MULTIPLE POINT LEAST SQUARES EQUALISATION IN A ROOM

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1. INTRODUCTION

In sound reproduction systems an equalisation filter is sometimes used to modify the frequency spectrum of the original source signal, before feeding it to the loudspeaker, in an attempt to compensate for unevenness in the frequency response of the loudspeaker and the listening room. Such equalisation filters can take many forms. One common form is a parallel combination of bandpass filters, the outputs of which have a manually adjustable gain and are added together to produce the output. Such filters can compensate for gross deficiencies in the frequency response of the sound reproduction chain, which includes the electroacoustic response of the loudspeaker and the acoustic response of the listening room. The transient properties of narrow bandwidth filters are, however, notoriously bad and this can lead to a degradation in the impulse response of the equalized reproduction chain.

Another approach is to design an equalisation filter by making the impulse response of the equalized sound reproduction chain as close as possible to that desired, a net impulse response of a delta function for example would mean that the sound reproduction chain had been perfectly equalised. It is, however, not possible in general to achieve such perfect inversion of the equalisation chain, since the acoustic path usually has delays and other non-minimum phase behaviour associated with it [1]. The ability of the equalisation filter to invert the response of the reproduction chain is much improved if the equalised output is compared with a delayed version of the original signal. Such a "modelling delay" is illustrated in the block diagram Figure 1. In Section 2 we will formalise the design of such single channel systems, and extend the theory to the case of multiple microphones in Section 3.

2. SINGLE CHANNEL EQUALISATION

We assume that the equalisation filter to be designed is digital and has an all zero (FIR) structure with coefficients h_0 to h_{I-1}. We also assume that the response of the unequalised reproduction chain is modelled by a digital FIR filter, with coefficients c_0 to c_{J-1}. If the sampled source signal is $x(n)$, the sampled signal fed to the loudspeaker is $y(n)$ and sampled output from the microphone is $\hat{d}(n)$, then:

$$y(n) = \sum_{i=0}^{I-1} h_i x(n-i), \qquad \hat{d}(n) = \sum_{j=0}^{J-1} c_j y(n-j)$$

so

$$\hat{d}(n) = \sum_{j=0}^{J-1} c_j \sum_{i=0}^{I-1} h_i x(n-i-j) = \sum_{i=0}^{I-1} h_i r(n-i) \tag{1}$$

where

$$r(n) = \sum_{j=0}^{J-1} c_j x(n-j) \tag{2}$$

The summation of equation (1) can be written in vector form as

$$\hat{d}(n) = \mathbf{r}^T(n)\mathbf{h} \tag{3}$$

where

$$\mathbf{r}^T(n) = [r(n),\ r(n-1),\ \ldots\ldots\ r(n-I+1)]$$
$$\mathbf{h}^T = [h_0,\ h_1\ \ldots\ldots\ h_{I-1}]$$

The most usual method of defining how $\hat{d}(n)$ is the 'best' approximation to $d(n)$ is to minimise the mean square difference between these two signals, i.e., to adjust the coefficients of the equalisation filter to minimise the "performance index":

$$J = E\{e^2(n)\} \tag{4}$$

where $e(n) = d(n) - \hat{d}(n)$, and E represents the expectation operator. It should be noted, however, that this performance index is not the only criterion which can be used to define the difference between the desired and equalised signals [2]. One advantage, however, of the mean square performance index, J, is that it is a quadratic function of each of the coefficients in the equalisation filter:

$$J = E\{d^2(n)\} + 2\mathbf{h}^T E\{\mathbf{r}(n)d(n)\} + \mathbf{h}^T E\{\mathbf{r}(n)\mathbf{r}^T(n)\}\mathbf{h} \tag{5}$$

which has a globally minimum value for some unique set of filter coefficients (since the matrix $E\{\mathbf{r}(n)\mathbf{r}^T(n)\}$ is positive definite). Using fairly standard optimisation methods this optimum set of filter coefficients can be shown to be given by

$$\mathbf{h}_{opt} = -[E\{\mathbf{r}(n)\mathbf{r}^T(n)\}]^{-1}E\{\mathbf{r}(n)d(n)\} \tag{6}$$

In practice, adaptive algorithms can be used to automatically adjust the coefficients of $\mathbf{h}$ to be a close approximation to $\mathbf{h}_{opt}$ [6,8,9]. We are concerned here with the physical consequences of designing an equalisation filter according to this criterion.

Previous studies of such equalising filters [3,4] have demonstrated that it is possible to obtain a flat response at the equalisation microphone position, but that the equalised response away from this point can be worse than the unequalised response. In order to illustrate this point, and to introduce the acoustic model which will be used in later sections, we consider using the equalisation strategy above in an enclosure with dimensions and acoustic damping typical of a car interior.

