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NONLINEAR VIBRATION ANALYSIS OF THIN RECTANGULAR MAGNETO-ELECTRO-ELASTIC COMPOSITE MATERIAL BY PERTURBATION TECHNIQUES

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This study attempts to analyze free and forced nonlinear vibration of rectangular thin Magneto-Electro-Elastic (MEE) plates. Considered MEE Plate is simply supported and the governing equations are derived based on the classical plate theory. Derived equations will be solved by two methods of perturbation, Iteration Perturbation Method (IPM), Homotopy Perturbation Method (HPM), and at the end the results of validation will be gained by a comparison between the obtained results and Finite Element technique and analytical solutions. 0.1% difference between perturbation techniques and other methods prove the accuracy of the resultant perturbation equations.

Keywords: Magneto-Electro-Elastic (MEE) material, Nonlinear Vibration, Iteration Perturbation Method (IPM), Homotopy Perturbation Method (HPM)

1. Introduction

There are few materials in nature which can couple several properties such as magnetic, electric, elastic, thermometric, etc., all together. In such materials, the appropriate responses are very week or they become applicable only if the temperature is very low. Even so, they are used only for very special applications. The benefits of these materials, make them prominent and urge the researchers to find ways for merging these properties into a single substance.

Composite materials have become a solution for this purpose. A composite material, by arranging different layers with different properties, can combine several characteristics in one place, consequently leading to the emergence of multi- property materials. Ferric material is introduced as a composite material with at least two ferric characteristics (FerroElectric, FerroMagnetic and FerroElastic)[1-3]. Since they can be utilized in multifunctional equipment, they are quite attractive and have increasingly been subjected to recent studies. In these materials, the interaction between different parameters can produce a new phenomenon like Magneto-Electricity [4-8].

MEE material is a good means to reach the compound benefits of magnetic, mechanical and electrical field for the previously mentioned composites. Piezoelectric and piezomagnetic smart material can be used in the form of woven or layer combinations [9]. With a glance at its important abilities, benefits and applications, a plethora of studies have been conducted in a fast ascending rate to explore them and to extract their governing equations.

Through a continuous harmonic movement in a magnetic and/or electric field, one can employ all of composite abilities together. Therefore, in many applications, vibration is the main activity in equipment and crucial to analyzing. Although linearization can lead to simplicity, its actuality and thus accuracy may be reduced, so nonlinear analysis is critical and undeniable for a high precision system analysis.

Garcia Lage and Mota Soares presented a layer wise partial mixed finite element analysis of MEE plates for static and free vibrational state [10-11]. Chen and his coworkers studied the free vibration of non-homogeneous transversely isotropic MEE plates [12]. Bhangale and Ganesan worked on the free vibration of simply supported non-homogeneous functionally graded MEE finite cylindrical shells and represented a finite element model for the evaluation of magnetic and piezoelectric effects on natural frequencies of material [13]. Ansari and Gholami analyzed size-dependent nonlinear forced vibration of magneto-electro-thermo-elastic Timoshenko nanobeams based upon the nonlocal elasticity theory [14]. Pakam and Arockiarajan investigated the effective properties of 1-3-2 type MEE composites [15]. This paper tries to study nonlinear vibration of this smart composite material by perturbation techniques. The precise prediction of nonlinear oscillations has been of significant importance in mechanical dynamics. Apart from a limited number of these problems, most of them do not have a precise analytical solution, so these nonlinear equations should be solved using approximate methods.

HPM and IPM are two of these approximation techniques. These procedures are convenient computational techniques which yield analytical solutions with a high accuracy. Unlike the traditional numerical methods, they do not need discretization and linearization. In addition, these techniques are quite straightforward to write computer code with and do not require a large computer memory. As one of their most remarkable features is that usually a few perturbation terms are adequate to obtain a reasonably accurate solution.

