Proceedings of The Institute of Acoustics

AN ADAPTIVE ALGORITHM FOR MULTICHANNEL ACTIVE CONTROL

S.J. Elliott and P.A. Nelson

Institute of Sound and Vibration Research
University of Southampton

1. INTRODUCTION

There are a large number of noise and vibration control problems in which the excitation is periodic, or nearly so. The noise and vibration from reciprocating or rotating machines, or from electrical transformers are obvious examples. It is often the case that the higher harmonics of such sources can be attenuated by conventional, passive control methods, which leave the lower frequencies relatively unaffected. It is in these situations that active noise or vibration control can prove to be very beneficial. The mechanical structure or acoustic space to which active control is to be applied is also often large compared to the wavelength of the disturbance being considered. In this case it is necessary to use multiple secondary sources to achieve suppression and the active control becomes multichannel. The fundamental excitation frequency is usually known, so the problem reduces to one of adjusting the amplitude and phase, or equivalently the in-phase and quadrature parts, of each harmonic component of the signal driving each secondary source.

This paper addresses the general problem of adjusting the in-phase and quadrature components of a pure tone fed to a number of secondary sources in order to minimise some error criterion. The extension to a harmonic excitation can then be made by superposition in the simplest case. The problem is first formulated in the frequency domain with an acoustic example, and the concept of a quadratic error surface introduced.

The in-phase and quadrature components of the source strengths of a number of secondary acoustic sources can be represented by the complex vector $\underline{q} = \underline{q}_R + j\underline{q}_I$. The error criterion, J, can be either the total acoustic potential energy in an enclosure [1], or the sum of the squares of the pressures at a number of locations [2], or the total output power of an array of sources including the secondary sources [1]. In all these cases the relationship between J and $\underline{q}$ can be expressed in the complex quadratic form:

$$J = \underline{q}^H \underline{A}\, \underline{q} + \underline{b}^H \underline{q} + \underline{q}^H \underline{b} + c \qquad (1)$$

where H denotes the Hermitial transpose and the interpretation of the matrix $\underline{A}$, vector $\underline{b}$ and scalar c will depend on the problem considered. This equation implies that if J is plotted against any two components of the variables represented by $\underline{q}$, then a bowl shaped surface is produced, as illustrated in Figure 1. It should be noted that the minimum value of J, at the bottom of this surface, and values of q_{1R} and q_{1I} necessary to achieve this minimum will, in general, depend on the values of all the other components in $\underline{q}$. In order to simultaneously minimise J with respect to all of the variables in $\underline{q}$, a multidimensional error surface can be postulated, by analogy with the three dimensional one above, which will have a unique global minimum for some particular set of parameters in $\underline{q}$, equal to $\underline{q}_o$, say. The existence of such a global minimum depends on the matrix $\underline{A}$ in equation (1) being positive definite, a property which can be shown to be true for the cases considered above.

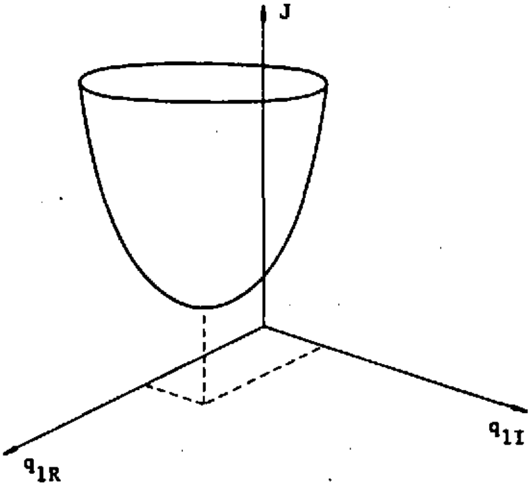

Figure 1. The quadratic error surface generated by plotting the error criterion, J, against any two variables, in this case the in-phase and quadrature components of source q_1.

