ACTIVE CONTROL IN DIFFUSE SOUND FIELDS

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1. INTRODUCTION

The success of active sound control in enclosed sound fields will, in general, depend on the spatial distribution and time histories of the original, primary, sources of sound, on the size of the enclosure and its acoustic absorption properties, and on the number and distribution of the secondary sources and error microphones. If we restrict our discussion to deterministic sources of sound, some general guidelines on the active control of enclosed sound fields excited at frequencies at which the modal density is sufficiently low to give isolated resonances, have been presented using a modal formulation in [1]. It has been demonstrated that global reduction, i.e., reductions in the total acoustic potential energy in the enclosure, can be achieved if a number of remote secondary sources are positioned so as to give independent control of each of the modes significantly excited by the primary source distribution. Although the use of this deterministic model of the sound field is still valid at frequencies for which the number of significantly contributing modes is greater than the number of secondary sources, no generally applicable results can be obtained since the behaviour of the active control system then depends on the exact details of primary and secondary excitation of the enclosure. At high modal densities in the enclosure, well above the Schroeder frequency [2], the sound field is best described statistically and probabilistic models may give a more useful description of the effects of various active control strategies.

In order to give some idea of the practical applicability of results in this frequency regime, it is useful to note that the Schroeder frequencies in two enclosures in which active control is currently being contemplated (the interior of cars and the interior of medium size propeller aircraft) have both been experimentally estimated as being between 100 and 150 Hz.

Using statistical concepts it has been demonstrated [1] that *global* control at high modal densities is only generally possible if secondary sources can be placed within half a wavelength of a compact primary source. *Local* reductions in pressure, however, can always be achieved in a pure tone sound field using a remote secondary source to drive the pressure to zero at any one point in the enclosure. This paper, which is an extension of reference [3], examines the consequences of such a control strategy both near the point of cancellation (the 'zone of quiet') and on the pressure field well away from this point.

2. THE ZONE OF QUIET

The instantaneous pressure at some distance from a reference point ($\underline{x}_o$) in a pure tone diffuse sound field can be considered to be the sum of two components, one perfectly correlated and one completely uncorrelated with the pressure at $\underline{x}_o$;

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$$P(\underline{x}_o + \Delta x) = P_C(\underline{x}_o + \Delta\underline{x}) + P_U(\underline{x}_o + \Delta\underline{x}) \quad (1)$$

where $$\langle P(\underline{x}_o)P_U(\underline{x}_o + \Delta\underline{x})\rangle = 0 \quad (2)$$

and $$P_C(\underline{x}_o + \Delta\underline{x}) = R_{\underline{x}_o}(\Delta\underline{x})P(\underline{x}_o) \quad (3)$$

in which $\langle * \rangle$ denotes spatial averaging in the field and $R_{\underline{x}_o}(\Delta\underline{x})$ is the spatial cross correlation function at the point $\underline{x}_o$ in the sound field. The space averaged value of the squared pressure at $\underline{x}_o + \Delta\underline{x}$ is given by

$$\langle P^2(\underline{x}_o + \Delta\underline{x})\rangle = \langle P_C{}^2(\underline{x}_o + \Delta\underline{x})\rangle + \langle P_U{}^2(\underline{x}_o + \Delta\underline{x})\rangle \quad (4)$$

since $$\langle P_C(\underline{x}_o + \Delta\underline{x})P_U(\underline{x}_o + \Delta\underline{x})\rangle = 0. \quad (5)$$

In a spatially homogeneous field

$$\langle P^2(\underline{x}_o + \Delta\underline{x})\rangle = \langle P^2(\underline{x}_o)\rangle = \langle \bar{P}^2\rangle \quad (6)$$

i.e., the uniform mean square pressure. Using equations (3), (4) and (6) we can thus express the space average squared component of pressure at $\underline{x}_o + \Delta\underline{x}$ uncorrelated with the pressure at $\underline{x}_o$ as

$$\langle P_U{}^2(\underline{x}_o + \Delta\underline{x})\rangle = \langle \bar{P}^2\rangle(1 - R_x{}^2(\Delta\underline{x})) \quad (7)$$

