONAL METHOD

Let us consider a plane sound wave incidence on shell, which consists of elastic cylindrical part and two absolutely solid hemospherical ends:

$$P_{inc} = A_0 \exp\left[i\left(k_z z + K_y y + K_x x\right)\right]$$

(A_0 - amplitude of the incidence wave; $k_0 = \{k_x; k_y; k_z\}$; $K = \omega / c_0$ - wave number in surrounding media; ω -frequence in rad.; c_0 - sound speed in media; $K_z = K_0 \cos(\theta_{inc})$; $K_x = 0$; $K_y = -K_0 \sin(\theta_{inc})$; θ_{inc} - the sliding angle of incident wave). Besides rectangular coordinate sistem we introduce a cylindrical and spherical sistem too.

Let us split the diffraction problem on two stages. The first- diffraction of absolutely solid body. The second - diffraction with taking into account the elasticity of the shell. We shall use the iterational method for the diffraction problem on the absolutely solid body. The sence of the method is following: let us consider three bodies: the sphere $S_1 = S_1^1 + S_1^2$, the sphere $S_2 = S_2^1 + S_2^2$ and the infinite cylinder $C = C_0 + C_1 + C_2$. On the surface of the cylinder C the normal velosity $V_c(z)$ is given. When the other bodies are absent: $V_{c_0}(z) \neq 0$, $V_{c_1}(z) = 0$,

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$V_{C_2}(z) = 0$. On the surfaces S_1 and S_2 normal velocities $V_{S_1}(\theta_1) \neq 0$ and $V_{S_2^1}(\theta_2) \neq 0$, $V_{S_1^2}(\theta_1) = V_{S_2^2}(\theta_2) = 0$ are given too. Let us difine these velocities in such way, that produced by S_1 , S_2 and C the full field satisfies to the condition of absolut hardness of the surfaces S_1^1 , S_2^1 , C_0 . If these velocities are found the diffraction problem on absolutely hard shell will be solved.

The zero-approuch we will take in form: $V_{C_0}^0 = - V_{inc}|_{C_0}$; $V_{S_1^1}^0 = - V_{inc}|_{S_1^1}$; $V_{S_2^1}^0 = - V_{inc}|_{S_2^1}$, where V_{inc} - is the incident wave velocity.

The next step is the calculation of the velocity $V_{C_0, S_1^1}^1$ produced by the cylinder C_0 on the surface S_1^1 . Adding to $V_{S_1^1}$ the quantity $- V_{C_0, S_1^1}^1$ we obtain a new distribution of velocity on S_1^1 which produces a correction to the velocity $V_{C_0}^0$ on the surface of the cylinder in its turn. Changing the sign of this correction we radiate by this velocity on S_1^1 again and so on. This procedure is useful in the case when the sphere S_2 is absent and we have a hemiinfinite cylinder. When the shell is limited, the iterational procedure is analogical to the described above with taking into account the mutual influence of the spheres S_1 and S_2 . For this method it is essential that the bodies S_1 , S_2 and C have canonical surfaces, which leads to devide the variables when solving the Helmgoltz equation. This iterational procedure is like the Schwarzshild's method for two bodies.

Let us define the direct and inverse Fourier-transform as following:

$$\Phi(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ikx}\,\Phi(x)\,dx \quad ; \quad \Phi(x) = \int_{-\infty}^{+\infty} e^{ikx}\,\Phi(k)\,dk \, .$$

Let the pressure $P_{rad}(\rho, z, \varphi)$ produced by cylinder have the form ($\varkappa = (k_0^2 - k_z^2)^{1/2}$) :

$$P_{rad}(\rho, z, \varphi) = \sum_{m=0}^{\infty} \cos(m\varphi) \int_{-\infty}^{+\infty} e^{ik_z z}\, H_m^{(1)}(\varkappa\rho)\, A_m(k_z)\, dk_z \, . \qquad (1)$$