The gradient of this surface with respect to each element of $\underline{q}$ can be calculated analytically by differentiating J with respect to the real and imaginary parts of $\underline{q}$ and defining [1]

$$\frac{dJ}{d\underline{q}} = \frac{dJ}{d\underline{q}_R} + j\,\frac{dJ}{d\underline{q}_I} = \underline{A}\,\underline{q} + \underline{b} \qquad (2)$$

If all the elements of this vector are set equal to zero, the elements of $\underline{q}$ must be those which correspond to the minimum of this error surface, thus

$$\underline{q}_o = -\,\underline{A}^{-1}\underline{b} \qquad (3)$$

The minimum value of J which corresponds to this set of source strengths is given by

$$J_o = c - \underline{b}^H\underline{A}^{-1}\underline{b} \qquad (4)$$

This formulation of the minimisation is thus analytically tractable and allows a clear evaluation of the performance of a control system from acoustical considerations only, but cannot be directly used in a practical control system. The properties of these equations, however, are implicitly used in all practical control systems, and the remainder of this paper illustrates this in the case of two approaches which have already been presented in the literature. We also introduce a third method which has grown out of the formulation presented above.

2. MATRIX INVERSION

Consider the case in which the outputs of an array of L sensors represented by the complex elements of the vector $\underline{p}$, are composed of the linear superposition of contributions from a primary source, $\underline{p}_p$ say, and those from an array of M secondary sources with inputs represented by the complex elements of $\underline{q}$, where L > M. Then

$$\underline{p}(\omega) = \underline{p}_p(\omega) + \underline{C}(\omega)\underline{q}(\omega) \qquad (5)$$

where the single frequency nature of the problem has been represented by the evaluation of all quantities at a single frequency, ω. The matrix $\underline{C}$ denotes the matrix of transfer functions, evaluated at ω, between each of the secondary source inputs and sensor outputs in the absence of the primary field. The sum of the squared moduli of the sensor outputs is taken as the error criterion, and may be denoted

$$J = \underline{p}^H\underline{p} \qquad (6)$$

The elements in the general complex quadratic form above, equation (1), can now be identified as:

$$\underline{A} = \underline{C}^H\underline{C}, \quad \underline{b} = \underline{C}^H\underline{p}_p, \quad \underline{c} = \underline{p}_p{}^H\underline{p}_p \qquad (7)$$

and the set of inputs to the secondary sources which minimises this ereror criterion is thus [3]:

$$\underline{q}_o = -(\underline{C}^H\underline{C})^{-1}\underline{C}^H\underline{p}_p \qquad (8)$$

This formulation has been generalised in the study of a related problem, that of "higher harmonic control" of helicopter vibration, in which the secondary input signals are applied as perturbations to the pitch angles of the rotor blades. In this case the error criterion also includes terms proportional to the sum of the squares of the secondary input signals and the sum of the squares of the differentials of these signals [4].

Returning to the acoustic case, consider the special case of equation (5), in which L = M, so that $\underline{C}$ becomes a square matrix and equation (8) reduces to:

$$\underline{q}_o = -\underline{C}^{-1}\underline{p}_p \qquad (9)$$

In this case the minimum in J corresponds to all the sensor outputs being driven to zero.

The direct measurement of the elements of $\underline{C}$ and $\underline{p}_p$, together with the use of equation (9), has been suggested as a practical method of evaluating $\underline{q}_o$ [5]. However, in practice, the measurement of $\underline{C}$ and $\underline{p}_p$ cannot be achieved without some error, and the application of equation (9) with these estimated parameters will result in some residual output from the sensors. An iterative algorithm has been suggested for further reducing these outputs [5] which will clearly compensate for errors in the estimation of $\underline{p}_p$, and even allow for slow changes in the primary excitation.