By adjusting the amplitude and phase of a remote secondary source the pressure at $\underline{x}_o$ due to some primary source can be driven to zero. We seek to describe the average behaviour of the sound field at positions close to $\underline{x}_o$ due to the two sources. By driving $P(\underline{x}_o)$ to zero, the correlated part of the pressure at $\underline{x}_o + \Delta\underline{x}$ is also driven to zero and we are left with the components of the pressure fields due to both the primary and secondary sources which are uncorrelated with the pressure at $\underline{x}_o$:

$$\langle P_{UT}{}^2(\underline{x}_o + \Delta\underline{x})\rangle = \langle \bar{P}_T{}^2\rangle(1 - R_x{}^2(\Delta\underline{x})) \quad (8)$$

where $\langle \bar{P}_T{}^2\rangle$ is the sum of the mean square pressures in the enclosure due to the primary and secondary sources. If the primary and secondary sources are simple monopoles then in a space average sense $\langle \bar{p}_T{}^2\rangle$ will be proportional to the sum of the squares of their source strengths.

A computer simulation of a three dimensional pure tone diffuse sound field has been performed in which a randomly located secondary source is used to drive the pressure from a randomly located primary source to zero at another randomly located observation point. The averaged result for the mean square pressure at various distances away from the cancellation point are shown in Figure 1. Also shown in this figure is the theoretical curve of equation (8), in which the spatial cross correlation appropriate to a three dimensional diffuse field has been used

$$R_x(\Delta\underline{x}) = \sin(k|\Delta\underline{x}|)/k|\Delta\underline{x}| \quad (9)$$

The simulated and theoretical results for the field close to the point of cancellation show good agreement and suggest that the zone of quiet, within which the pressure has been reduced by at least 10 dB with respect to the primary field, has a diameter of about one tenth of a wavelength. The total mean square pressure well away from the point of cancellation was observed to rise to about four times that due to the primary source alone in this simulation, although this was not repeatable from one simulation to another. The reasons for this lack of statistical stability are discussed in the next section.

3. THE INCREASE IN PRESSURE WELL AWAY FROM THE POINT OF CANCELLATION

The complex pressure at $\underline{x}_o$ due to both the primary and secondary sources, of source strengths q_p and q_s respectively, is given by

$$P(\underline{x}_o) = Z_p q_p + Z_s q_s \qquad (10)$$

where Z_p and Z_s are the acoustic transfer impedances between the primary and secondary sources, and the observation point $\underline{x}_o$. Clearly the secondary source strength required to drive $P(\underline{x}_o)$ to zero is

$$q_{so} = \frac{Z_p}{Z_s} . q_p \qquad (11)$$

Therefore

$$\frac{|q_{so}|^2}{|q_p|^2} = \frac{|Z_p|^2}{|Z_s|^2} \qquad (12)$$

The statistical variation in $|q_{so}|^2$ can now be described in terms of the statistical properties of $|Z_p|^2$ and $|Z_s|^2$. The probability density function of the in-phase and quadrature components of the pressure in a pure tone diffuse field are Gaussian [2]. The probability density function of the mean square pressure, and hence of $|Z_p|^2$ and $|Z_s|^2$, will therefore be of Chi-squared form with 2 degrees of freedom. If the primary and secondary sources are a good deal further than a wavelength apart, then their pressure fields will be uncorrelated. The probability density function we are seeking will thus be that of the ratio of two independent random variables each distributed according to a Chi-squared probability density function with two degrees of freedom. Such a function is given by [4]

$$PDF(v) = \frac{1}{(1 + v)^2} \qquad (13)$$

where v is equal to $|q_{so}|^2/|q_p|^2$.

Figure 2 shows the probability distribution function of the mean square secondary source strength divided by the mean square primary source strength observed in the computer simulation described above, compared with the predicted distribution described by equation (13). The curves are seen to be in good agreement. An important property of equation (13) is that the theoretical mean and variance of the distribution are infinite. Since the total mean square pressure in the enclosure depends on the mean square source strengths, this quantity too must in principle have an infinite mean and variance. This explains the lack of repeatability in the mean square pressure estimates in the simulations described above.

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It is demonstrated in [5] that equation (13) can be considered as a special case of the more general Pareto distribution whose density function is given by

$$P(v,N) = \frac{N}{(1 + v)^{N+1}} \qquad (14)$$

Using a single remote source to minimize the pressure at a single microphone results in a Pareto distribution with $N = 1$, equation (13). If M remotely placed sources are used to minimise the pressure at L well separated sensors, it is found that the observed probability density function of the mean square secondary source strengths is still well described by the Pareto distribution, with $N = L/M$.

