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SOUND PROPAGATION THROUGH PARTICULATE SUSPENSIONS IN AIR

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1. ABSTRACT

Predictions of the acoustic effect of solid suspensions in air may be based either on a multiple scattering analysis, or on a continuum approach using a particulate relaxation method. This paper compares the theoretical predictions for a suspension of alumina spheres in air. Both theories predict a frequency of maximum attenuation per wavelength that, for spheres of radius > 1μm, falls in the audible or sub-audible range. The relative merits of the models are discussed, and a technique is proposed to validate them experimentally.

2. LIST OF SYMBOLS

a	monopole scattering constant
a_0	velocity of sound in gas
a_s	velocity of sound in suspension
b	dipole scattering constant
c_p	specific heat capacity of gas
c'_p	specific heat capacity of particles
f	frequency
i	$\sqrt{-1}$
k	propagation constant of gas
k'	propagation constant of particles
k_s	propagation constant of suspension
Pr	Prandtl number of gas $= \mu c_p / \kappa$
r	radius of particles
T	absolute temperature
α	specific attenuation of suspension
α_s	attenuation of suspension
β	dispersion of suspension
γ	ratio of specific heats of gas
δ_v	viscous skin depth
δ_t	thermal skin depth
ϕ_m	mass fraction of particles
ϕ_v	volume fraction of particles
κ	thermal conductivity of gas
κ'	thermal conductivity of particles
μ	dynamic viscosity of gas
ν	kinematic viscosity of gas
ρ	density of gas
ρ'	density of particles
τ_d	dynamic relaxation time
τ_t	thermal relaxation time
Θ	coefficient of volume expansion - gas
Θ'	coefficient of volume expansion - particles
ω	angular frequency

3. INTRODUCTION

The propagation of sound through suspensions of particulates in air is influenced by thermal and viscous transport mechanisms occurring at the interface of the inhomogeneities. Several formulations have been devised to model the resulting sound propagation.

Early work, most notably by Epstein and Carhart [1], concentrated only on the attenuation of sound, making no comment on the dispersion associated with a frequency dependent phase velocity. This work calculated both thermal and viscous scattering effects for each single, (liquid) particle, and scaled up according to the number of particles. As such it was valid only when the aerosol was very disperse so that multiple scattering events were

statistically unlikely. Experimental work was performed on artificial fogs of water in air by Knudsen, Wilson and Anderson [2], although there was only limited comparison with existing theory.
Zink and Delsasso [3] made the first measurements of both attenuation and dispersion, using a suspension of solid particles in air. They also presented a somewhat crude theory to account for the observed dispersion, which relied on the modification to sound velocity caused by the particles' heat capacity and density contributions. Frequency dependence was modelled as the varying fraction of particles "fully participating" in sound propagation at different frequencies.

Scattering theory has been extended to higher concentration fractions by accounting for multiple scattering effects. Waterman and Truell [4] made the first notable contribution. Lloyd and Berry [5] further extended the theory by including higher order scattering statistics, as did Ma et al [6]. However, the latter work treated the particles as lossless scatterers, ignoring the important effects of viscosity and thermal transfer.

A different approach was taken by Temkin and Dobbins [7]. They modelled the medium as a pseudo-homogeneous continuum, with characteristic momentum and thermal (or energy) relaxation times. This relaxation theory is again restricted to disperse aerosols, because non-overlapping thermal and viscous waves are assumed.

Temkin and Dobbins validated their theory initially by drawing on the results of Zink and Delsasso, and shortly afterwards, by their own experimentation [8], which confirmed their theory both qualitatively and quantitatively.

The relaxation formulation is restricted to frequencies below the characteristic relaxation frequencies of the processes involved. The scattering theories may be restricted to the long wavelength limit, where the particles are very small in comparison to the wavelength, (a condition usually expressed as $kr < 0.1$, where k is the propagation constant of the continuous phase, and r is the radius of the scatterers). Multiple scattering formalisms may extend far above this limit, into the high ultrasonic regions for practically-sized scatterers.