We consider here the question of the stability of the algorithm to errors in the measurement of the matrix $\underline{C}$. Let the measured estimate of this matrix be denoted by $\hat{\underline{C}}$. Assuming a good estimate of the stationary primary excitation is obtained, the inputs to the secondary sources will be $\underline{q}_1$, say, where

$$\underline{q}_1 = -\hat{\underline{C}}^{-1}\,\underline{p}_p \qquad (10)$$

so that the residual output of the sensors, $\underline{p}_1$, say, are given by

$$\underline{p}_1 = [\underline{I} - \underline{C}\,\hat{\underline{C}}^{-1}]\underline{p}_p \qquad (11)$$

The proposed iterative algorithm involves adding an additional input to the secondary sources given by

$$\underline{q}_2 = -\hat{\underline{C}}\,\underline{p}_1 \qquad (12)$$

so the new residual field is given by

$$\underline{p}_2 = [\underline{I} - \underline{C}\,\hat{\underline{C}}^{-1}]\underline{p}_1 = [\underline{I} - \underline{C}\,\hat{\underline{C}}^{-1}]^2\underline{p}_p \qquad (13)$$

It can be seen that if this process is repeated n times, we obtain

$$\underline{p}_n = [\underline{I} - \underline{C}\,\hat{\underline{C}}^{-1}]^n\underline{p}_p \qquad (14)$$

If the matrix $[\underline{I} - \underline{C}\hat{\underline{C}}^{-1}]$ is expressed in normal form [10], it can be seen that the elements of $\underline{p}_n$ will only tend to zero with increasing n if the modulus of all the eigenvalues of the matrix $[\underline{I} - \underline{C}\,\hat{\underline{C}}^{-1}]$ are less than unity.

If we express $\hat{\underline{C}}$ as

$$\hat{\underline{C}} = \underline{C} + \underline{\Delta C} = [\underline{I} + \underline{\epsilon}]\underline{C} \qquad (15)$$

where $\underline{\Delta C}$ is the absolute error matrix and $\underline{\epsilon}$ is the fractional error matrix, then

$$\underline{I} - \underline{C}\,\hat{\underline{C}}^{-1} = \underline{\epsilon}[\underline{I} + \underline{\epsilon}]^{-1} \qquad (16)$$

By assuming the real and imaginary parts of the complex elements of $\underline{\epsilon}$ are randomly distributed with an approximately Gaussian distribution, a simulation has been performed for various standard deviations of these elements to examine the eigenvalue spread of eqn.(16) and hence the probability of the algorithm being unstable. Results of such a simulation involving the evaluation of 1000 such matrices with orders 2,4,8 and 16 are given in Fig. 2. It can be seen, for example, that if the order of the matrix is 16 and the standard deviation of the elements of $\underline{\epsilon}$ is 0.15, then there is a 60% chance of the iterative algorithm being unstable.

It should be noted, however, that the fractional error matrix may be written as

$$\underline{\epsilon} = \underline{C}^{-1}\underline{\Delta C} \qquad (17)$$

The magnitude of the elements of $\underline{\epsilon}$ thus depend not only on those of $\underline{\Delta C}$ but also on the inverse of $\underline{C}$. Consequently, if $\underline{C}$ is ill conditioned, large values of $\underline{\epsilon}$ will result. This situation can occur, for example, if two sensors are placed close together, or if the response of the system is dominated by a single, lightly damped mode.

