TRANSMISSION LINE MATRIX MODELLING APPLIED TO NEAR AND FAR FIELD TRANSDUCER BEAM PATTERNS

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1. INTRODUCTION

A system to model acoustic wave propagation in two dimensions, based on the Transmission Line Matrix Modelling method (TLM) has been developed. It has been used to predict the beam patterns of a number of transducer configurations. These beam patterns have been compared with experimental data, and with analytical solutions for some simple geometries. There is close agreement between the modelled and experimental data. There are disagreements between the modelled data and the analytical solutions, although these are associated with regions where the analytical solutions are subject to invalid assumptions. TLM requires no such assumptions, and appears to be valid over a wider region than the analytical solutions used, and provides information on the whole field. TLM thus has potential as a powerful technique for the characterisation of beam patterns for a wide range of transducer geometries.

2. TRANSMISSION LINE MATRIX MODELLING (TLM)

2.1 The development of TLM

Transmission Line Matrix (TLM) modelling was developed in the early seventies[1], first as a technique for the investigation of electromagnetic wave propagation[2] and later for other electronic and electrical problems[3]. It has been applied to a wide variety of wave propagation problems in homogeneous and inhomogeneous media, including electromagnetic compatibility studies of motor vehicles[4] and the modelling of semiconductor lasers[5]. It has also been used in diffusion problems such as water diffusion in rice[6], heat propagation in turbine blades[7], and chemical reaction kinetics[8]. TLM has also been applied to acoustics[9,10].

Details of the basis of TLM are given in the references[1,2,11,12]. TLM is a method which physically models wave propogation and is ideally suited to simulating sound waves travelling through a medium. In the original work on TLM in electromagnetics, the properties being modelled were voltage and current. The equivalent quantities in acoustics are velocity and either displacement or pressure. The TLM method can be applied in one, two or three dimensions. In this work, a two dimensional system has been produced.

2.2 The basis of TLM

Fig. 1 illustrates a section of TLM mesh and the various concepts discussed in this section. The space being modelled is represented as a cartesian mesh of electrical transmission lines. The transmission lines are joined where they cross, the junctions being termed nodes. It can be demonstrated[1] that as long as the spacing between adjacent nodes is less than one tenth of a wavelength of the ultrasound frequency being studied, waves will propagate on the mesh. The model is discretised in space by the mesh and in time by modelling the waves as impulses representing the instantaneous value of the quantity being modelled at each node.

An accurate numerical model for the propagation of voltage impulses is then implemented in software. In electrical terms the parameters of the connecting transmission lines and their manner of connection are governed by Kirchoff's laws. An impulse incident on a node from a particular direction will see an impedance one third of the characteristic impedance of the connecting transmission lines. This gives rise to a reflection coefficient of -1/2, resulting in a negative impulse of half the incident amplitude being reflected in the incident element. Positive impulses of the same amplitude are transmitted in the other three directions.

The four reflected impulses are then incident on the surrounding nodes, and are themselves scattered. The process is discretized in time and space so that impulses reflected from a node in one iteration are incident on its neighbours in the next. Combining the calculations for impulses incident in all four branches connected to a node leads to a very simple calculation for the voltage at that node after each iteration given by equation (1).

$$ {}_{k+1}V_n^r = \frac{1}{2}\left[\sum_{m=1}^{4} {}_kV_m^i\right] - {}_kV_n \qquad (1) $$

In the next iteration the scattered pulses will be incident on four new nodes. At each node the process will be repeated and the pulses being injected will disperse through the network. This calculation is very simple, and can be computed very rapidly.

Boundaries in the propagating medium are achieved by placing resistive loads at nodes. When sound travelling in air interacts with a solid the result is a total reflection, so object boundaries are modelled as open circuits. The edges of the mesh should behave as closely as possible to a free space boundary, and are modelled with a matching impedance to give a zero reflection coefficient.

If impulses are injected at a number of adjacent nodes a wavefront will be generated. Thus transmitters can be modelled using sets of adjacent driving nodes corresponding to the geometry of the transducers. Changing the amplitude of the injected pulse over successive iterations allows modelling of the effect of applying a varying driving voltage.

Receiving transducers are modelled by summing the total energy incident on the nodes corresponding to the geometry of the transducer being modelled.

