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ERROR SURFACES IN ACTIVE NOISE CONTROL

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1. INTRODUCTION

The objective of an active noise control system can normally be expressed as being the minimisation of some well defined error criterion. For example, in systems designed to control low frequency noise in ducts such an error criterion may be the mean square value of the pressure at a single point downstream of the secondary source. In order to actively control the sound in a three dimensional space the criterion which perhaps is the most desirable to minimise is the total acoustic potential energy in the space, although in practice the sum of the squares of the outputs from a finite number of microphones may be the only available error criterion.

The magnitude of this error criterion will depend on the values of all the adjustable parameters in the active control system. There will in general be many such adjustable parameters and the graph of the error as it varies with each of these, the error surface, will thus be multidimensional.

It is found that this error surface has a similar shape for a wide variety of active noise control problems and so similar strategies can be employed for minimising the error in different active noise control systems. It is the purpose of this paper to discuss some of the properties of this error surface in the types of active noise control system applied to various problems.

2. HARMONICALLY EXCITED ENCLOSED SOUND FIELDS

A useful practical error criterion in this case appears to be the sum of the squares of the outputs of a number of pressure microphones distributed in the space [1,2]. Assuming the field is excited at a single, known, frequency complex notation can be used to describe the inphase and quadrature outputs from each of a number (L) of microphones; $V_{out}(\ell)$, and the inphase and quadrature inputs to a number (M) of loudspeakers $V_{in}(m)$.

Defining the vectors,

$$\underline{V_{out}} = [V_{out}(1) \quad V_{out}(2) \quad V_{out}(3) \quad \ldots \quad V_{out}(L)]^T$$

$$\underline{V_{in}} = [V_{in}(1) \quad V_{in}(2) \quad V_{in}(3) \quad \ldots \quad V_{in}(M)]^T$$

which are related via the L x M matrix $\underline{C}$, where each element of $\underline{C}$ is the complex electrical transfer function from each loudspeaker input to each microphone output at the excitation frequency. We further assume that if all of the voltages to the loudspeakers are zero the vector of microphone outputs is equal to $\underline{V_{out}}'$, which is due to the primary noise sources in the enclosure.

We then have only to assume <u>linearity</u> of the acoustic field and the transducers which allows the application of the principle of superposition. This leads to

$$\underline{V_{out}} = \underline{V_{out}}' + \underline{C}\,\underline{V_{in}} \qquad (1)$$

ERROR SURFACES IN ACTIVE NOISE CONTROL

The sum of the squares of the microphone outputs may be expressed as $\underline{V}_{out}^{H}\,\underline{V}_{out}$ (where the superscript H denotes the conjugate of the transpose). This is a real scaler, and is equal to

$$E = \underline{V}_{out}^{H}\,\underline{V}_{out} = \sum_{\ell=1}^{L} |V_{out}(\ell)|^2 =$$

$$\underline{V}_{out}'^{H}\,\underline{V}_{out}' + \underline{V}_{out}'^{H}\,\underline{C}\,\underline{V}_{in} + \underline{V}_{in}^{H}\,\underline{C}^{H}\,\underline{V}_{out}' + \underline{V}_{in}^{H}\,\underline{C}^{H}\,\underline{C}\,\underline{V}_{in} \qquad (2)$$

In which the first and last terms are real and the second and third are complex conjugates. All terms except the first contain the real and imaginary components of the terms of $\underline{V}_{in}$, i.e. the amplitudes of the inphase and quadrature input voltages to each of the loudspeakers. The highest power of any of these terms is the second and so the scaler $\underline{V}_{out}^{H}\,\underline{V}_{out}$ must be a quadratic function of these voltages. If these are taken as the variable parameters in the control system the error surface is thus bowl shaped with a unique global minimum. Note that if the controllable variables of the active system are the amplitude and phase of each of the loudspeaker input voltages, the error surface also monotonically falls to a unique minimum, assuming the phase is constrained to be within some 360 degree range, although it is not quadratic in this case.