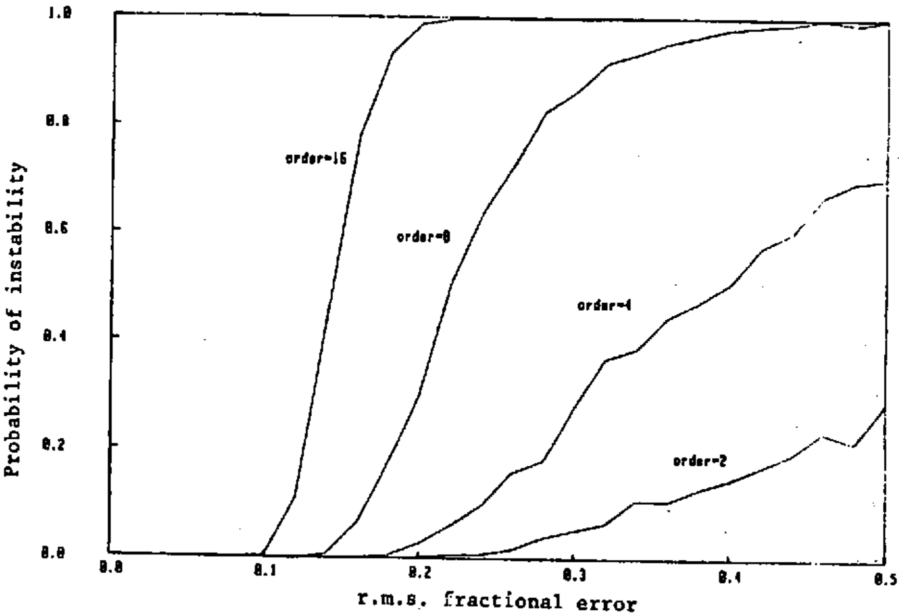

Figure 2. The probability of the iterative matrix algorithm being unstable against the standard deviation of the elements of the fractional error matrix for various matrix orders.

3. TRIAL AND ERROR

Rather than use the analytic equation describing the optimal source strengths, another practical approach to minimization would be to make use of the *shape* of the error surface. It has been shown above that the sum of the mean squared sensor outputs is a quadratic function of the in-phase and quadrature signals applied to each of the secondary sources, assuming only that the system is linear. This is true whether or not the sensor outputs contain components at other harmonic frequencies, since these components will be unaffected by the level of the excitation at the harmonic of interest. However, such a 'broad band' error criterion will be less sensitive to adjustments in the source strengths than the sum of the mean squared error signals evaluated only at the harmonic being adjusted.

A variety of algorithms which exploit the shape of this error surface have been presented by Chaplin and his co-workers [6]. Some of these algorithms perturb the in-phase and quadrature components of one harmonic while monitoring the 'broad band' or 'narrow band' error criterion. If the error criterion decreases as a result of this perturbation, another adjustment is made to the output in the same fashion. This is a form of trial and error control which relies for its convergence on the fact that the error surface has a unique global minimum. If the output of each secondary source in a multichannel system is adjusted separately in this way, the fact that cross coupling exists will not affect the stability of the algorithm. It will, however, be necessary to sequentially adapt all sources in turn a number of times before the true global minimum is approached.

Another type of trial and error algorithm discussed by Chaplin [6] is the adjustment of the amplitude of a periodic signal at a number of fixed points in the cycle. These are combined together using a synchronous waveform synthesiser to produce the outputs for the secondary sources. This process is exactly the same as adjusting the coefficients of a digital FIR filter fed by a periodic impulse train [7]. Such algorithms are often referred to as working in the 'time domain' to distinguish them from the 'frequency domain' algorithms described above.

In order to discuss the error surface for such algorithms it is useful to reformulate the problem in the sampled time domain rather than the continuous frequency domain. This turns out to be a more powerful description of a dynamic control system since it can describe and account for transient behaviour. The frequency domain formulation above is essentially a steady state description of the system to be controlled. After any adjustments are made to the secondary sources using such frequency domain algorithms, all transients must be allowed to die away before the new state of the system can be measured [5].

The nomenclature used here is slightly different from that used above in order to be more consistent with the signal processing literature [8]. Let the sampled output of the ℓ'th sensor be $e_\ell(n)$, which is equal to the sum of the contribution from the primary source, $d_\ell(n)$, and contributions from each of the secondary sources. The input to the m'th actuator is produced by passing a reference signal, $x(n)$, containing the correct frequencies, through an FIR filter whose i'th coefficient is w_{mi}. If the electrical transfer function between the m'th secondary source and ℓ'th sensor is itself modelled as a J'th order FIR filter, whose j'th coefficient is $c_{\ell mj}$, then

$$e_\ell(n) = d_\ell(n) + \sum_{m=1}^{M} \sum_{j=0}^{J-1} c_{\ell mj} \sum_{i=0}^{I-1} w_{mi}\, x(n-i-j) \qquad (18)$$

The error criterion can now be defined as

$$J = \sum_{\ell=1}^{L} \overline{e_\ell^2(n)} \tag{19}$$

where the overbar denotes that a time average has been taken. Since $e_\ell(n)$ is a linear function of each w_{mi}, J must be a quadratic function of each of these filter coefficients. Becuse of the quadruple summation involved in expressing J as an explicit function of w_{mi}, it is difficult to cast this formulation in terms of the general quadratic form of equation (1). However, since J can never be negative, and is obviously a quadratic function of the filter coefficients, this error surface must also have a unique global minimum. Consequently each of the w_{mi} coefficients may be adjusted using trial and error algorithms to minimise J, provided they are adjusted separately, as discussed above.