In the next sections, the basic relations and dynamic equation of motion will be derived first. Then nonlinear vibrational analysis will be performed by using perturbation methods. Comparisons between the answers, and the analytical and the FEM solutions serve to prove the validity and accuracy of the model.

2 .Modeling

Suppose a thin plate of MEE composite as shown in Figure 1. For such a material, the basic Equations are presented as follows [16]:

$$\sigma = C\,\varepsilon - eE - qH \tag{1}$$

$$D = e^T \varepsilon + \eta E + dH \tag{2}$$

$$B = q^T \varepsilon + dE + \mu H \tag{3}$$

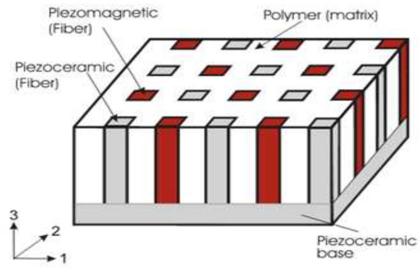

Figure 1: Schematic representation of an MEE composite [15]

Where, σ, D and B are the stress, electric displacement, and magnetic induction (i.e., magnetic flux) respectively; ε, E and H are the strain, electric field, and magnetic field, respectively, C, η and μ are the elastic, dielectric, and magnetic permeability coefficients, respectively e, q and d are the piezoelectric,piezomagnetic,and magneto electric coefficients, respectively. The related matrices for MEE material, are introduced in[16].

Also the relationship between the electrical field, the electrical potential, and the magnetic potential is:

$$E_k = -\phi_{,k} \qquad H_k = -\psi_{,k} \quad (k = x, y, z) \tag{4}$$

Thus Eq. 1 to Eq. 3 becomes[17]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{11} & 0 \\ 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \phi_{,z} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & q_{31} \\ 0 & 0 & q_{32} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \psi_{,z} \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} D_z \\ B_z \end{Bmatrix} = \begin{bmatrix} e_{31} & e_{31} \\ q_{31} & q_{31} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \end{Bmatrix} - \begin{bmatrix} \eta_{33} \\ d_{33} \end{bmatrix} \phi_{,z} - \begin{bmatrix} d_{33} \\ \mu_{33} \end{bmatrix} \psi_{,z} \tag{6}$$

2.1 Equation of motion

For thin rectangular plates, and based on classical plate theory, strain-nonlinear displacement relation is [18]:

$$\begin{aligned} \varepsilon_x &= u_{,x} + \frac{1}{2} w_{,x}^2 \\ \varepsilon_y &= v_{,y} + \frac{1}{2} w_{,y}^2 \\ \varepsilon_{xy} &= \frac{1}{2}(u_{,x} + v_{,y} + w_{,x} w_{,y}) \\ \varepsilon_z &= \varepsilon_{xz} = \varepsilon_{yz} = 0 \end{aligned} \tag{7}$$

The motion equation of the thin plate, based on the classical theory of von Karmen [19] is:

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} + q = I_0 \ddot{w} + I_2 (\ddot{w}_{,xx} \ddot{w}_{,yy}) \tag{8}$$

And the compliance relation is [20]:

$$\varepsilon_{x,yy} - \gamma_{xy,xy} + \varepsilon_{y,xx} = w_{,xy}^2 - w_{,xx} w_{,yy} \tag{9}$$

in which N represents forces, M shows moments and I is the inertial momentum and is defined as:

$$\begin{aligned}\{N_x \quad N_y \quad N_{xy}\}^T &= \int_{-h/2}^{h/2}\{\sigma_x \quad \sigma_y \quad \sigma_{xy}\}^T dz\\ \{M_x \quad M_y \quad M_{xy}\}^T &= \int_{-h/2}^{h/2}\{\sigma_x \quad \sigma_y \quad \sigma_{xy}\}^T zdz \\ \{I_0 \quad I_2\}^T &= \int_{-h/2}^{h/2}\{1 \quad z^2\}^T \rho_0 dz\end{aligned} \tag{10}$$