Because both the incident field and shell have a plane of symmetry we have in this expantion only terms with $\cos(m\varphi)$. $H_m^{(1)}(\varkappa\rho)$ - Hankel-function of number m . $A_m(k_z)$ - the corresponding Fourier-amplitude. For the term of number m of the normal part of the velosity on the surface of cylinder we obtain :

$$V_{rad}^{m}\Big|_{C_0} = (i\omega\rho_0)^{-1} \int_{-\infty}^{+\infty} e^{ik_z z}\, H_m^{(1)\prime}(\varkappa R)\, \varkappa\, A_m(k_z)\, dk_z \, ,$$

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$H_m^{(1)\prime}(\varkappa R)$ - the derivative of Hankel-function with respect to its argument, ρ_0 - the medium density. The cylinder impedance we define as [1] :

$$Z^{m}_{rad}(k_z) = \frac{P^{m}_{rad}(k_z)}{V^{m}_{rad}(k_z)} = \frac{(i\omega\rho_0 R)\ H_m^{(1)}(\varkappa R)}{(\varkappa R)\ H_m^{(1)\prime}(\varkappa R)} \quad (2)$$

It is easy to obtain an expression for $A_m(k_z)$ in (1) :

$$A_m(k_z) = \frac{(i\omega\rho_0 R)\ V^{m}_{rad}(k_z)}{(\varkappa R)\ H_m^{(1)\prime}(\varkappa R)} \quad (3)$$

Let us find the velocity $V^{m,1}_{c_0,S_1^1}(\theta_1)$ produced by cylinder C_0 on the surface S_1^1 . Using (1) and (3) we obtain:

$$V^{m,1}_{c_0,S_1^1}(\theta_1) = \sin\theta_1\left(\int_{-\infty}^{+\infty} e^{ik_z z}\frac{H_m^{(1)\prime}(\varkappa\rho)}{H_m^{(1)\prime}(\varkappa R)}V^{m}_{rad}(k_z)\,dk_z\right) + \\ +\cos\theta_1\left(\int_{-\infty}^{+\infty} e^{ik_z z}\cdot\frac{(ik_z)\ H_m^{(1)}(\varkappa\rho)}{\varkappa\ H_m^{(1)\prime}(\varkappa R)}V^{m}_{rad}(k_z)\,dk_z\right), \quad (4)$$

where $z = R\cos\theta_1$; $\rho = R\sin\theta_1$; $\pi \le \theta_1 \le \pi/2$.

The velocity $V^{m,1}_{c_0,S_2^1}(\theta_2)$ produced by cylinder C_0 on the surface S_2^1 has the form:

$$V^{m,1}_{c_0,S_2^1}(\theta_2) = \sin\theta_2\cdot\left(\int_{-\infty}^{+\infty} e^{ik_z z}\frac{H_m^{(1)\prime}(\varkappa\rho)}{H_m^{(1)\prime}(\varkappa R)}V^{m}_{rad}(k_z)\,dk_z\right) + \\ +\cos\theta_2\left(\int_{-\infty}^{+\infty} e^{ik_z z}\frac{(ik_z)\ H_m^{(1)}(\varkappa\rho)}{H_m^{(1)\prime}(\varkappa R)\cdot\varkappa}V^{m}_{rad}(k_z)\,dk_z\right), \quad (5)$$

where $z = R\cos\theta_2 + L$; $\rho = R\sin\theta_2$; $0 \le \theta_2 \le \pi/2$.

The velocities (4) and (5) have peculiarities $\sim \ell_n(\varkappa R\sin\theta_{(1,2)})$ by $\theta_1 \to \pi$ and $\theta_2 \to 0$. These peculiarities are being square integrated and do not influate on the essence of the method.