4. STOCHASTIC GRADIENT

Another method of adjusting each of the filter coefficients would be in proportion to the negative of the gradient of the error criterion with respect to that coefficient, i.e., the method of steepest descent. For the k'th iteration, this corresponds to

$$w_{mi}(k+1) = w_{mi}(k) - \mu \frac{\partial J}{\partial w_{mi}} \tag{20}$$

where μ is the convergence coefficient. Note that in this case,

$$\frac{\partial J}{\partial w_{mi}} = \sum_{\ell=1}^{L} \overline{\frac{\partial e_\ell^2(n)}{\partial w_{mi}}} = \sum_{\ell=1}^{L} \overline{2e_\ell(n) \frac{\partial e_\ell(n)}{\partial w_{mi}}} \tag{21}$$

and that from equation (18)

$$\frac{\partial e_\ell(n)}{\partial w_{mi}} = \sum_{j=0}^{J-1} c_{\ell mj}\, x(n-i-j) \tag{22}$$

This is a sequence equal to the one which would be obtained at the ℓ'th sensor if the reference signal, delayed by i samples, was applied to the m'th secondary source. We denote this sequence by $r_{\ell m}(n-i)$, so that

$$\frac{\partial J}{\partial w_{mi}} = 2 \sum_{\ell=1}^{L} \overline{e_\ell(n) r_{\ell m}(n-i)} \tag{23}$$

Because of the time averaging in this expression for the gradient, the filter coefficients can only be updated slowly by equation (20) as it stands.

If, however, the instantaneous value of equation (23) is taken as an approximation to its true value, then each of the filter coefficients can be updated for every new sample of $e_\ell(n)$ and the algorithm becomes [9]:

$$w_{mi}(n + 1) = w_{mi}(n) - \alpha \sum_{\ell=1}^{L} e_\ell(n) r_{\ell m}(n - i) \qquad (24)$$

where $\alpha = 2\mu$. Although the estimate of the gradient will probably be incorrect at any one instant, it must be correct on average. The instantaneous gradient may thus be considered to be the true gradient contaminated by some zero mean measurement noise. The algorithm is thus said to use a 'stochastic gradient' estimate.

The derivation of the algorithm above places no constraint on the form of the reference signal, except that it is not affected by the output of the secondary sources. It is, however, extremely simple to apply the algorithm in the case of a synchronously sampled sinusoidal reference signal of the form

$$x(n) = \cos(\pi n/2) \qquad (25)$$

which has exactly 4 samples/cycle. In this case only two point adaptive filters need by used since the input to the m'th secondary source $(s_m(n))$, for example, may then be expressed as

$$s_m(n) = w_{mo}x(n) + w_{m1}x(n - 1)$$

$$= w_{mo}\cos(\pi n/2) + w_{m1}\sin(\pi n/2) \qquad (26)$$

It is clear that in this case the two filter coefficients are proportional to the in-phase and quadrature parts of the secondary signal. This special case of the time domain formulation does then have a frequency domain interpretation. In a similar way the filtered reference signal, $r_{\ell m}(n)$, can be generated by passing $x(n)$ through a two point FIR filter which exactly matches the transfer function between source and sensor at the frequency of interest.

In order to demonstrate the properties of the algorithm for this type of reference signal, a simple simulation has been performed of a system with two secondary sources coupled to each of four sensors by transfer functions of the form

$$c_{\ell m}(z) = a_{\ell m} z^{-P_{\ell m}} / (1 - b_{\ell m} z^{-Q_{\ell m}})$$

as illustrated in Figure 3.

numerator in this expression represents an overall delay, the average value of which is 14 samples, or 3½ periods of the reference signal. The denominator represents a simple 'reverberant' behaviour by a recursive term which has an average time constant of about 10 samples or 2½ periods of the reference signal.