After defining stress function (F), we have [21]:

$$\begin{aligned}\sigma_x &= F_{,yy} + e_{31}\phi_{,z} + q_{31}\psi_{,z}\\ \sigma_y &= F_{,xx} + e_{31}\phi_{,z} + q_{31}\psi_{,z}\\ \sigma_{xy} &= -F_{,xy}\end{aligned} \tag{11}$$

Compliance Eq. 9 is simplified as

$$\frac{c_{11}}{c_{11}^2 - c_{12}^2}(F_{,xxxx} + F_{,yyyy}) + (\frac{1}{c_{66}} - \frac{2c_{11}}{c_{11}^2 - c_{12}^2})F_{,xxyy} = w_{,xy}^2 - w_{,xx}w_{,yy} \tag{12}$$

Simply supported rectangular plate boundary condition is defined as:

$$\begin{aligned}w &= w_{,xx} = 0 \quad (x = 0, a)\\ w &= w_{,yy} = 0 \quad (y = 0, b)\end{aligned} \tag{13}$$

and the boundary condition for closed-circuit is:

$$\phi(z, \pm h/2) = \psi(z, \pm h/2) = 0 \tag{14}$$

If we get the transverse displacement as

$$w(x, y) = hf(t)\,\mathrm{Sin}(\pi x/a)\,\mathrm{Sin}(\pi y/b) \tag{15}$$

then by replacing Eq. 15 into 12, eq. 16 is derived:

$$\frac{c_{11}}{c_{11}^2 - c_{12}^2}(F_{,xxxx} + F_{,yyyy}) + (\frac{1}{c_{66}} - \frac{2c_{11}}{c_{11}^2 - c_{12}^2})F_{,xxyy} = \frac{\pi^4 h^2 f^2(t)}{2a^2 b^2}\{\mathrm{Cos}(\pi x/a)\,\mathrm{Cos}(\pi y/b)\} \tag{16}$$

To solve this PDE, we should get F as:

$$F = \sum_{q=0}^{N}\sum_{p=0}^{N} B_{pq}\,\mathrm{Cos}(p\,\pi x/a)\,\mathrm{Cos}(q\,\pi y/b) \tag{17}$$

By substituting Eq. 17 to Eq. 16 and Eq.15 with Eq. 12 and by applying Galerkin method for PDE, we arrive at a nonlinear ODE:

$$\ddot{f} + \omega_0^2 f + \beta f^2 + \alpha f^3 = \bar{Q}\,\mathrm{Cos}\,\Omega t \tag{18}$$

in which ω_0 is the natural frequency, $\bar{Q}$ is the excitation amplitude, and α and β are nonlinear stiffness matrix coefficients represented as

$$\omega_0 = \pi^2 \left\{ \frac{h^2 (a^2 + b^2)^2 (\mu_{33} e_{31}^2 - c_{11} d_{33}^2 - 2 d_{33} e_{31} q_{31} + \eta_{33} q_{31}^2 + c_{11} \eta_{33} \mu_{33})}{a^2 b^2 \rho_0 [12 a^2 b^2 - \pi^2 h^2 (a^2 + b^2)](\eta_{33} \mu_{33} - d_{33}^2)} \right\}^{1/2} \quad (19)$$

$$\beta = 0$$

$$\alpha = \frac{3\pi^4 h^2 (a^4 + b^4)(c_{11}^2 - c_{12}^2)}{4a^2 b^2 c_{11} \rho_0 [12a^2 b^2 - \pi^2 h^2 (a^2 + b^2)]} \quad (20)$$

$$\bar{Q} = \frac{192 a^2 b^2 q_0}{\pi^2 h^2 \rho_0 [12 a^2 b^2 - \pi^2 h^2 (a^2 + b^2)]}$$

3. Nonlinear equation solving

Primary resonance of the system is studied analytically by applying HPM and IPM methods on Equation (18),