The physical reason for the large values of secondary source strength, which cause the lack of convergence in the single source single sensor case above, is clear from equation (11). If the secondary source happens to be placed at a point in the room where at the operating frequency it has little influence on the pressure at $\underline{x}_o$, then $|Z_s|^2$ will be very small and $|q_{so}|^2$ correspondingly large. The next section considers ways in which this increase in mean square secondary source strength, and thus total mean square pressure, may be avoided.

4. POSSIBLE METHODS OF LIMITING THE INCREASE IN MEAN SQUARE PRESSURE AWAY FROM THE POINT OF CANCELLATION

4.1 Hard Limiting

In any practical active control system the secondary source will only be able to supply a finite maximum output, q_{smax}, say. If the position of the secondary source and the frequency dictate that the modulus of the optimum secondary source, q_{opt} from equation (11), exceeds this limit, it is assumed that active control is abandoned or that the secondary source position can be adjusted slightly so that the limit is not exceeded. The probability density function for the mean square secondary source strength will now be a modified form of equation (13) with a truncated upper limit and a normalisation factor to ensure that the integral of the density function over the interval is still unity. This may be expressed [5] as

$$P_H(v) = \frac{1 + v_{max}}{v_{max}} \frac{1}{(1 + v)^2} \qquad 0 \leq v \leq v_{max}$$

$$P_H(v) = 0 \qquad v > v_{max} \qquad (15)$$

in which $v = |q_s|^2/|q_p|^2$ and $v_{max} = |q_{smax}|^2/|q_p|^2$. In contrast to equation (13) the mean value of this distribution does converge for any finite v_{max}, and this expectation value of the mean square secondary source strength ($\langle v \rangle$) is plotted against the upper limit imposed on its value (v_{max}) in Figure 3.

4.2 Soft Limiting

Consider the general case in which the vector of complex pressures at L microphones (p) is equal to the sum of the pressures due to the primary sources ($\mathbf{p_p}$) and those due to M secondary sources, whose complex strengths are given by the elements of the vector $\mathbf{q_s}$, coupled to $\mathbf{p}$ by the matrix of complex transfer impedances $\mathbf{Z_s}$, so that

$$\mathbf{p} = \mathbf{p_p} + \mathbf{Z_s q_s} \tag{16}$$

A cost function frequently used in optimal control which penalises control effort as well as total error, and which effectively "soft limits" the secondary source strengths is:

$$J_T = \mathbf{p}^H\mathbf{p} + \beta \mathbf{q_s}^H\mathbf{q_s}$$

$$\therefore \quad J_T = \mathbf{p_p}^H\mathbf{p_p} + \mathbf{p_p}^H\mathbf{Z_s q_s} + \mathbf{q_s}^H\mathbf{Z_s}^H\mathbf{p_p} + \mathbf{q_s}^H[\mathbf{Z_s}^H\mathbf{Z_s} + \beta I]\mathbf{q_s} \tag{17}$$

This is a Hermitian quadratic function of $\mathbf{q_s}$ which, since $[\mathbf{Z_s}^H\mathbf{Z_s} + \beta I]$ is positive definite for $\beta > 0$, has a unique global minimum for

$$\mathbf{q_s} = \mathbf{q_{so}} = -[\mathbf{Z_s}^H\mathbf{Z_s} + \beta I]^{-1}\mathbf{Z_s}^H\mathbf{p_p} \tag{18}$$

Applying this result to the single sensor, single source case (L = M = 1) considered above, and assuming $p_p = Z_p q_p$, the optimum soft limited source strength becomes

$$q_{so} = -\frac{Z_s^* Z_p}{(|Z_s|^2 + \beta)} q_p \tag{19}$$

This result reduces to equation (11) for $\beta = 0$, although the probability density function for $|q_{so}|^2/|q_p|^2$ now has a convergent mean for all $\beta > 0$. This mean value of $|q_{so}|^2/|q_p|^2$ is plotted against β in Figure 4.