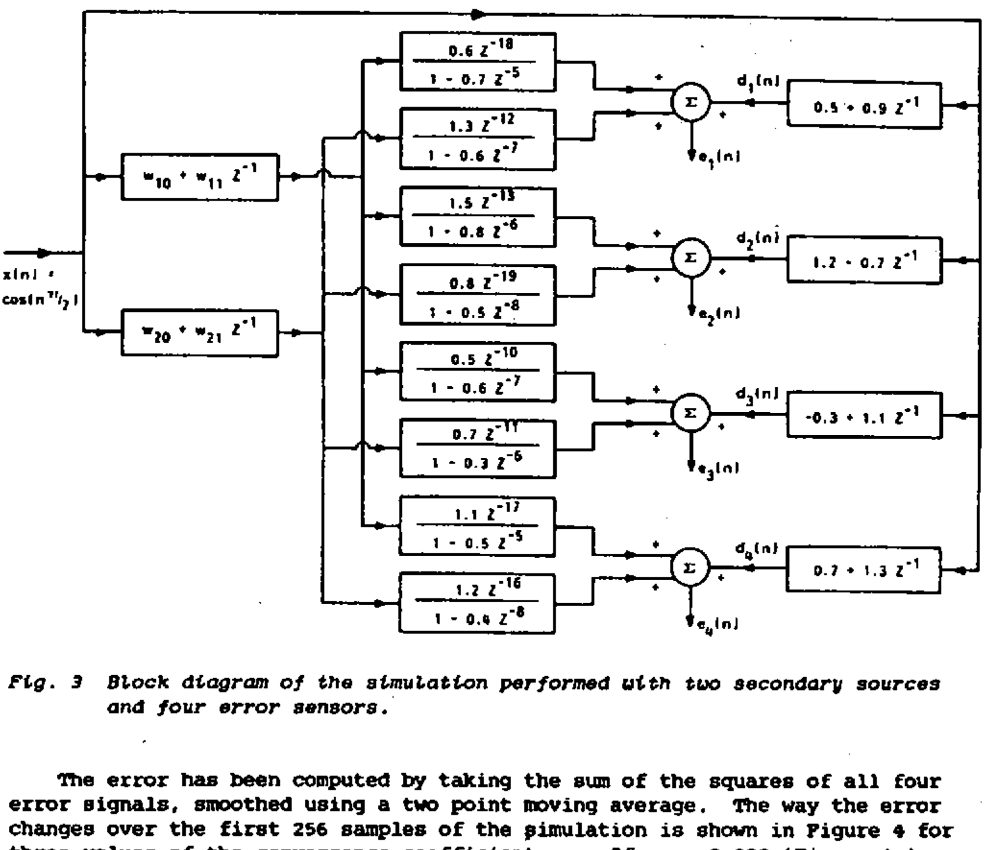

Fig. 3 Block diagram of the simulation performed with two secondary sources and four error sensors.

The error has been computed by taking the sum of the squares of all four error signals, smoothed using a two point moving average. The way the error changes over the first 256 samples of the simulation is shown in Figure 4 for three values of the convergence coefficient α. If $\alpha = 0.002$ (Figure 4a), the algorithm is seen to converge, although not as quickly as when $\alpha = 0.005$ (Figure 4b). The overall delay in the response, due to the delay in the models for each $c_{\ell m}$, is clearly seen in these figures. If $\alpha = 0.005$, the algorithm is seen to have converged within about 100 samples, or 25 cycles of the reference signal. If the reference signal were at 100 Hz, this corresponds to a convergence time of 250 ms. This compares favourably with the 'reverberation time' of the system being controlled, which may be estimated to be the overall delay plus three time constants of the recursive part, which is about 100 ms in this case.