3.1 HPM

The main feature of HPM is as follow [22]:

$$L(v) - L(f_0) + PL(f_0) + P(\beta v^2 + \alpha v^3) = 0 \quad (21)$$

in which L(f) is

$$L(f) = \frac{d^2 f}{dt^2} + \omega_0^2 f \quad (22)$$

By assuming that the first approximation is

$$f_0(t) = A \text{ Cos } \omega_0 \lambda t \quad (23)$$

We can find the ratio of nonlinear frequency to natural frequency as:

$$\lambda = \sqrt{1 + \frac{3\alpha A^2}{4\omega_0^2} - \frac{\bar{Q}}{A\omega_0^2}} \quad (24)$$

3.2 IPM

In this method, a nonlinear equation is first arranged with the standard form [23]:

$$\ddot{f} + h(f, \dot{f}, \ddot{f}, t) = 0 \quad (25)$$

Then we get $\dot{f}(t) = y(t)$ and rewrite the equation $\dot{y}(t) = -h(f, y, \dot{y}, t)$ (26)

Now, by getting the initial guess function as bellow

$$f(t) = A \cos(\omega t) \quad (27)$$

If we follow the method, the ratio of nonlinear frequency to the natural frequency will be represented as:

$$\lambda=\sqrt{1+\frac{8A\beta}{3\omega_0^2}+\frac{3\alpha A^2}{4\omega_0^2}-\frac{4\bar{Q}\cos\Omega t}{A\omega_0^2}} \tag{28}$$

4. Results and discussion

4.1 Free vibration

In order to investigate mentioned solution methodologies, a 50x50 rectangular isotropic plate with the properties specified in Table 2 is considered.

Table 2: Thin rectangular MEE plate Properties [24]

$C_{11}=21.3\times10^{10}\ Nm^{-2}$	$C_{12}=11.3\times10^{10}\ Nm^{-2}$
$C_{66}=5\times10^{10}\ Nm^{-2}$	$e_{31}=-2.71Cm^{-2}$
$q_{31}=222\,N(Am)^{-1}$	$\eta_{33}=6.37\times10^{-9}\,C(Vm)^{-1}$
$\mu_{33}=0.839\times10^{-4}\ NS^2C^{-2}$	$d_{33}=2750\times10^{-12}\ NS(VC)^{-1}$
$\rho_0=5550\,Kgm^{-3}$	

In Table 3, the ratios of nonlinear frequency to the natural frequency obtained by IPM and HPM are presented and compared with multiple time scales method [25] and finite element method [26]

Table 3: comparison between methods

A=W_{max}/h				
	0.4	0.6	0.8	1.0
HPM	1.02027	1.04505	1.07880	1.12069
IPM	1.02027	1.04505	1.07880	1.12069
FEM [26]	1.02049	1.04559	1.07959	1.12239
Analytical [25]	1.02032	1.04525	1.07936	1.12197

As clearly shown in Figure 2, there is a very good agreement between HPM and IPM results in the free vibration analysis with analytical and FE and only 0.15% difference has been viewed in comparison with the FEM and 0.11% with the analytical responses.

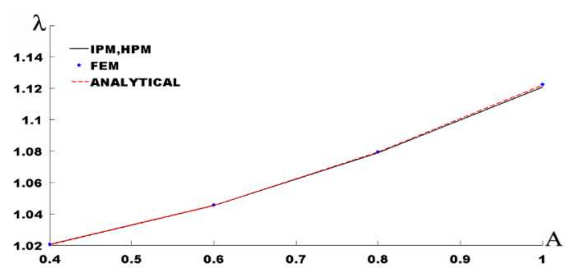

Figure 2: methods comparison.