The problems caused by the chance of a particularly small transfer impedance from a single source to a single sensor can also be removed by using two independently positioned secondary sources to control the pressure at a single point (i.e., L = 1, M = 2). This problem is overdetermined with no effort parameter, β, since the matrix $\mathbf{Z_s}^H\mathbf{Z_s}$ is singular, but even very small values of β allow equation (18) to be computed to define a unique pair of secondary source strengths, q_{s1} and q_{s2}, which each couple as best they can to the observation point via the relevant transfer impedances z_{s1} and z_{s2}. These source strengths may be written:

$$q_{s1} = \frac{-Z_{s1}^* Z_p}{(|Z_{s1}|^2 + |Z_{s2}|^2 + \beta)} q_p, \quad q_{s2} = \frac{-Z_{s2}^* Z_p}{(|Z_{s1}|^2 + |Z_{s2}|^2 + \beta)} q_p \tag{20}$$

The expectation value of $(|q_{s1}|^2 + |q_{s2}|^2)/|q_p|^2$ is plotted against β in Figure 5, which shows that the total source strength required to minimise the pressure at a point is considerably less than for the case of a single source (Figure 4). It is interesting to note that equation (20) is valid as $\beta \to 0$, in which case perfect cancellation at the control microphone will be achieved.

4.3 Minimisation of the Output of Two Closely Spaced Microphones

The disturbance of the sound field away from the point of control can also generally be reduced by *minimising* the sum of the squares of the pressures at more than one location rather than effecting *cancellation* at a point. If the vector of complex pressures at two such microphones is $\mathbf{P}$, this is the sum of the contributions from the primary source, q_p, via a vector of transfer impedances $\mathbf{Z}_p$ and the secondary source q_s via a vector of transfer impedances $\mathbf{Z}_s$:

$$\mathbf{P} = \mathbf{Z}_p q_p + \mathbf{Z}_s q_s \tag{21}$$

The cost function to be minimised by q is now $J_p = \mathbf{P}^H\mathbf{P}$ and the optimum secondary source strength is

$$q_{so} = -\frac{\mathbf{Z}_s^H \mathbf{Z}_p}{\mathbf{Z}_s^H \mathbf{Z}_s} q_p \tag{22}$$

The probability density function of $|q_{s2}|^2/|q_p|^2$ now depends on the relationship beween the pressure at the two observation points. If the spacing is considerably less than a wavelength, the two pressures are highly correlated and the experiment reduces to that of cancellation at a point. As the two microphones are moved further apart, however, they will become less correlated until, when the spacing is much greater than a wavelength, they are statistically independent. The distribution function of $|q_{so}|^2/|q_p|^2$ is thus expected to smoothly change from a Pareto distribution with $N = 1$ to a Pareto distribution with $N = 2$, as the microphones are moved apart. In fact by again considering the correlated and uncorrelated parts of the two pressures it can be argued [5] that if the microphones are spaced a distance Δr apart the effective Pareto parameter (N) is

$$N = 2 - \sin^2(k\Delta r)/(k\Delta r)^2 \tag{23}$$

and this has been found to be in good agreement with simulation results. A property of the Pareto distribution is that it has a finite mean for all N greater than unity. The mean of the distribution of $|q_{so}|^2/|q_p|^2$ can now be calculated for various microphone separations Δr, using the value of N in equation (23). This is shown in Figure 5.

The computer simulation described above was also used to determine the average reductions close to a pair of microphones with various spacings, when J_p was minimised with a remote secondary source. These results are shown in Figure 6 [5] in the form of an isometric plot of the average reductions at various distances from the centre point of the microphones ($\Delta \underline{x}$) for various microphone separations (Δr). The rise in mean square pressure well away from the microphones as Δr is reduced is demonstrated, but the interesting effect is that instead of a single zone of quiet, two separate reductions in the pressure are produced at the two microphones when $\Delta r > 0.4\lambda$, i.e., for the separation at which the pressures at the microphones are becoming uncorrelated.

5. CONCLUSIONS

The Schroeder frequencies of various enclosures such as the cabins of aircraft or cars in which active sound control is currently being contemplated have been measured as being between 100 and 150 Hz. Above this frequency in these enclosures statistical models may give a more useful description of the sound field than deterministic models. This paper considers the effects of cancellation at a point in such a pure tone diffuse sound field both on the field close to the cancellation microphone, the zone of quiet, and on the field far from the cancellation microphone. Good agreement is observed between theoretical results for these two regions and computer simulations. A potentially large increase in the mean square pressure away from the point of cancellation is seen to occur, although this can be controlled to some extent by: (a) hard limiting the secondary source strength, (b) soft limiting the secondary source strength(s), or (c) minimising the sum of the squares of the pressures at two closely spaced microphones.