If the microphones have equal sensitivities and their output voltages are proportional to the pressures at various points in the field, $\underline{P}$ say, where $\underline{P}'$ are those due to the primary alone, and the volume velocities of the loudspeakers $\underline{q}$ are in proportion to their input voltages, equation (1) is equivalent to

$$\underline{P} = \underline{P}' + \underline{Z}\,\underline{q} \qquad (3)$$

in agreement with [2] where $\underline{Z}$ is now an L x M matrix of transfer impedances. This interpretation is not necessary however, since all acoustic and electro-acoustic effects are accounted for in equation (1), provided they are linear. For example the acoustic loading effects on the loudspeakers, if they have a finite internal impedance, are automatically accounted for in equation (1).

The error surface is only defined for steady state sinusoids at a single frequency, so if any changes are made to the inputs to the loudspeakers, all transient effects must have decayed away before the state of the system can properly be expressed by the equations above and the concept of the quadratic error surface can again be used. Because the shape of the error surface is particularly simple, with a unique minimum, a variety of strategies may be employed to vary the loudspeaker input voltages in such a way as to move down the error surface. An obvious example is the method of steepest decent, although even a simple trial and error algorithm will, eventually, succeed.

3. BROADBAND NOISE PROPAGATING IN DUCTS

Consider a duct system in which plane waves generated by a primary, broadband, noise source are picked up by a "detection" sensor whose output is fed to a secondary electroacoustic source via some electrical filter called the controller. A measure of the attenuation achieved with such a system can be obtained by taking the mean square output of an "error" sensor placed downstream of the secondary. In order to derive an expression for this error criterion as

ERROR SURFACES IN ACTIVE NOISE CONTROL

a function of the coefficients of the filter used as the controller, assume that the primary source is itself electroacoustic [3] and is driven by a voltage V_p. Such a system is illustrated in fig. 1

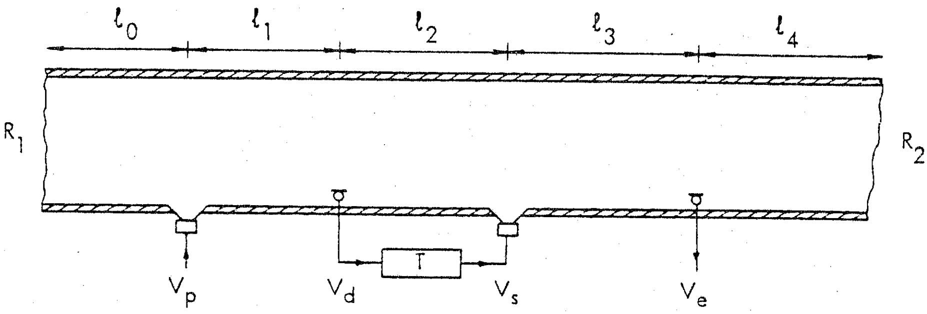

Figure 1. Active noise control system in a duct with an electroacoustic primary source driven by a voltage V_p.

Although the assumption of an electroacoustic primary source is rather artificial it is a useful one for purposes of discussion. If the input voltage to the secondary source is V_s and the output voltages from the detection and error sensors are V_d and V_e respectively, we may define four <u>electrical</u> transfer functions as:

$$A \triangleq \left[\frac{V_e}{V_p}\right]_{V_s = 0} \qquad B \triangleq \left[\frac{V_d}{V_p}\right]_{V_s = 0} \qquad C \triangleq \left[\frac{V_e}{V_s}\right]_{V_p = 0} \qquad F \triangleq \left[\frac{V_d}{V_s}\right]_{V_p = 0}$$

We assume that the V_d and V_e are not corrupted by noise uncorrelated with V_p. If all elements in the system are linear, then, by superposition;

$$V_e = A\,V_p + C\,V_s \quad \text{and} \quad V_d = B\,V_p + F\,V_s \qquad (4,5)$$

Assuming $V_s = T\,V_d$, where T is the transfer function of the controller, the behaviour of the entire system may be described by the equivalent block diagram of fig. 2.