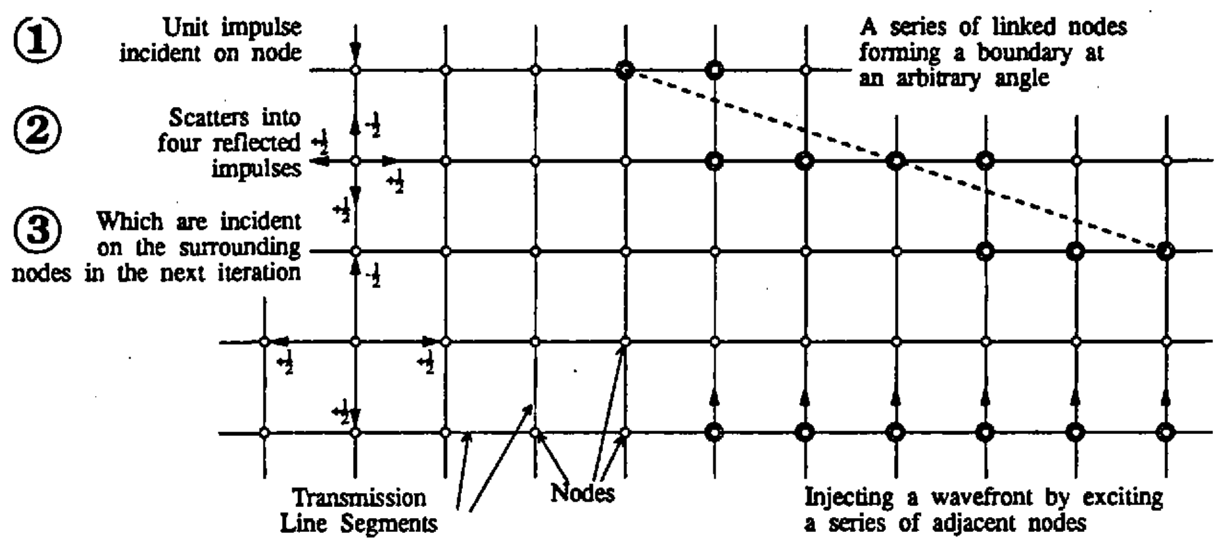

Fig. 1: A section of a transmission line matrix

The system currently implemented operates on a Sun SPARC Station, and will accommodate up to 2048 by 1024 nodes. It is usually used to produce an animated series of frames of data showing the progress of propagating wavefronts at successive instants, in order to analyse the interaction of sound waves with objects. It can be used to analyse beam patterns by recording the peak sound intensity measured at each point in the matrix as a CW waveform at the desired frequency is transmitted.

3. COMPARISON OF RESULTS FROM ANALYTICAL EXPRESSIONS AND TLM

The TLM system described here operates in two dimensions. In order to assess the accuracy of the model, a three dimensional case that corresponded to the model was sought. The simplest was a single point in the modelling plane, corresponding to a line source infinite in extent in the direction normal to the plane of the model. The model was run to determine its accuracy in modelling the expected $1/r^2$ dependence of source pressure. Graphs of $\log_{10}$ P vs $\log_{10}$ r are given in Fig. 2, and exhibit the expected gradient of -0.5.