The field of the spere S_1^1 let us introduce in form:

$$P_{sF}(r,\theta_1,\varphi) = \sum_{n=0}^{\infty} h_n^{(1)}(k_0 r)\sum_{m=0}^{n} P_{m,n}(\cos\theta_1)\,\beta^{(1)}_{m,n}\cdot\cos(m\varphi) \quad , (6)$$

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where $h_n^{(1)}(k_0 r) = (\pi/(2k_0 r))^{1/2} H^{(1)}_{n+1/2}(k_0 r)$ - spherical Hankel-function ; $P_{m,n}(\cos\Theta_1)$ - Lejandre-polynom.

Using the ortogonality of Lejandre polinoms for $\beta^{(1)}_{m,n}$ we obtain:

$$\beta^{(1)}_{m,n} = \frac{(2n+1)\left[(n-m)!\right]}{2\left[(n+m)!\right]}\left(\int_0^{\pi} V^m_{S_1}(\Theta_1) P_{m,n}(\cos\Theta_1)\sin\Theta_1 d\Theta_1\right)\frac{i\omega\rho_0}{k_0 h_n^{(1)\prime}(k_0 R)} \tag{7}$$

The velocity $V^{m,1}_{S_1^1,C_0}(z)$ produced by the sphere S_1 on the surface C_0 has the form:

$$V^{m,1}_{S_1^1,C_0}(z) = \sin\Theta_1\left(\sum_{n=0}^{\infty}\frac{h_n^{(1)\prime}(k_0 r_1)}{h_n^{(1)\prime}(k_0 R)} P_{m,n}(\cos\Theta_1)\cdot\beta^{(1)}_{m,n}\right) -$$

$$-\frac{\sin^2\Theta_1\cdot\cos\Theta_1}{(k_0 R)}\left(\sum_{n=0}^{\infty}\frac{h_n^{(1)}(k_0 r_1) P'_{m,n}(\cos\Theta_1)}{h_n^{(1)\prime}(k_0 R)}\beta^{(1)}_{m,n}\right),$$

where $P'_{m,n}(\cos\Theta_1)$ - the derivative of the Lejandre polinom with respect to argument; $r_1 = R/\sin\Theta_1$; $\Theta_1 = \mathrm{arctg}(R/z)$. For the second sphere we obtain:

$$V^{m,1}_{S_2^1,C_0}(z) = \sin\Theta_2\left(\sum_{n=0}^{\infty}\frac{h_n^{(1)\prime}(k_0 r_2)}{h_n^{(1)\prime}(k_0 R)} P_{m,n}(\cos\Theta_2)\cdot\beta^{(2)}_{m,n}\right) -$$

$$-\frac{\sin^2\Theta_2\cos\Theta_2}{(k_0 R)}\left(\sum_{n=0}^{\infty}\frac{h_n^{(1)}(k_0 r_2) P'_{m,n}(\cos\Theta_2)}{h_n^{(1)\prime}(k_0 R)}\beta^{(2)}_{m,n}\right),$$

where $r_2 = R/\sin\Theta_2$; $\Theta_2 = \pi/2 + \mathrm{arctg}(z'/R)$; $z' = L - z$.

Analogical formulae may be obtained for the mutual influence of the spheres. For the initial distribution of the normal velocity on the surfaces of cylinder and spheres we take the incident velocity with inverse sign:

$$V^{m,0}_{C_0}(z) = -V^m_{inc}(z)\big|_{C_0} = -(i\omega\rho_0)^{-1} A_0 e^{ik_z z}(i)^m \varepsilon_m J'_m(k_y R)\, k_y \quad ;$$

$$V^{m,0}_{S_1^1}(\Theta_1) = -V^m_{inc}(\Theta_1)\big|_{S_1^1} = (i\omega\rho_0)^{-1} A_0 e^{i(k_z R\cos\Theta_1)}(i)^m \varepsilon_m \times$$

$$\times\left(J_m(k_y R\sin\Theta_1)(ik_z\cos\Theta_1) + J'_m(k_y R\sin\Theta_1)\cdot(k_y\sin\Theta_1)\right) \quad ;$$

$$V^{m,0}_{S_2^1}(\Theta_2) = -V^m_{inc}(\Theta_2)\big|_{S_2^1} = (i\omega\rho_0)^{-1} A_0 (i)^m \varepsilon_m \exp\left\{i(k_z R\cos\Theta_2) + ik_z L\right\}\times$$

$$\times\left(J_m(k_y R\sin\Theta_2)(ik_z\cos\Theta_2) + J'_m(k_y R\sin\Theta_2)\cdot(k_y\sin\Theta_2)\right),$$

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where $\varepsilon_0 = 2$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 ... = 1$.