The final, residual, error cannot be zero when there are more sensors than sources. It can be shown however that the total error after 100 samples, in the case where $\alpha = 0.005$, is within 2% of the value of the total error corresponding to the theoretically optimal source strengths, computed using methods similar to those outlined in the introduction. If a value of α

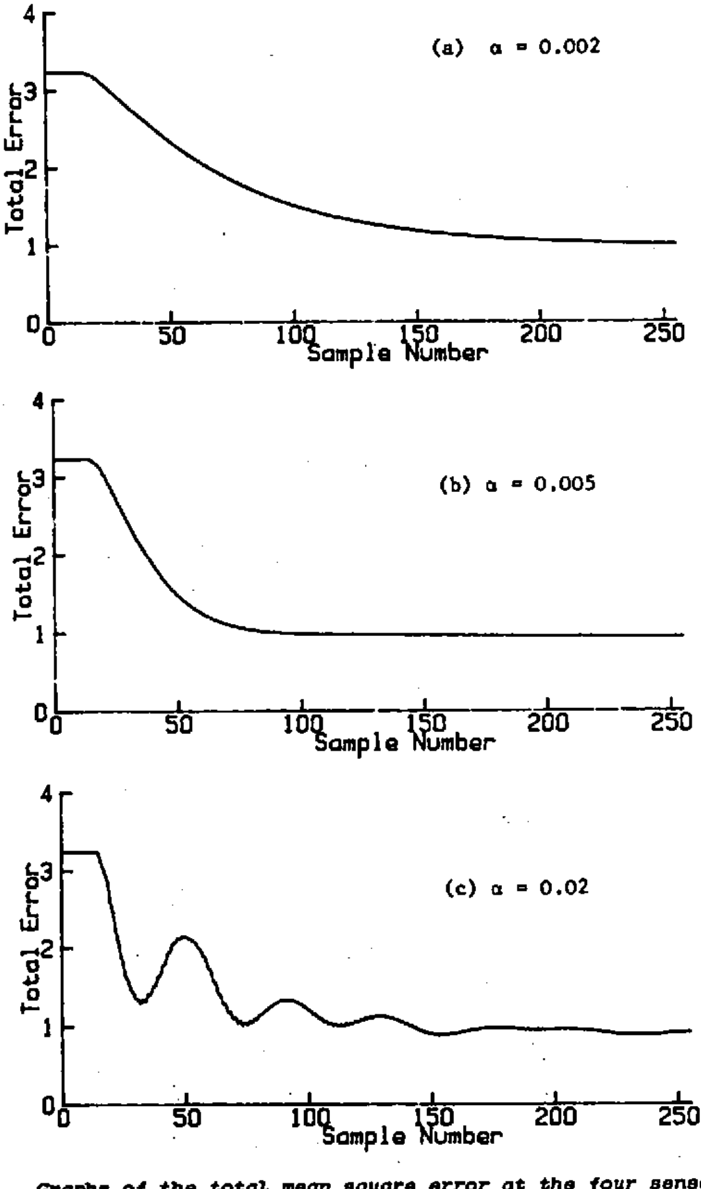

Figure 4. Graphs of the total mean square error at the four sensors against time for the simulation, with three values of the convergence coefficient; (a) α = 0.002, (b) α = 0.005, (c) α = 0.02.

larger than about 0.005 is used the convergence time begins to increase again and the error begins to show a characteristic oscillatory behaviour with time as illustrated in Figure 4c. The amplitude of these oscillations increases as α is further increased until the system becomes unstable for values of α greater than about 0.05.

Other properties of this algorithm are that it is very robust to errors in the generation of the filtered reference signals. In fact the algorithm can be made stable with nearly 90° phase error on these signals, indicating that the algorithm is very tolerant to errors made in the measurement of the transfer characteristics of the system to be controlled. The algorithm is also robust to uncorrelated noise contamination of the sensor outputs and to mild non- linearities in the response of the system to be controlled.

The fast adaption time of the algorithm can be further demonstrated by periodically modulating the magnitude of the primary excitation in the simulation. The way in which the total mean square error varies with time with and without adaption is illustrated in Figure 5, where the period of the modulation is 200 samples or 50 cycles of the reference signal. It can be seen that after an initial transient the algorithm is able to 'track' changes in the level of the excitation and still provide substantial attenuation.

The properties of the stochastic gradient algorithm which are illustrated in this simulation have all been observed in a practical implementation of the algorit80 a fast microprocessor. This system has been used to drive two loudspeakers in a room so as to minimise the sum of the squares of four microphones.