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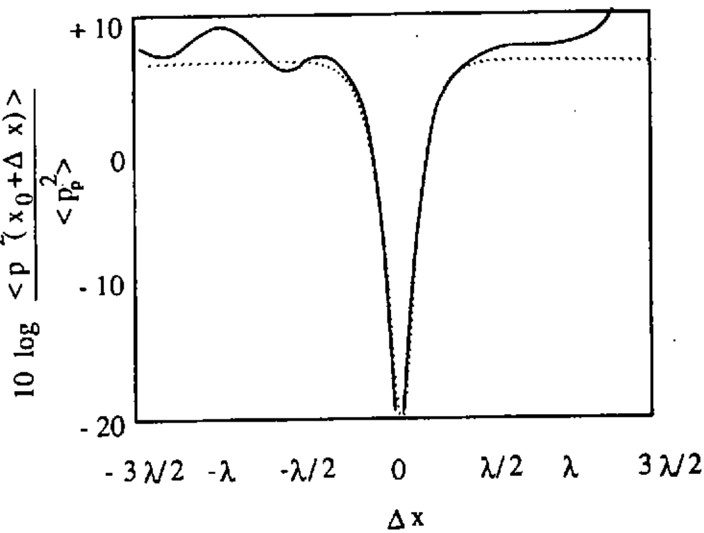

Figure 1. The mean square pressure at various distances from a single point of cancellation in a pure tone diffuse sound field, normalised by the mean square pressure in the field before cancellation. Simulation results over 200 averages (solid) and theory (dashed).

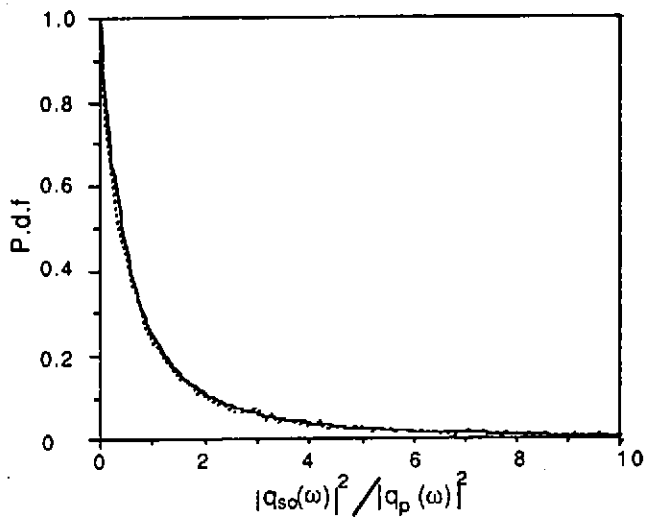

Figure 2. Probability density function for the mean square secondary source strength required to cancel the pressure at a point in a pure tone diffuse sound field. Computer simulation (solid) and theory (dashed).

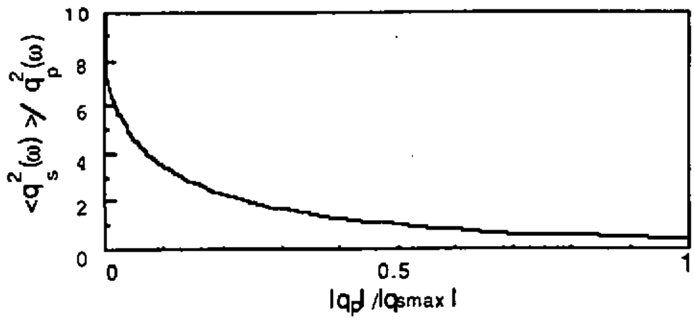

Figure 3. Expectation value of the mean square secondary source strength for various values of hard limiting.

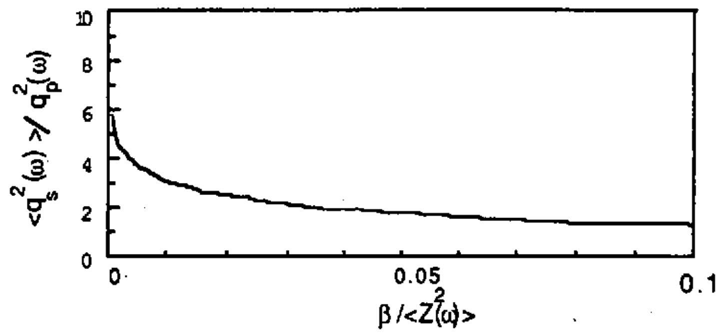

Figure 4. Expectation value of the mean square secondary source strength for various values of the soft limiting parameter β (normalised by the expectation value of the mean square transfer impedance in the room).