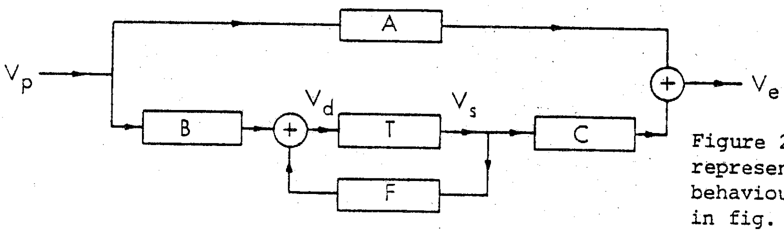

Figure 2. Block diagram representing the behaviour of the system in fig. 1.

The spectrum of the output from the error microphone can thus be written

$$V_e = \left(A + \frac{BCT}{1 - TF}\right) V_p \qquad (6)$$

It should be emphasised that, provided all components of the system are linear, all loading effects are again automatically accounted for in this formulation.

We initially assume that the transfer function of the feedback path, F, is identically zero, and that the controller is implemented as a digital FIR filter so that T is the Fourier transform of its impulse response sequence. In this case the spectrum V_e is linearly proportional to the response T in the equation above. If the Fourier transform of this expression is taken and the linear properties of this transform considered, then it is found that the instantaneous value of the output from the error sensor is directly proportional to each of the values of the impulse response sequence of the controller, in other words directly proportional to the coefficients of the FIR filter. Thus the mean square value of the output of the error microphone must be a quadratic function of these coefficients, the error surface is again bowl shaped and simple gradient decent algorithms can be used to adjust the coefficients to reach the unique global minimum in the error surface.

In practice the transfer function of the feedback path, F, is very rarely zero, as will be discussed below. Assuming the product TF is less than unity however, the expression for V_e, above may be expanded as

$$V_e = \left[A + BCT\ (1 + TF + (TF)^2 + \ldots)\right] V_p \qquad (7)$$

Clearly V_e is no longer a linear function of T, so that the mean square output from the error microphone will be dependent on the filter coefficients raised to powers greater than two. Consequently the error surface will no longer have the simple bowl shape described above.

Measurements have been made of the mean square output from a pressure microphone used as the error sensor in a practical realisation of fig. 1, as the value of one coefficient of a programmable digital FIR filter, used as the controller, was varied. In the first experiment a random noise source was used to drive both the primary source (directly) and the secondary source (via the controller). The detection sensor was not used in this case, and F is zero. The graph of the variation of the mean square error with the first filter coefficient, h_o, which may be considered as a slice of the hyperdimensional error surface, is presented in fig. 3. It is seen to be a good approximation to a quadratic function, giving a minimum error at some well defined coefficient value. This is in contrast however to the case in which the feedback path is made deliberately large by using a pressure microphone as the detection sensor, placed close to the secondary source, whose output drives the controller. The error curve in this case (Fig 4) shows little systematic variation of the error with the first filter coefficient until it is large enough to make the open loup gain (TF) equal to unity at some frequency in which case the system becomes unstable and the error tends to infinity.

ERROR SURFACES IN ACTIVE NOISE CONTROL

In order to assess the likely size of the feedback term F, the acoustics of the wave propagation in the duct can be analysed [4]. An expression for F has been derived in terms of the parameters below;

1) the ratio of the amplitude of the 'downstream' travelling wave produced by the secondary source to its input voltage (H_s),

2) the ratio of the 'upstream' travelling wave produced by the secondary to the 'downstream' travelling wave produced (D_s),

3) the ratio of the output voltage of the detection sensor to the amplitude of the 'downstream' wave incident upon it (H_d),

4) the ratio of the output voltages of the detection sensor for equal amplitude incident 'upstream' and 'downstream' travelling waves (D_d) and

5) the geometry and reflection coefficients of the duct, as defined in fig 1.

It is assumed for purposes of this discussion that the internal acoustic impedance of both sources is high.