4-2 Forced vibration

For the mentioned rectangular MEE with the given dimensions and properties, the frequency Responses are derived by IPM and HPM techniques. Eq. 29 and Eq. 30 show the Resulting relations.

$$(\sigma - \frac{3\alpha A^2}{8})^2 A^2 = \frac{4Q}{\omega_0^2} \tag{29}$$

$$(\sigma - \frac{3\alpha A^2}{8})^2 A^2 = \frac{Q}{\omega_0^2} \tag{30}$$

And Eq. 31 shows the frequency response solved by the multiple scale technique [25]

$$(\sigma - \frac{3\alpha A^2}{8})^2 A^2 = \frac{Q^2}{4\omega_0^2} \tag{31}$$

Frequency response curves for HPM and IPM based on eq. respectively and all the curves are contrasted in Fig. 3. Comparing the resulting curves with the multiple scale technique's response shows very low diversion between the results at the amplitude values above 0.2, but in the vicinity of zero, there is a slight difference, although it is not at a significant level. Also, for the forced vibration, IPM and HPM results are very close to each other in almost all ranges of amplitude.

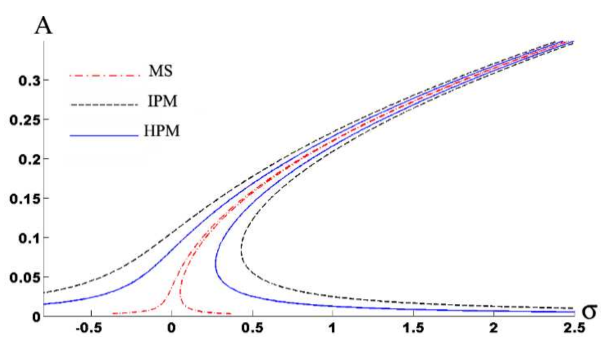

Figure 3: Frequency response curve of MEE. Comparisons between HPM, IPM and Multiple Scale techniques

5. Conclusion

In the present paper, nonlinear vibration behavior of MEE rectangular composite smart material was studied. Since most nonlinear equations do not have a precise analytical solution, they should be solved using approximate methods. Therefore, using IPM and HPM perturbation techniques, frequency equations were derived for the general forced condition. Comparisons between the results of these two methods and those of other research was performed by utilizing the analytical and the FEM methods for the free vibration situation. The results show an adaptation between IPM and HPM and only 0.15% diversion was seen in comparison with the FEM. Also, 0.11% difference for analytical responses with perturbation techniques was viewed. Lower 1% diversity between all the results show the good agreement between these different relations and prove the highly accurate resultant relations in free vibrational conditions. Moreover, forced vibrational analysis was conducted. Similar to previous conditions, there is a very good agreement between IPM and HPM results in all ranges of amplitude although there is a small difference in the results of these two techniques and those of the multiple scale in low amplitudes.

REFERENCES

(1) Schmid, H. (1994). Introduction to the Proceedings of the 2nd International Conference on Magnetoelectric Interaction Phenomena in Crystals. The 2nd International Conference on Magnetoelectric Interaction Phenomena in Crystals. 161, pp. 1-28. Ferroelectrics.

(2) Fiebig, M. (2005). Revival of the magnetoelectric effect. Journal of Physics(38), 123-152.

(3) Eerenstein, W., Mathur, N., & Scott, J. (2006). Multiferroic and magnetoelectric materials. Nature(442), 759-765.

(4) Schmid, H. (1994). Multiferroic Magnetoelectrics. Ferroelectrics, 162, 317-338.

(5) Bichurin, M. (1997). Magnetoelectric Multiferroic Composites. MEIPIC-3. 204, pp. 1-356. Ferroelectrics.

(6) Bichurin, M. (2002). Proceeding of the fourth conference on magneto electric interaction phenomena in crystals (MEIPIC-4) , veliky Novgorod , Russia, Ferroelectric 279(2002) 1-166; Ferroelectric 280(2002) ,[167-384] 1-218,

(7) Fiebig, M., Eremenko, V., & Chupis Kluwer, I. (2003). Magnetoelectric Interaction Phenomena in Crystals. MEIPIC-5, (pp. 21-24).