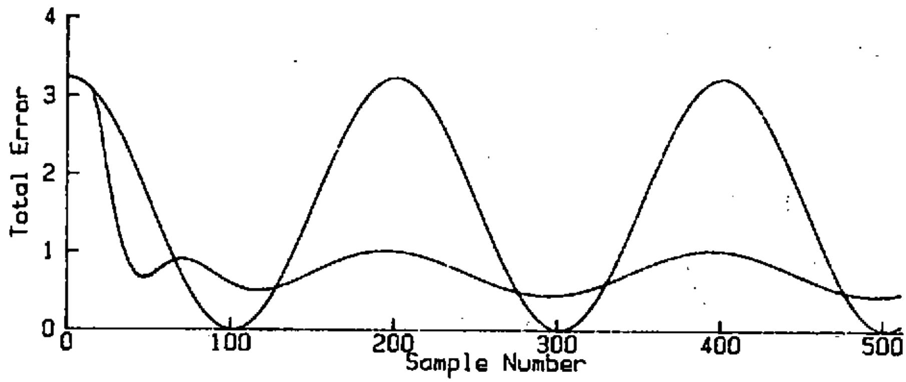

Figure 5. Graphs of the total mean square error against time for the simulation with the primary signals periodically modulated. The graph with the larger amplitude is without active control, and the other with control.

5. CONCLUSIONS

Three possible algorithms for the multichannel active control of periodic sound and vibration have been reviewed. They all rely on a knowledge of the frequency of the primary source and are thus all 'synchronous' with it. They may be distinguished by a number of factors which include speed of convergence, complexity of the algorithm, and thus cost of the hardware, and their ability to react to non-stationary fields. Another important distinction between them is the accuracy with which the characteristics of the system to be controlled must be measured in order to ensure that the control system remains stable. The matrix inversion algorithm is seen to be very sensitive to such inaccuracies, particularly for systems with a large number of channels. At the other extreme, some of the trial and error algorithms make no assumptions at all about the system to be controlled, but pay the price in their relatively slow convergence properties. The stochastic gradient algorithm has been found to require only a rather crude estimate of the characteristics of the system to be controlled in order to rapidly descend down the error surface and achieve a stable convergence.

REFERENCES

1. P.A. Nelson, A.R.D. Curtis and S.J. Elliott 1985 *Proc. IOA.* 7, 45–53. Quadratic optimisation problems in the active control of free and enclosed sound fields.

2. A.J. Bullmore, P.A. Nelson and S.J. Elliott 1985 *Proc. IOA.* 7, 55–63. Active control of harmonic enclosed sound fields of low modal density; a computer simulation.

3. S.J. Elliott and P.A. Nelson 1985 *Proc. Inter-noise 85*, 583-587. The active minimisation of sound fields.

4. W. Johnson 1982 *NASA TP-1996* Self-tuning regulators for multicycle control of helicopter vibration.

5. A.D. White and D.G. Cooper 1984 *Applied Acoustics* 17, 99-109. An adaptive controller for multivariable active control.

6. R.A. Smith and G.B.B. Chaplin 1983 *Proc. Inter-noise 83*, 407–410. A comparison of some Essex algorithms for major industrial applications.

7. S.J. Elliott and P. Darlington 1985 *IEEE transactions in Acoustics, Speech and Signal Processing* ASSP-33(3), 715-717. Adaptive cancellation of periodic, synchronously sampled, interference.

8. B. Widrow *et al* 1975 *Proc. IEEE* 63, 1692-1716. Adaptive noise cancelling: Principles and applications.

9. S.J. Elliott and P.A. Nelson 1985 *Electronics Letters* 21, 979-981. Algorithm for multichannel LMS adaptive filtering.

10. B. Noble 1969 Applied Linear Algebra. Prentice Hall, New Jersey.

ACKNOWLEDGEMENTS

The authors are supported by the Science and Engineering Research Council under the special replacement scheme. We would like to thank the numerous people who have contributed to the work presented in this paper, in particular Mr. A. Curtis (who performed the simulations for Figure 2), Mr. P Darlington and Mr. I. Stothers, who have all provided valuable insight.