The acoustic response from an acoustic source in one position in the enclosure to a microphone in another, was modelled as the sum of the contributions of a finite number of acoustic modes in the enclosure [5]. The size of the enclosure was 1.9 m long by 1.1 m high by 1.0 m wide, and all modes with a natural frequency below 1200 Hz were included in the modal summation (about 500 modes), even though the response was only calculated for frequencies up to 512 Hz, the sample rate being 1024 Hz. The damping ratio of all the modes was set to 0.1. This purely acoustic response was then convolved with a filter which had a zero at d.c., and a zero at half the sample rate, which represented the high pass filtering action of a loudspeaker and the low pass filtering action of the anti-aliasing and reconstruction filters which would be used in any practical system. In the coordinate system used for the computer simulation the origin was in the front bottom right hand corner of the enclosure (as seen from the interior) and the coordinates (x1, x2, x3) represent the distance back, across and up, respectively, from this origin. The loudspeaker was represented by a point acoustic source at (0.0, 0.9, 0.7) which is approximately the position of a front dashboard loudspeaker on the left hand side of a real car. The frequency response was calculated from this loudspeaker to a microphone at (0.1, 0.1, 0.9) which corresponds approximately to the position of the driver's right hand ear in a real car.

A 50 coefficient FIR filter has been used to equalise this frequency response using a modelling delay of 15 samples. This filter was adapted to minimise the mean square modelling error, using an algorithm discussed in [6,9]. The frequency response and impulse response of the equalisation filter after convergence are shown in Figure 2. The results are illustrated in Figure 3 which shows the original (solid) and equalised (dashed) response at the equalisation microphone (microphone 0), and also the original response and the effect of this equalisation filter at three other microphones: microphone 1, at position (0.9,0.9,0.9), corresponding approximately to the front passenger's left hand ear; microphone 2, at (1.9,0.1,0.9), corresponding approximately to the right hand rear passenger's position; and microphone 3, at (1.9,0.9,0.9), corresponding approximately to the left hand rear passenger's position. It is clear that although the frequency response has been significantly improved at microphone 0, and somewhat improved at microphone 1, this equalistion filter makes the responses more peaky at the rear microphone positions. This is lagely due to the presence of the first longitudinal acoustic mode in the enclosure, with a natural frequency of about 90 Hz. This has little effect at microphones 0 and 1, since they are close to the nodal plane of this mode, and these microphones have a relatively low response at about 90 Hz, which is boosted by the equalisation filter. The microphones in the rear of the enclosure (2 and 3) pick up this mode strongly, however, even before boosting by the equalisation filter, so the effect of the equalisation filter is to produce a peak of some 15 dB above the average response at 90 Hz.

3. MULTIPLE POINT EQUALISATION

The failure of single point equalisation schemes to control the response at points away from the equalisation microphone within the enclosure suggests that the problem of equalisation at a number of points might be cast as a more general least squares problem. This is illustrated in Figure 4, in which the output of a single equalisation filter is coupled to multiple microphones via multiple room impulse responses, and each microphone output is subtracted from a desired signal, formed by passing the source signal through an individual modelling delay (of Δ_ℓ samples for the ℓ'th microphone), to obtain an error signal at each microphone.

The vector of output signals can now be represented [6,7] as:

$$\mathbf{e}(n) = \mathbf{d}(n) + \mathbf{R}(n)\mathbf{h} \tag{7}$$

where

$$\mathbf{e}^T(n) = [e_1(n), e_2(n) \quad \ldots \quad e_L(n)]$$
$$\mathbf{d}^T(n) = [d_1(n), d_2(n) \quad \ldots \quad d_L(n)]$$
$$\mathbf{R}^T(n) = [\mathbf{r}_1(n), \mathbf{r}_2(n) \quad \ldots \quad \mathbf{r}_L(n)]$$

and $\mathbf{r}_\ell(n)$ and $\mathbf{h}$ are defined similarly to the vectors in the previous section. The object of the equalising filter is now to minimise the sum of the squares of each of the errors, and this new performance index may be written as:

$$J = E\{\mathbf{e}^T(n)\mathbf{e}(n)\},$$

MULTIPLE POINT LEAST SQUARES EQUALISATION IN A ROOM

so that

$$J = E\{\mathbf{d}^T(n)\mathbf{d}(n)\} + 2\mathbf{h}^T E\{\mathbf{R}^T(n)\mathbf{d}(n)\} + \mathbf{h}^T E\{\mathbf{R}^T(n)\mathbf{R}(n)\}\mathbf{h} \qquad (8)$$

This performance index again has a globally minimum value for a unique set of equalisation filter coefficients given by

$$\mathbf{h}_{opt} = -[E\{\mathbf{R}^T(n)\mathbf{R}(n)\}]^{-1} E\{\mathbf{R}^T(n)\mathbf{d}(n)\} \qquad (9)$$

An adaptive algorithm has been presented [6,9] for automatically adjusting the coefficients of $\mathbf{h}$ to be a close approximation to $\mathbf{h}_{opt}$, and this has been used to obtain an equalising filter for the enclosure described in the last section. This equalising filter, however, now attempts to do the best job of equalising at all four microphone positions by minimising the sum of the squares of the differences between the microphone outputs and delayed source signals.

