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MULTI-OBJECTIVE MODELING OF ACOUSTIC SANDWICH PANELS

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INTRODUCTION

The sound insulation characteristics of partitions or panels separating adjoining spaces in housing and industry is often an important aspect of their design. In many cases principal concerns will include support of static loads and adherence to severe panel deflection requirements [1]. Thus, the intent of their design is to provide the required load-bearing capability and sound attenuation characteristics with a minimum of weight and cost. It is generally agreed that single-layered panels, governed by limp-mass behavior, are lacking in terms of efficient utilization of material and ease of handling in many structural applications. By employing multi-layered panels comprised of several materials, it is possible to obtain stronger, lighter panels with a combined performance not available from single-layered panels. Constructions consisting of thick, lightweight cores sandwiched between thin, dense facings are an example in which structural and sound insulation advantages can be obtained while minimum amounts of materials are utilized.

In addition, consideration must often be given to other physical properties of the materials, such as flammability, toxicity, settling and compaction, etc. All the above considerations lead to the conclusion that the optimal acoustic design of sandwich panels is a multi-objective optimization problem. As such it must be studied in a comprehensive way with an attempt to optimally balance the often conflicting multiple objectives.

STATEMENT OF THE PROBLEM

The task is thus defined as designing a sandwich panel with good sound insulation characteristics, at low cost, and meeting or exceeding minimum performance requirements with respect to load bearing, deflection, and relevant building code specifications. Mathematically this is a multi-objective optimization problem of the following form:

$$\text{Extremize } F(\overline{x}) = \{f_1(\overline{x}),\ f_2(\overline{x}),\ \ldots,\ f_p(\overline{x})\}$$

$$\text{subject to: } g_i(\overline{x}) \leq 0\ \forall_i \text{ constraints.}$$

where the $\overline{x} = (x_1, x_2, \ldots x_n)$ are the decision variables and the $f_i(\overline{x})$ are the individual objective functions. For the present study, explicit objectives are formulated for the transmission loss (which is to be maximized), the panel end-load capability (to be maximized), the mid-span deflection ratio (to be minimized), and the cost of the panel core (to be minimized).

The calculation of transmission loss (*TL*) is complicated, and even for, a plane-wave analysis the calculation depends upon frequency, angle of incidence, and all the mass, stiffness and geometric properties of the panel [2,3]. Further, the choice of a good measure for sound insulation or transmission loss is not unique [3,4,5] and may strongly influence the outcome of any optimization study.

The problem is thus further complicated by the inclusion of objectives for the compressive load (per unit width) sustainable by a panel,

$$N_x = \tilde{D}\left(\frac{\pi}{L}\right)^2 - \frac{\tilde{F}^2\left(\frac{\pi}{L}\right)^4}{\tilde{C}\left(\frac{\pi}{L}\right)^2 + 4\kappa^2\ \left(\frac{C_{66}}{h_c}\right)} \tag{1}$$

and for the ratio of the mid-span (maximum) panel deflection to its length,

$$\frac{w(L/2)}{L} = \left(\frac{4}{\pi^3}\right)(p_o L)\left[\tilde{D}\left(\frac{\pi}{L}\right)^2 - \frac{\tilde{F}^2\left(\frac{\pi}{L}\right)^4}{\tilde{C}\left(\frac{\pi}{L}\right)^2 + 4\kappa^2\ \left(\frac{C_{66}}{h_c}\right)}\right] \tag{2}$$

In eqs. (1) and (2), $\tilde{D}$, $\tilde{F}$ and $\tilde{C}$ are bending and extensional stiffnesses of a sandwich panel of length L[2]. Clearly these equations also represent complex relationships between the various panel properties and the design objectives, and these would be very difficult to handle in a formal optimization study.

Finally, in regard to cost considerations, one of the decisions made in the present problem formulation was to use the Component Design Categories (CDC) concept introduced by Weber, *et al.* [4], rather than try to optimize TL over the complete range of physical properties of a panel. Indeed, when the diverse designs are grouped into more closely defined homogeneous categories, the direct effect of acoustical performance on cost becomes quite apparent. These groups of homogeneous designs, the CDC's, are formed by limiting the range of variation of key design characteristics, such as density, physical structural characteristics, and so on.

METHODOLOGY

This paper presents a model for analyzing the trade-offs between the acoustic performance of sandwich panels, their structural requirements, and their material costs. The first phase of the model derivation is an acoustic optimization study [3] based on the sandwich panel TL model of Dym and Lang [2], this study having as its sole objective function the TL calculated from an A-weighted, discrete-frequency-average transmission coefficient [3]. The weighting and discrete frequencies chosen in the band 1-4kHz are identical to those used to define a single insulation measure in an ASTM standard [5]. This optimum TL is then used to obtain an explicit relation for the TL as a function of the optimization variables (e.g., the core thickness and core density in a panel core optimization design), such as that given by Dym, Makris and Smith [3] for hardboard skins (see Panel E in [2]) and isotropic foam cores:

$$TL = 41.85\ \rho_c^{0.116}\ h_c^{0.256}; \quad [\rho_c] = \mathrm{kg/m^3},\ [h_c] = \mathrm{m} \tag{3}$$

The next phase of the study is the conversion of eqs. (1) and (2) to forms paralleling eq. (3) for the particular type of optimization being carried out (e.g., the core optimization described earlier). This is accomplished in a straightforward manner with standard multiple regression techniques. The results are, for the end-load on a panel whose width is 1.2m, with $[N_x] = \mathrm{N/m}$,

$$(1.2)N_x = -4.71 \times 10^5 + (4.93 \times 10^3)\rho_c + (7.45 \times 10^6)h_c \tag{4}$$

and for the mid-span deflection ratio,

$$\frac{w(L/2)}{L} = \frac{h_c - 0.181 + (0.264 \times 10^{-2})\rho_c}{-27.4 + (0.536)\rho_c} \tag{5}$$

It is to be noted that the core density, ρ_c, occurs in the static formulae of eqs. (4) and (5) because it is assumed that material moduli are linearly proportional to the mass density of the material.

Finally, a similar formulation for the cost of a 1.2m x 2.4m panel is found, for example, for a family of rigid polyisocyanurate foam cores with a density range of 30-50 kg/m^3, from manufacturer's data [6]:

$$C = [-416 + (19.54)\rho_c](1.2)(2.4)(h_c); \quad [C] = \$/\text{panel core} \tag{6}$$

Eqs. (3), (4), (5), and (6) represent the closed form expressions $f_1(\overline{X})$, $f_2(\overline{X})$, $f_3(\overline{X})$, and $f_4(\overline{X})$ respectively. These (multiple) objectives are used along with a Generalized Reduced Gradient (GRG) nonlinear programming package [7] to generate the Pareto optimal solutions for the panel design problem.

CONCLUSIONS

These results detail an explicit model which can be used to optimize the TL of a sandwich panel as well as minimizing the cost while optimally meeting performance requirements with respect to load and deflection. Explicit realizations for the TL, the end loading, the mid-span deflection, and the cost have been presented for a sandwich panel with a foam core between hardboard skins, these formulae being the objective functions sought for the multi-objective optimization study.

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