Recent work has been directed at a more complete description above the long wavelength limit, especially towards descriptions of very concentrated aerosols with volume fractions of 0.5 or more [11]

This paper will refer only to the long wavelength regime and will concentrate on diffuse aerosols. It will compare the predictions of the multiple-scattering formalism, principally that by Lloyd and Berry, with those of the relaxation theory of Temkin and Dobbins.

4. THEORY

4.1 Relaxation Theory

Temkin and Dobbins [7] begin their analysis with an intuitive argument. The velocity of a massive particle suspended in a fluid exposed to an acoustic field will lag behind the particle velocity of the fluid (at all except the lowest frequencies), causing viscous drag at the interface. Similarly, the particles will be out of thermal equilibrium with the acoustically-induced temperature fluctuations of the fluid. The authors proceed to solve the Navier-Stokes and heat transfer equations in a manner calculated to demonstrate the dual relaxation nature of the problem.

Attenuation and phase velocity are expressed in terms of two relaxation time constants:

$\tau_d = 2r^2\rho' / 9\mu$, the dynamic relaxation time (due to viscous processes) and:

$\tau_t = \left(3\mathrm{Pr}\, c_p' / 2c_p\right).\tau_d$ the thermal relaxation time

The theory is restricted by two important criteria:

$\omega\tau_d \le 1$ and $(\omega\rho r^2/2\mu)^{1/2} << 1$

Essentially these allow representation of the viscous forces by first order Stokes' drag, and negect higher orders caused by non-uniform particle velocity in the acoustic field. Furthermore, the particles are considered far more dense than the surrounding fluid: $\rho'/\rho >> 1$

The theory is divided into two regimes: "low mass concentrations", where the mass concentration of particulates is infinitesimal, and "finite mass concentrations", where the volume concentration, although small, may be higher by the factor ρ'/ρ .

For infinitesimal mass fraction, where $\phi_m^2 << 1$, we state the results of theory in terms of α, the dimensionless specific attenuation (attenuation per wavelength $\alpha_s a_0 / f$) and β, the frequency dispersion:

$$\alpha = \pi\phi_m\omega\left[\frac{\tau_d}{1+\omega^2\tau_d^2} + (\gamma-1)\left(c_p'/c_p\right)\frac{\tau_t}{1+\omega^2\tau_t^2}\right]$$

$$\beta = \left(\frac{a_0}{a_s}\right)^2 - 1 = \phi_m\left[\frac{1}{1+\omega^2\tau_d^2} + \frac{(\gamma-1)\left(c_p'/c_p\right)}{1+\omega^2\tau_t^2}\right]$$

4.2 Multiple Scattering Theory

The complex propagation constant for an aerosol of particles in air is $k_s = \omega/a_s + i\alpha_s$

Lloyd and Berry calculate K from :

$$(k_s/k)^2 = (1+\phi_v a)(1-3\phi_v b) + 6\phi_v^2 b^2$$

Using Allegra and Hawley's [10] expression for single scattering coefficients, and McClements' [9] long wavelength limit approximation:

$$a = \left[\frac{\rho k'^2}{\rho' k^2} - 1\right] + \frac{3i}{kr^2}a_0 T\rho\kappa H\left(\frac{\Theta}{\rho c_p} - \frac{\Theta'}{\rho' c_p'}\right)^2$$

$$b = -\left\{2\left[1 + 3(1+i)\frac{\delta_v}{2r} + 3i\frac{\delta_v^2}{2r^2}\right]^{-1} + \frac{3\rho}{(\rho'-\rho)}\right\}^{-1}$$

where $H = \left\{\frac{1}{(1-iz)} - \frac{\kappa}{\kappa'}\frac{\tan(z')}{\tan(z')-z'}\right\}^{-1}$, $z = (1+i)r/\delta_t$, $\delta_t = \sqrt{\frac{2\kappa}{\omega\rho C_p}}$, and $\delta_v = \sqrt{\frac{2\mu}{\omega\rho}}$, and similarly for primed quantities.

Now α and β can be derived from the complex propagation constant of the suspension:

$\alpha = 2\pi a_0 \, \mathrm{Im}(k_c) / \omega$

$\beta = \left[a_0 \, \mathrm{Re}(k_c) / \omega\right]^2 - 1$

The value a is related to the monopole scattered field caused by particle pulsation, and b to the dipole scattered field caused by particle oscillation relative to the fluid. The variables δ_t and δ_v are the thermal and viscous skin depths, respectively, which represent the distances for the thermal and viscous waves to decay by $1/e$.