(8) Wen Nan, C., Bichurin, M., Shuxiang, D., Viehland, D., & Srinivasan, G. (2008). Multiferroic Magnetoelectric Composites: Historical Perspective, Status, and Future Directions. Journal of Applied Physics, 103(031101).

(9) José, M., Moita, S., Cristóvão, M., Mota, S., & Carlos, A. (2009). Analyses of magneto-electro-elastic plates using a higher order finite element model. Composite Structures(91), 421–426.

(10) Lage, G., & Soares, M. (2004). Layerwise partial mixed finite element analysis of magneto-electro-elastic plates. Computer Structure(82), 1293-1301.

(11) Lage, G., & Soares, M. (2005). Static and free vibration analysis of magneto-electro-elastic laminated plates by a layerwise partial mixed finite element model. The 3rd international conference on structural stability and dynamics. Florida, USA.

(12) Huang, X.-L., & Shen, H.-S. (2005). Nonlinear free and forced vibration of simply supported shear deformable laminated plates with piezoelectric actuators. International Journal of Mechanical Sciences(47), 187-208.

(13) Bhangale, R., & Ganesan, N. (2006). Free vibration of functionally graded non-homogeneous magneto-electro-elastic cylin-drical shells. International Journal for Computational Methods in Engineering Science and Mechanics, 7(3), 191-200.

(14) Ansari, R., Gholami, R., & Rouhi, H. (2015). Size-Dependent Nonlinear Forced Vibration Analysis of Magneto-Electro-Thermo-Elastic Timoshenko Nanobeams Based upon the Nonlocal Elasticity. Composite Structures, 126, 216-226.

(15) Pakam, N., & Arockiarajan, A. (2014). Study on effective properties of 1-3-2 type magneto-electro-elastic composites. Sen-sors and Actuators, 209, 87-99.

(16) Pan, E. (2001). Exact solution for simply supported and multilayered magneto-electro-elastic plates. Journal of Applied Mechanics, 68(4), 608-618.

(17) Liu, M. F., & Chang, T. P. (2010). Closed form expression for the vibration problem of a transversely isotropic magneto-electro-elastic plate. Journal of Applied Mechanics, 77(2), 245-260.

(18) Reddy, J. (2004). Mechanics of laminated composite plates and shells: Theory and analysis. CRC press.

(19) Nayfeh, A., & Pai, P. (2008). Linear and nonlinear structural mechanics. John Wiley & Sons.

(20) Chia, C. (1980). Nonlinear analysis of plates. New York: McGraw-Hill.

(21) Xue, C., Pan, E., Zhang, S., & Chu, H. (2011). Large deflection of a rectangular magnetoelectroelastic thin plate. Mechanics Research Communications, 38(7), 518-523.

(22) He, J.-H. (2000). A coupling method of a homotopy technique and a perturbation technique for non-linear problem. Inter-national Journal of Non-Linear Mechanics(35), 37-43.

(23) Ganji, S., Barary, A., Karimpour, S., & Dommairy, G. (2012). Motion of a rigid rod rocking back and forth and cubic-quintic duffing oscillators. Journal of theoretical and applied mechanics, 50(1), 215-229.

(24) Sladek, J., Sladek, V., Krahulec, S. and Pan, E. (2013), "The mlpg analyses of large deflections of magnetoelectroelastic plates",Engineering Analysis with Boundary Elements, Vol. 37, No.4, 673-682.

(25) Shooshtari, A., & Razavi, S. (2014). Nonlinear free and forced vibrations of anti-symmetric angle-ply hybrid laminated rec-tangular plates. Journal of Composite Materials, 48(9), 1091-1111.

(26) Singha, M., & Daripa, R. (2009). Nonlinear vibration and dynamic stability analysis of composite plates. Journal of Sound and Vibration, 328(4), 541-554.