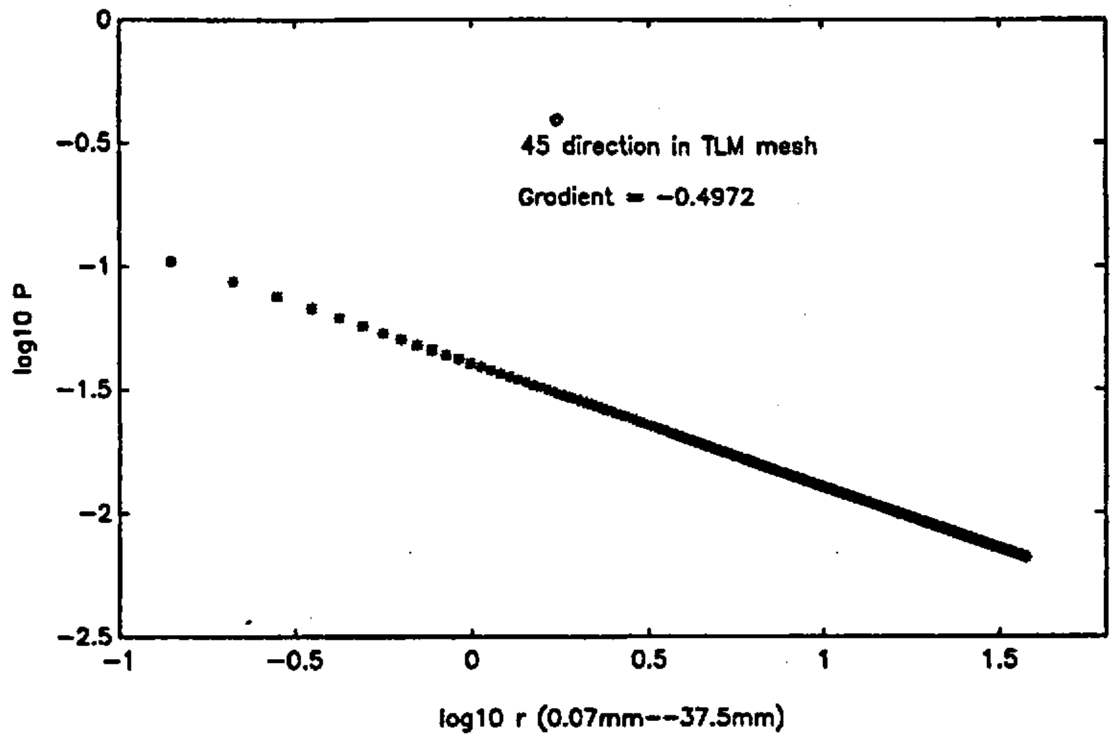

Fig. 2 Pressure versus range for an infinite line source

The analysis was then extended to a line in the modelling plane, corresponding to a strip source of infinite extent in the plane normal to the model, shown in Fig. 3.

Expressions for axial and lateral distributions of pressure within the Fresnel region were derived, based on the Helmholz integral, resulting in (2)

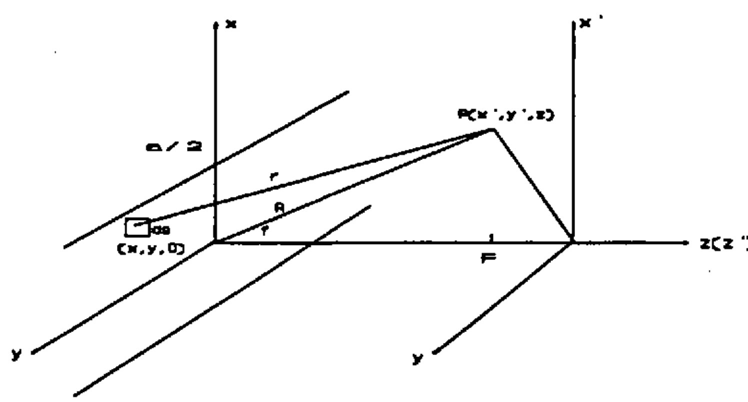

Fig. 3 An infinite strip source

$$|P_{axis}(z)| = \sqrt{2}P_0\left(C^2\left(\frac{a}{\sqrt{2\lambda z}}\right) + S^2\left(\frac{a}{\sqrt{2\lambda z}}\right)\right)^{\frac{1}{2}} \qquad (2)$$

Here C(u) and S(u) are the Fresnel integrals which are

$$C(u) = \int_0^u \cos\frac{\pi}{2}t^2 dt \qquad S(u) = \int_0^u \sin\frac{\pi}{2}t^2 dt \qquad (3)$$

Theoretical and modelled results are shown in figure Fig. 4. The two agree closely over most of the range under consideration, with the exception of the extreme near field. This corresponds to a region within which the assumptions made in extracting the analytical expression are not valid. In this region, TLM is likely to be giving more reliable predictions of the actual response than the analytical expression.

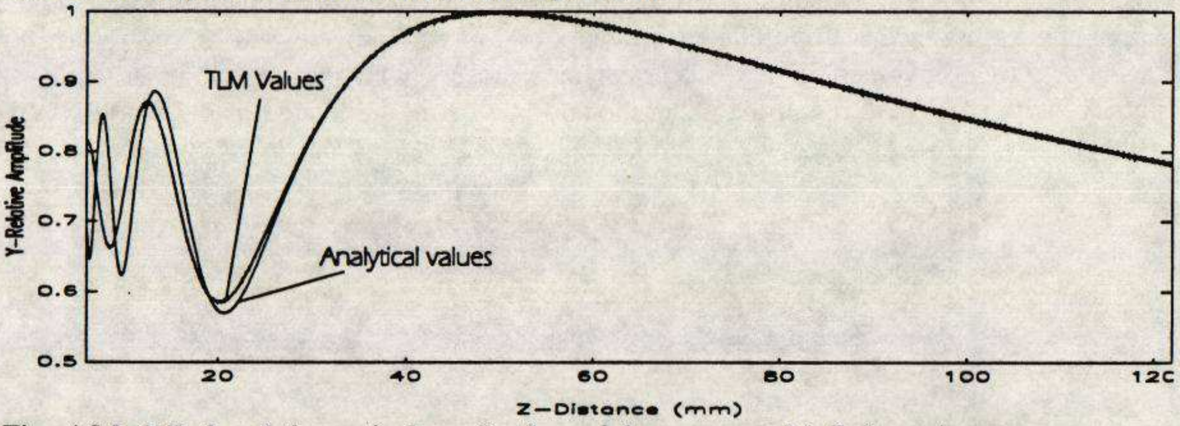

Fig. 4 Modelled and theoretical results for axial response of infinite strip source

4. COMPARISON OF TLM MODELLING OF A PHASED ARRAY WITH EXPERIMENTAL DATA

The TLM system has also been used to compare experimental measurements of sound fields with modelled predictions. The sound field of a 16 element linear array of ultrasonic transducers operating in air was measured, and plotted as shown in Fig. 5. Individual element sensitivities were also measured to be fed into the model.