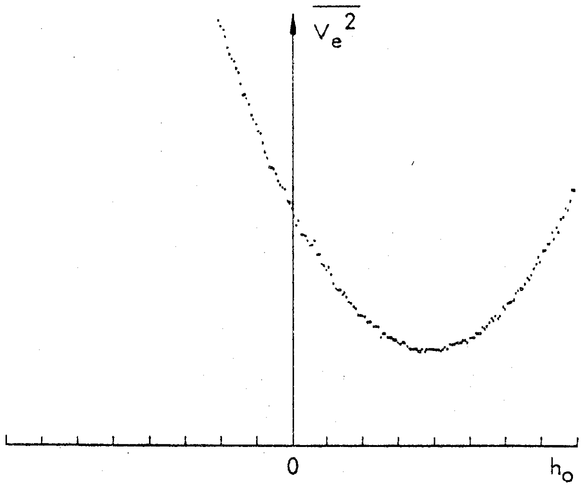

Figure 3. Error curve for noise in duct with FIR controller and no feedback path.

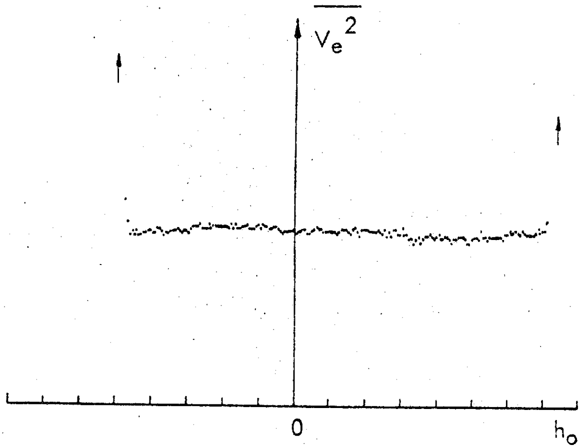

Figure 4. Error curve for noise in duct with FIR controller and appreciable feedback path.

Such an analysis gives:

$$F = H_d H_s \times \Big[D_d D_s e^{-\gamma \ell_2} + D_d R_2 e^{-\gamma(\ell_2 + 2(\ell_3 + \ell_4))} + D_s R_1 e^{-\gamma(\ell_2 + 2(\ell_0 + \ell_1))} + R_1 R_2 e^{-\gamma(2\ell - \ell_2)} \Big] / (1 - R_1 R_2 e^{-2\gamma \ell}) \quad (8)$$

in which $\ell = \ell_o + \ell_1 + \ell_2 + \ell_3 + \ell_4$, the total length of the duct and γ is the complex propagation constant. Individual feedback terms in the square brackets may be identified from fig 1. This expression indicates that F is only zero if either D_d and R_1, or D_s and R_2 are zero, i.e. one transducer must be perfectly directional and one end of the duct perfectly anechoic. Since this is unlikely to be the case in practice, various authors [5,6,7] have suggested that a special form of controller be used in which the normal filter connecting V_d to V_s has an extra electrical feedback filter in parallel, connecting V_s back to V_d. If this latter filter is adjusted to have a transfer function exactly equal to, but out of phase with, the electroacoustic feedback path, F, then the output of this filter will exactly cancel the response due to F and the original filter can be adjusted to minimise the error using simple gradient decent algorithms.

One disadvantage of this approach is that the complexity of the two filters required in the controller will, in general, be much greater than that of an equivalent single filter. The reason for this is apparent if the form of the transfer function for F, quoted above, is compared to the theoretical form for the transfer function of the controller needed to reduce V_e to zero. This can be calculated by setting equation (6) to zero to give an expression for the "ideal" controller, as derived by Ross [3]:

$$T_i = A/(AF - BC) \tag{9}$$

The form of the ideal controller thus depends only on C, F and the ratio A/B Which is equal to $(V_e/V_d)_{V_s=o}$. Each of these quantities can be measured without the need to observe V_p and so T_i can be estimated by purely electrical measurements of V_e, V_d and V_s. Using the analysis of the propagating acoustic waves in the duct discussed above, expressions for A, B and C have been derived [4]. It is assumed that the transfer functions and directivities for the primary source and error sensor. (H_p, D_p, H_e, D_e) are defined by analogy with those of the secondary source and detection sensor (H_s, D_s, H_d, D_d respectively). This gives,

$$A = \left[\frac{V_e}{V_p}\right]_{V_s=o} = \frac{H_e H_p e^{-\gamma(\ell_1+\ell_2+\ell_3)}\left[1 + D_e R_2 e^{-2\gamma\ell_4}\right]\left[1 + D_p R_1 e^{-2\gamma\ell_o}\right]}{\left[1 - R_1 R_2\, e^{-2\gamma\ell}\right]} \tag{10}$$