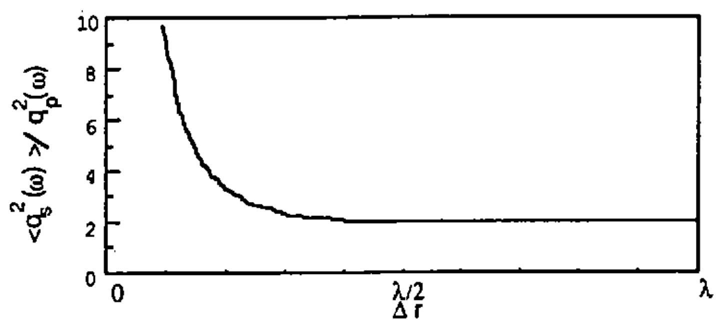

Figure 6. Expectation of the mean square secondary source strength required to minimise the sum of squared pressures at two points separated by a distance Δr apart in a pure tone diffuse sound field.

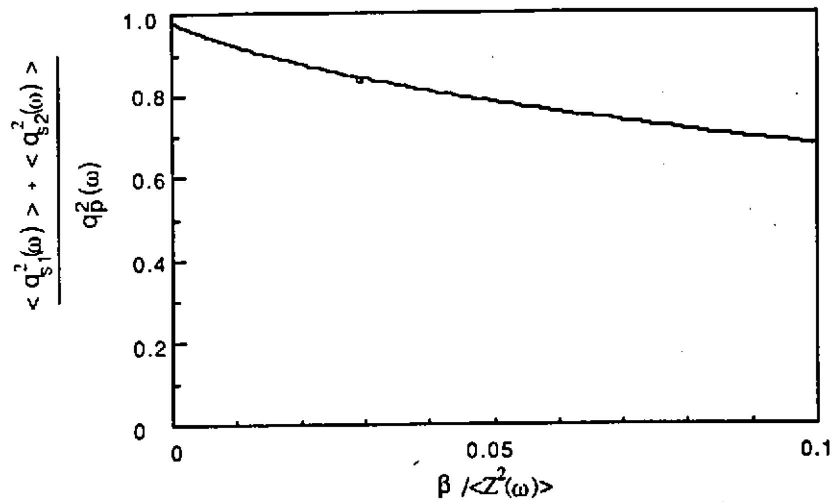

Figure 5. Expectation of the sum of mean square strengths of two sources driven so as to minimise the pressure at one point in a pure tone diffuse field with soft limiting parameter β.

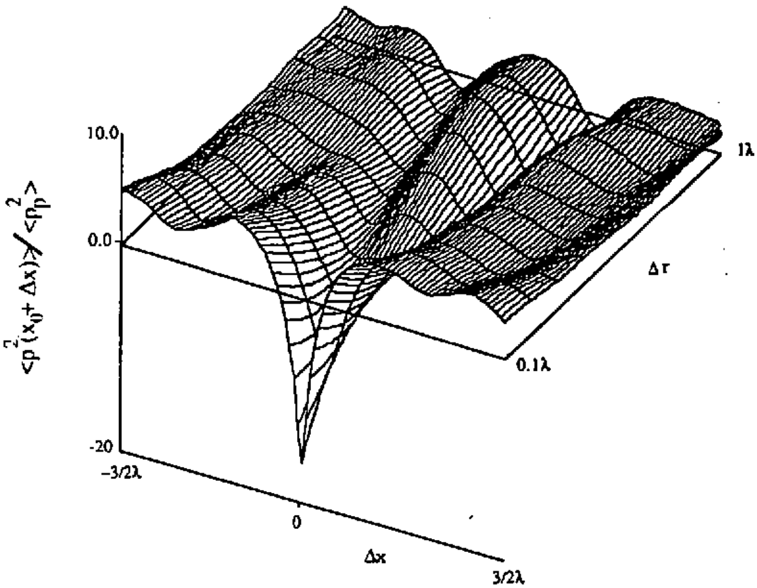

Figure 7. Expectation value of the mean square pressure in a pure tone diffuse sound field after minimising the sum of square pressures at two microphones (distances Δr apart), against the distance Δx from the mid-point of the two microphones.