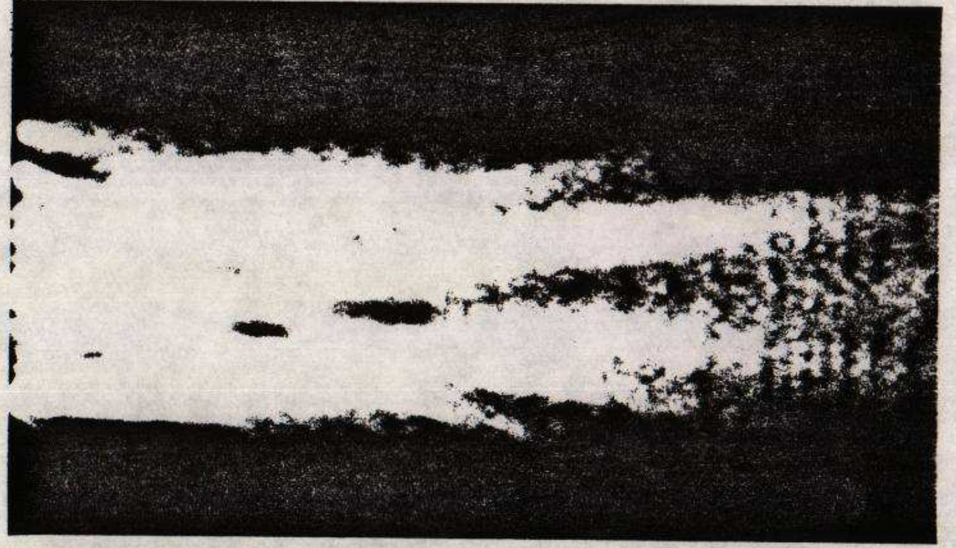

Fig. 5 Measured beam pattern from a 16 element airborne ultrasonic transducer array

The array was modelled assuming zero inter-element gaps (valid for the transducer technology used) and applying the amplitude sensitivities measured from the real array. The resulting beam pattern is given in Fig. 6. It corresponds closely with the experimentally measured pattern, with most features in the experimental data being duplicated in the model.

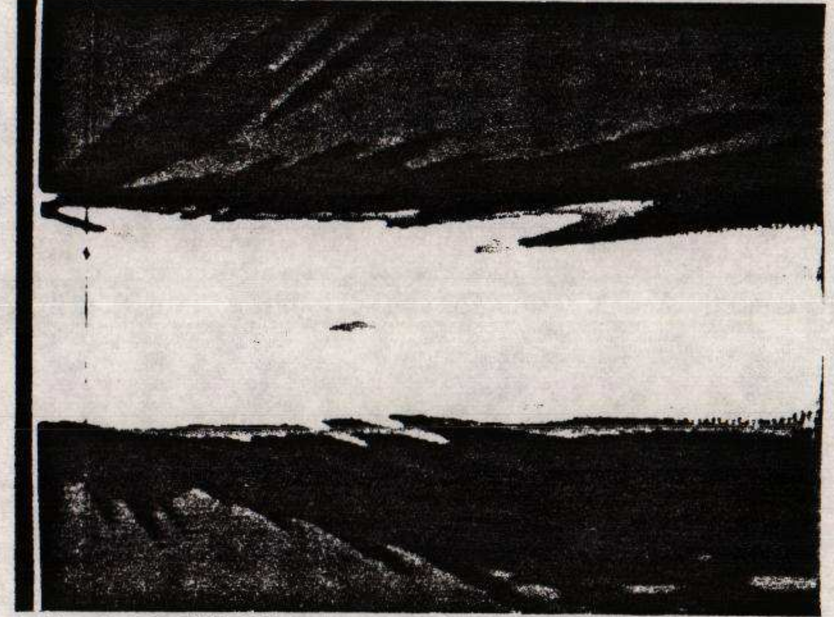

Fig. 6 Modelled beam pattern for 16 element array

5. CONCLUSION

The potential of TLM modelling has been demonstrated as a useful, accurate tool in the analysis of transducer beam patterns. The method calculates the entire field produced by the transducer, and makes none of the assumptions neccessary to extract analytical expressions. The model has the capability to do this for arbitrary transducer geometries. To be really useful, the system needs extending to three dimensions, which will give rise to difficulties in accommodating useful model sizes, although increases in computing power will assist. It is also possible to couple existing, more sophisticated models of transducers to the TLM system, allowing accurate prediction of the response of devices before construction.

6. REFERENCES

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