$$B = \left[\frac{V_d}{V_p}\right]_{V_s=o} = \frac{H_d H_p e^{-\gamma\ell_1}\left[1 + D_d R_2 e^{-2\gamma(\ell_2+\ell_3+\ell_4)}\right]\left[1 + D_p R_1 e^{-2\gamma\ell_o}\right]}{\left[1 - R_1 R_2 e^{-2\gamma\ell}\right]} \tag{11}$$

$$C = \left[\frac{V_e}{V_s}\right]_{V_p=o} = \frac{H_e H_s e^{-\gamma \ell_3}\left[1 + D_e R_2 e^{-2\gamma \ell_4}\right]\left[1 + D_s R_1 e^{-2\gamma(\ell_o+\ell_1+\ell_2)}\right]}{\left[1 - R_1 R_2 e^{-2\gamma \ell}\right]} \qquad (12)$$

which when substituted into the expression for T_i above, gives

$$T_i = e^{-\gamma \ell_2} / \left[H_d H_s (1 - D_d D_s e^{-2\gamma \ell_2})\right] \qquad (13)$$

A result also derived by A. Roure [8].

The form of the ideal controller derived in [9] can be thought of as a special case of equation (13) in which H_d, H_s, D_d and D_s are unity.

It is surprising how simple this expression is considering the complexity of those for A,B,C and F individually. It should also be noted that equation (13) is only dependent on the 'local' properties of the active system between the detection sensor and secondary source, and is independent, for example, of the overall length of the duct or the transfer function or directivity of the error microphone.

The length of the impulse response of T_i will generally be determined by the recursive term in the denominator. At least one of the transducers can generally, be made highly directional so that the product $D_d D_s$ is small and the impulse response of T_i is well contained.

In contrast the length of the impulse response of the filter which matches the transfer function F will generally be determined by the term $(1 - R_1 R_2 e^{-2\gamma \ell})^{-1}$ in the denominator of equation (8). The reflection coefficients of the ends of practical ducts can be close to unity, especially at low frequencies. In this case the impulse response of the filter will take a considerable time to decay and if the filter has an FIR structure, as suggested in [6], it must have a considerable number of coefficients.

4. CONCLUSIONS

It has been shown that the error surface applicable to a practical active noise control system for a harmonic three dimensional sound field will be quadratic in form. The error will thus have a single minimum for a unique set of input voltages to the secondary sources. This form of error surface will only be found in active systems designed to control broad band noise in ducts if no feedback is present between the secondary source and detection sensor. This is unlikely to be the case in practice and one method of overcoming the problem, by electrically cancelling the feedback path, is shown to potentially lead to much higher order filters than would otherwise be needed.

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REFERENCES

[1] P.A. Nelson et al, 'Quadratic optimisation problems in the active control of free and enclosed sound fields'. This conference.

[2] A.J. Bullmore et al, 'Active control of harmonic enclosed sound fields of low modal density: A computer simulation'. This conference.

[3] C.F. Ross, 'An algorithm for designing a broadband active sound control system', J. Sound Vib. 80 pp. 373-380, (1982).

[4] S.J. Elliott and P.A. Nelson, 'Models for describing active noise control in ducts', I.S.V.R. Technical Report no.127, (1984).

[5] G.B.B. Chaplin and R.A. Smith, 'Improvements in and relating to 'active' methods for attenuating compressional waves', U.K. Patent no.1555760, (1979).

[6] L.A. Poole et al, 'The implementation of digital filters using a modified Widrow-Hoff algorithm for the adaptive cancellation of acoustic noise', paper 21.7 Proc. ICASSP, (1984).

[7] J.E. Ffowcs Williams et al, 'Anti-phase noise reduction', Phys. Technol. 16 p.19-31, (1985).

[8] A. Roure, 'Systeme d'absorption acoustique active large bande auto-adaptive', LMA report no.2040, CNRS, Marseille.

[9] Kh. Eghtesadi and M.G. Leventhall, 'Active attenuation of noise - The monopole approach', JASA 71 pp.608-611, (1982).