Using the formulae (1) , (3) and (6) , (7) , for the far field we obtain:

$$P^m(\theta_1,r_1) = (i\omega\rho_0)\left[\int_{-\infty}^{+\infty} \frac{e^{ik_z z} H_m^{(1)}(\varkappa\rho)\, V_{rad}^m(k_z)}{\varkappa\, H_m^{(1)\prime}(\varkappa R)}\, dk_z \; + \right.$$

$$\left. + \left(\sum_{n=0}^{\infty} \frac{h_n^{(1)}(k_0 r_1)}{h_n^{(1)\prime}(k_0 R) k_0} P_{m,n}(\cos\theta_1)\cdot\beta_{m,n}^{(1)}\right) + \left(\sum_{n=0}^{\infty} \frac{h_n^{(1)}(k_0 r_2)}{h_n^{(1)\prime}(k_0 R) k_0} P_{m,n}(\cos\theta_2)\cdot\beta_{m,n}^{(2)}\right)\right], \quad (8)$$

where $\rho = r_1 \sin\theta_1$; $z = r_1\cos\theta_1$; $r_2 = (r_1^2 + L^2 - 2Lr_1\cos\theta_1)^{1/2}$.

The scattering amplitude is:

$$f = \left(\sum_{m=0}^{\infty} P^m(\theta_1, r_1)\right) / \left(A_0 i\omega\rho_0 \left(\exp\{ik_0 r_1\}/r_1\right)\right).$$

Let us define $R_s = 20 \log(2|f|)$. The integration in (8) was produced by means of Stacionary phase method for large $k_0 r_1$. In the case $m = 0$ this integral may be easy calculated. Thus, the contribution in the scattering amplitude from cylindrical part of the body is equal:

$$f_y = \frac{(-2i)}{k_0 \sin\theta_1} \sum_{m=0}^{\infty} \frac{V_{rad}^m(k_0\cos\theta_1)}{H_m^{(1)\prime}(k_0 R\sin\theta_1)} \exp\left\{-im\frac{\pi}{2}\right\} .$$

It is necessary to take into account the eigenfunctions of the shell, when considering the diffraction on the elastic shell. These eigenfunctions depend on the boundary conditions. Let us suppose for the simplicity the conditions of hinges. On the ends of the shell the hinges are leaned on two absolutely solid hemispheres. In this case when $z = 0, L$ we have such end conditions:

$$W_3(z) = 0 \;; \quad \frac{\partial^2 W_3(z)}{\partial z^2} = 0 \;,$$

which means that the normal displacements and the bending moments are equal to zero. We use the equations of movement of thin cylindrical shell in form:

$$L_{ij}\, \dot{W}_j(z,\varphi) = \delta_{i3}\gamma_1 \left(P_{inc}(z,\varphi,R) + P_{rad}(z,\varphi,R)\right) , \quad (9)$$

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where L_{ij} -matrix of selfconjugated differential operators; δ_{i3} - Kronecer's symbol; $\hat{\gamma_1} = (1-\nu^2)/Eh$; E , ν - Joung's modulus and Poisson's coefficient of the material of the shell correspondingly.