The frequency response and impulse response of this new equalisation filter are shown in Figure 5, and Figure 6 shows the equalised response at all microphone positions, compared to the original responses. It is clear that the peaks which are common to all four microphone responses, for example that at about 200 Hz, have been largely removed. However, the equalising filter has to cope with conflicting requirements at about 90 Hz: of increasing the response in the front of the enclosure and of suppressing the response in the rear. In fact the equalisation filter does suppress the peak in the rear at the expense of creating a dip in the front at this frequency, since this strategy generates a smaller total residual error than boosting the response in the front and having the response in the back rise even further. Apart from the dips in the equalised responses at about 90 Hz in the front, and at about 180 Hz at microphone 2, the equalisation filter can be seen to be doing a reasonable job of equalisation at all points. The variation in the frequency response function from 2 Hz to 500 Hz, averaged across microphones, is about 15 dB when using this equalisation filter, compared to the original average variation in the frequency response function of some 28 dB over this frequency range.

The modelling delays used to generate the desired signals, $d(n)$, at each microphone for these results were chosen for microphones 0, 1, 2 and 3 to be 15, 14, 18 and 17 samples respectively. It was found that if all the modelling delays were set to be equal, a significantly poorer equalised response was obtained overall. It is interesting to note that the differences in the modelling delays used above are approximately equal to the differences in the propagation times of a direct acoustic wave from the loudspeaker to each of the microphones. This suggests that the equalisation filter can equalise the response at each microphone best by simulating a plane propagating wave in the enclosure.

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ACKNOWLEDGEMENTS

This work was supported in part by the National Aeronautics and Space Administration under NASA Contract No. NAS1-18107 while one of the authors (S.J. Elliott) was visiting the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665.

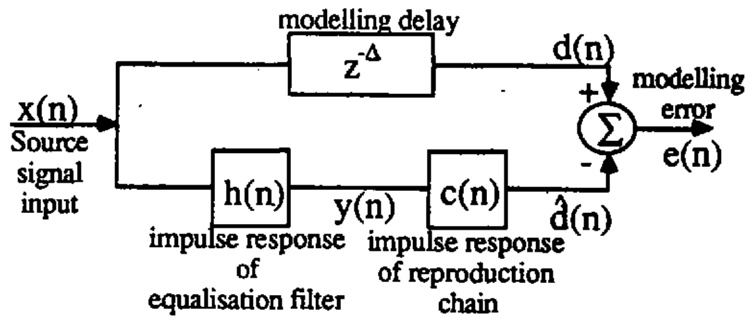
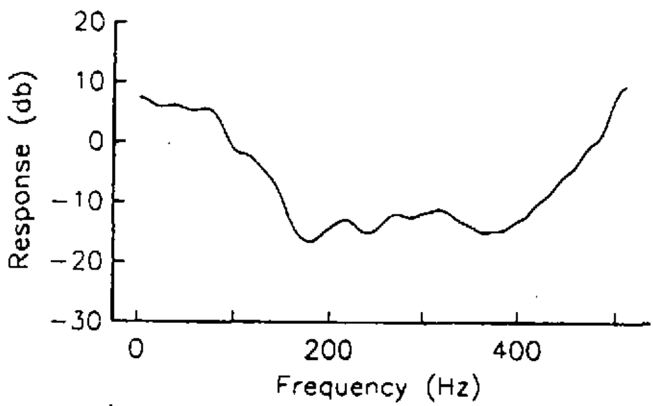


Figure 1: Block diagram of single point equalisation problem with sampled signals



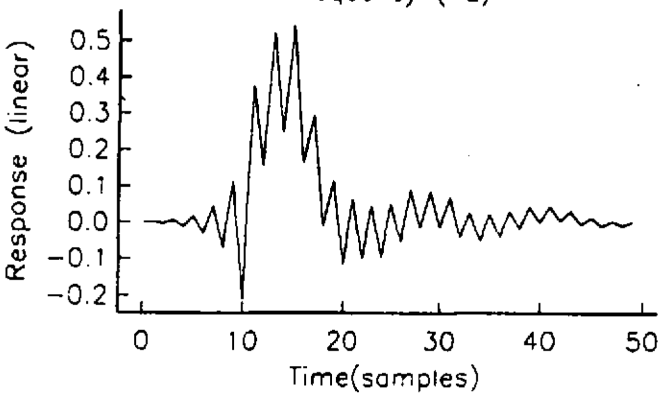


Figure 2:
The frequency and impulse responses of the filter which equalises the enclosure response of Figure 3 with a modelling delay of 15 samples.