This long wavelength approximation has been verified experimentally by McClements [9], who found reasonable agreement using emulsions of immiscible liquids (n-hexadecane in water). There was less agreement at high mass fractions ($\phi_m > 0.1$); but in this case the density ratio $\rho'/\rho \approx 1$, and so the volume fraction $\phi_v = \phi_m \rho / \rho'$ is equally high, stretching the limits of scattering theory. These experimental results cannot verify relaxation theory, due to the low density ratio. On the other hand such verification has been carried out [8] through measurements on an aerosol of oleic acid in nitrogen gas.

5. COMPARISON OF THEORIES

Initially, it is instructive to note the difference in the reported range of applicability of these two formulations. For a particle radius of 2μm, then the long wavelength limit applicable to this formulation of scattering theories is:

$\omega << a_0 / 10r \;\Rightarrow\; f << 2.7\,\mathrm{MHz}$

Taking figures for alumina particles in air (as used by Zink and Delsasso, and shown in Table 1), then the relevant limit for relaxation theory is given by:

$\omega \le 1/\tau_d \;\Rightarrow\; f \le 870\,\mathrm{Hz}$, a difference of over 2 decades.

Fig 1 compares the two theories for infinitesimally disperse aerosols of alumina in air, while fig 2 shows the divergence of the theories for higher concentrations ($\phi_m = 0.3$). Note that the specific attenuation shows a characteristic peak (at 870 Hz). The relaxation theory predicts this peak explicitly: it is composed of the sum of two relaxation curves with local peaks at $\omega = 1/\tau_d$ and $\omega = 1/\tau_t$. For the given aerosol, the peaks are closely located ($\tau_t / \tau_d = 0.89$) and the magnitude of the viscous contribution is dominant, so only a single maximum is noticable. This is the case for most practical aerosols.

At low frequencies ($\omega << 1/\tau_d$), dispersion approaches an asymptotic value, that may be found easily from relaxation theory:

$$\text{for } \omega = 0, \quad \beta_{max} = \phi_m \left[1 + (\gamma - 1) c_p' / c_p\right]$$

Above the specific attenuation peak ($\omega >> 1/\tau_d$), dispersion tends to zero, as the thermal and inertial fluctuations of the particles tend to zero.

Fig 3 shows the predicted relationship between frequency of maximum specific attenuation and particle radius, and the excellent agreement between theories; the relationship is an inverse square law, shown explicitly by relaxation theory, since $\tau_d \propto r^2$.

As most features are around the $\omega = 1/\tau_d$ peak, the $\omega \leq 1/\tau_d$ restriction on relaxation theory is apparently severe. However, the close agreement with scattering theory even above this limit — and Temkin and Dobbins' own experimentation [8]— show that this limit may be exceeded by at least one decade without severe errors.

6. PROPOSED EXPERIMENTS

We propose to investigate aerosols of alumina in air, especially at high particle concentrations, using the experimental rig shown in Fig 4 . It consists of a vertical steel tube of similar construction to a standing wave tube, coupled to an equal length of plastic tube. At the base of the steel tube is a clamped rigid plate. A probe microphone samples the acoustic field near the top of the steel tube. Coupled to the top of the plastic tube is a loudspeaker. The steel tube is filled with an aerosol of alumina particles, which are available in narrow distributions (near mono-dispersed) of known radius.

A pseudo-random noise generator drives the loudspeaker, via a power amplifier. The received and amplified microphone signal is digitized by a PC-controlled frequency analyzer (MLSSA), which also incorporates the noise generator. Cross-correlating the known noise input with the received signal yields the impulse response of the entire system [12]. A typical response is shown in Fig. 5.

Segment A corresponds to the downwards travelling, anechoic pulse, while B is the 1st pulse reflected from the rigid end. Segment B is extracted, and transformed into the frequency domain. This reading will be taken with the tube empty, and separately with a sample of dispersed particles. Equalising the particle-filled tube spectrum with respect to the empty tube spectrum thus yields the transfer function of two traverses of the aerosol within the steel tube; all effects due to external equipment, tube attenuation and non-perfect end reflection will be equalised out.