The solution of the system (9) we construct in the following form:

$$\dot{W}_1(z,\varphi) = \sum_{p=0}^{\infty}\sum_{m=0}^{\infty} \dot{W}_1^{mp} \cos(K_p z)\cos(m\varphi),$$
$$\dot{W}_2(z,\varphi) = \sum_{p=0}^{\infty}\sum_{m=0}^{\infty} \dot{W}_2^{mp} \sin(K_p z)\sin(m\varphi), \quad (10)$$
$$\dot{W}_3(z,\varphi) = \sum_{p=0}^{\infty}\sum_{m=0}^{\infty} \dot{W}_3^{mp} \sin(K_p z)\cos(m\varphi).$$

Because P_{inc} is event function of φ, P_{rad} is event too:

$$P_{inc}(z,\varphi,R) = \sum_{p=0}^{\infty}\sum_{m=0}^{\infty} P_{inc}^{mp} \sin(K_p z)\cos(m\varphi), \quad (11)$$
$$P_{rad}(z,\varphi,R) = \sum_{p=0}^{\infty}\sum_{m=0}^{\infty} P_{rad}^{mp} \sin(K_p z)\cos(m\varphi),$$

where $K_p = \pi p / L$.

It is easy to obtain an expression for mechanical impedance Z_y^{mp} by elastic vibrations of shell [1] :

$$Z_y^{mp} = -\frac{i C_{\ell_0}^2 h \rho_M}{\omega R^2}\left(\frac{|A_0^{mp}|}{|A_{33}^{mp}|}\right),$$
$$|A_0^{mp}| = \det|L_{ij}^{mp}| \;;\; |A_{33}^{mp}| = L_{11}^{mp} L_{22}^{mp} - L_{12}^{mp} L_{21}^{mp}.$$

For the solution of the problem it is necessary to define V_{rad}^{mp} through

We use equations: $P_{inc}^{mp} = V_{inc}^{mp} Z_{inc}^{mp}$; $P_{rad}^{mp} = V_{rad}^{mp} Z_{rad}^{mp}$;

$$P_y^{mp} = V_y^{mp} Z_{rad}^{mp} \;;\; P_{SF}^{mp} = V_{SF}^{mp} Z_{SF,S}^{mp} \;;$$
$$P_{rad}^{mp} + P_{inc}^{mp} + P_y^{mp} + P_{SF}^{mp} = -(V_{rad}^{mp} + V_{inc}^{mp} + V_y^{mp} + V_{SF}^{mp}) \cdot Z_y^{mp},$$

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where P_y^{mp} and V_y^{mp} - amplitudes of pressure and velocity which are additions because of the elasticity of the shell; P_{sf}^{mp} and V_{sf}^{mp} - amplitudes of pressure and velocity on the surface of cylinder due to the influence of the hemispheres.

Finally we obtain:

$$V_y^{mp} = -V_{inc}^{mp}\left(\frac{Z_{inc}^{mp} - Z_{rad}^{mp}}{Z_y^{mp} + Z_{rad}^{mp}}\right) - \frac{\left(P_{sf}^{mp} - V_{sf}^{mp} Z_{rad}^{mp}\right)}{\left(Z_y^{mp} + Z_{rad}^{mp}\right)} . \qquad (12)$$

The full vibrating velocity in the scattering wave:

$$V_{rad,f}^{mp} = -V_{inc}^{mp} - V_{sf}^{mp} + V_y^{mp} ,$$

where Z_{inc}^{mp} and Z_{rad}^{mp} we take in form (2).

When calculating impedance Z_{rad}^{mp} we take into account that the shell is limited [2] . The impedance of limited shell $Z_{rad,s}^{mp}$ is:

$$Z_{rad,s}^{mp}(k) = \frac{1}{\pi L}\int_{-\infty}^{\infty} Z_{rad}^{m}(k)\, F(k_p,k,L)\, F(k_p,(-k),L)\, dk ,$$

where

$$F(k_p,k,L) = \frac{1}{2i}\left\{\frac{\left(e^{i(k_p-k)L}-1\right)}{i\,(k_p-k)} + \frac{\left(e^{-i(k_p+k)L}-1\right)}{i\,(k_p+k)}\right\}.$$

The proposed iteration procedure is effective in calculation.

3. REFERENCES

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