The pulses may be separated in this way because care has been taken to ensure that there is no time aliasing from other reflections, and that tube length is sufficient to allow decay of individual impulses: the plastic tube serves only to separate the 1st reflection from the loudspeaker cone, and the desired end reflection.

Since plane waves are constrained within the tube (within a certain frequency range) the attenuation coefficient for the aerosol may thus be read off from the magnitude of the transfer function, and phase velocity may be derived from the phase of the transfer function.

7. CONCLUSIONS

Despite their different formulations, the two theories supply very similar results within the range of their applicability. In particular, their projected frequencies of maximum specific attenuation agree closely. The scattering theory is valid over larger ranges of both kr and particle concentration, even with the long wavelength simplifications; however relaxation theory stays in agreement beyond values of kr that assumptions strictly allow, suggesting that the neglected high-order drag forces are insignificant.

Relaxation theory is useful in several practical ways: it is computationally simple, especially to estimate frequencies of maximum specific attenuation; it requires less knowledge of material constants ($\Theta, \Theta', k', \kappa'$); and its simple formulation gives good intuitive insight into the processes involved.

Finally, it is interesting to note that, for 2μm alumina particles in air, the maximum specific attenuation is predicted to be near 1 kHz, thus making audio-frequency verification feasible.

8. REFERENCES

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9. ACKNOWLEDGEMENT

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	Alumina (Al_2O_3)	Air
k	$\omega/6000$ m^{-1}	$\omega/340$ m^{-1}
Θ	2.6×10^{-5} K^{-1}	$1/T$ K^{-1}
ρ	3800 kg m^{-3}	1.2 kg m^{-3}
c_p	770 J kg^{-1} K^{-1}	1005 J kg^{-1} K^{-1}
κ	20 J s^{-1} m^{-1} K^{-1}	2.4×10^{-2} J s^{-1} m^{-1} K^{-1}
μ	—	1.85×10^{-5} N s m^{-2}
γ	—	1.4

Table 1. Table of parameter values used for alumina particles and air.

SOUND PROPAGATION THROUGH PARTICULATE SUSPENSIONS IN AIR

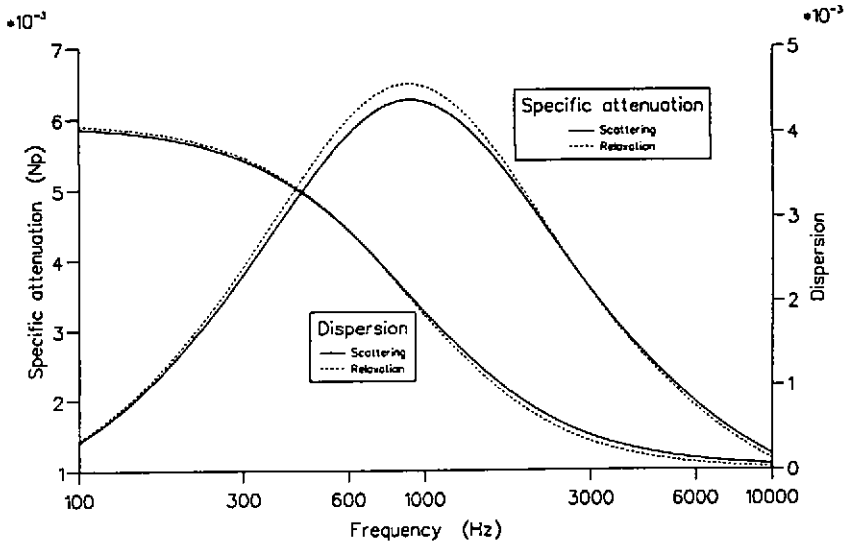

Fig. 1. Specific Attenuation & Dispersion — infinitesmal mass fraction ($\phi_m = 3\times10^{-3}$, $r = 2\mu m$)

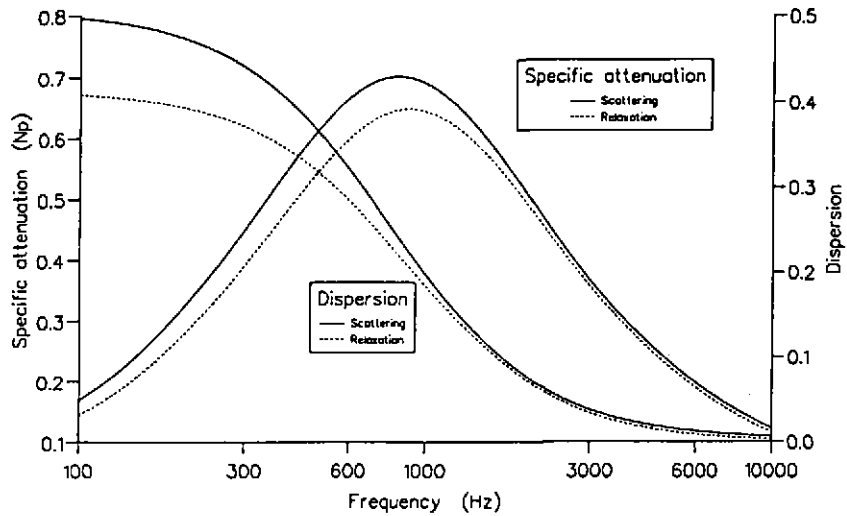

Fig. 2. Specific Attenuation & Dispersion — finite mass fraction ($\phi_m = 0.3$, $r = 2\mu m$)

SOUND PROPAGATION THROUGH PARTICULATE SUSPENSIONS IN AIR

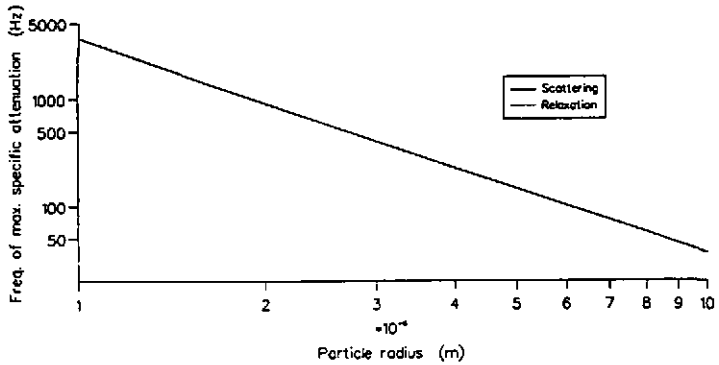

Fig. 3 Plot of frequency of maximum specific attenuation vs. alumina particle radius

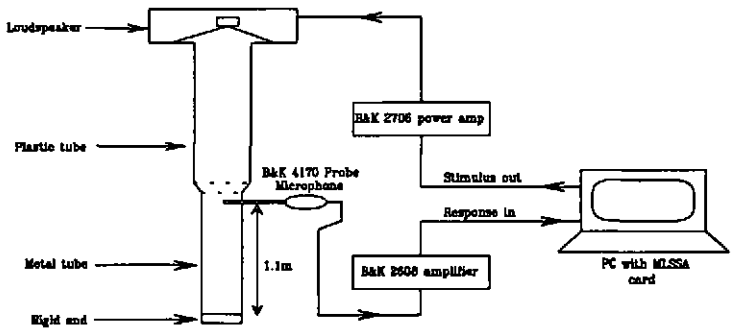

Fig. 4 Experimental apparatus

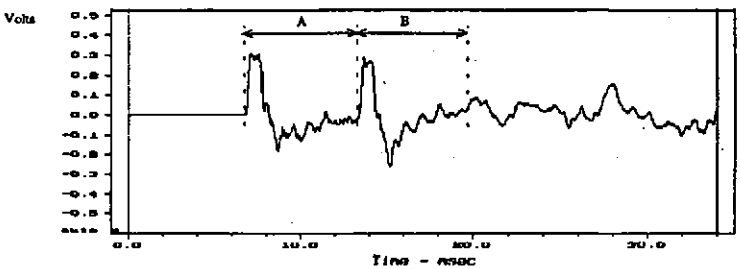

Fig. 5 Typical impulse response of apparatus (without particles)