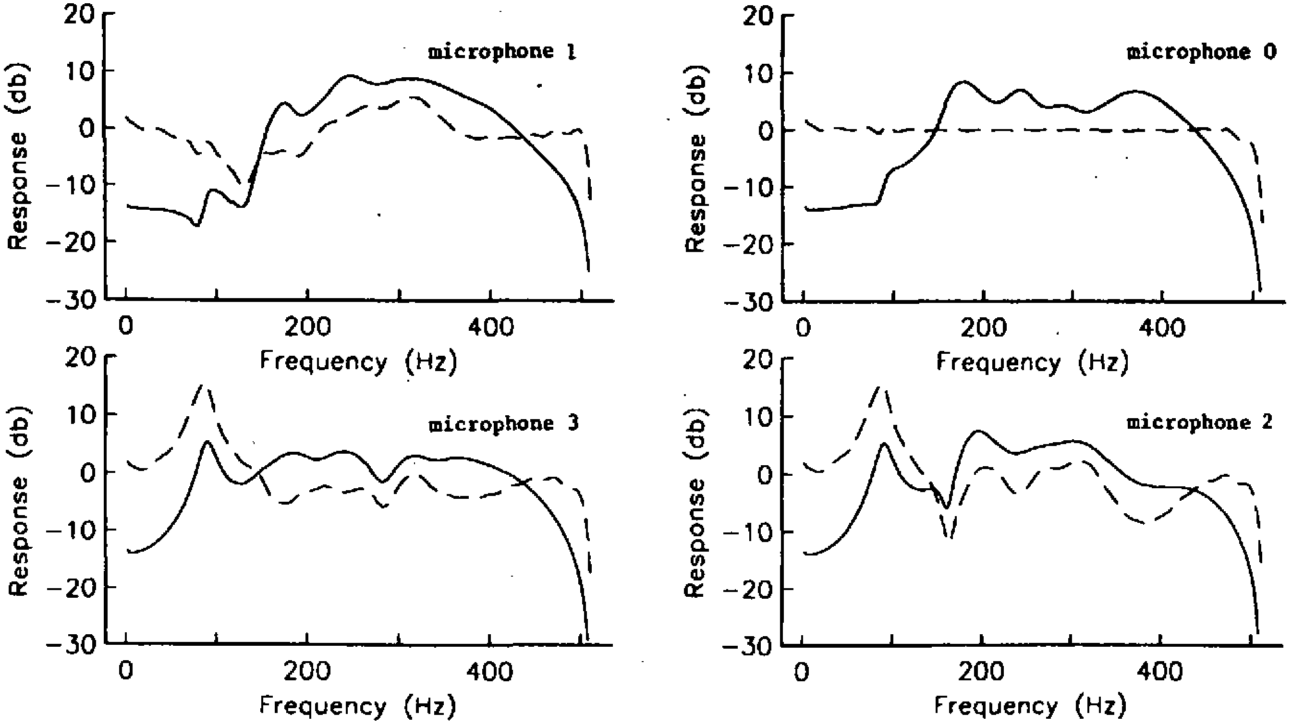


Figure 3: The original frequency response from the loudspeaker to four microphone position in the enclosure (solid line) and the response after the introduction of an equalisation filter designed to equalise the response at microphone 0 (dashed line).

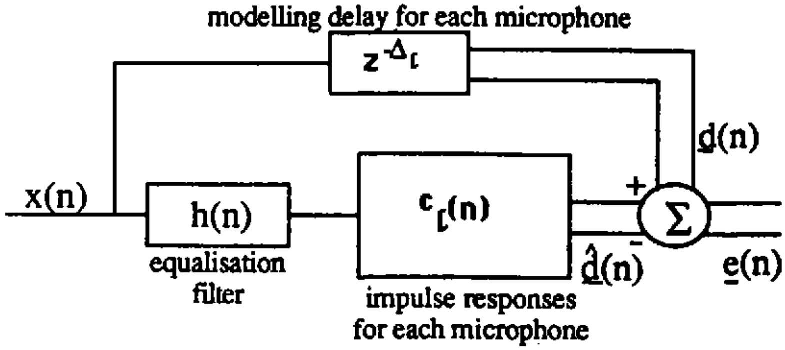


Figure 4: Block diagram of the multiple point equalisation problem with sampled signals.

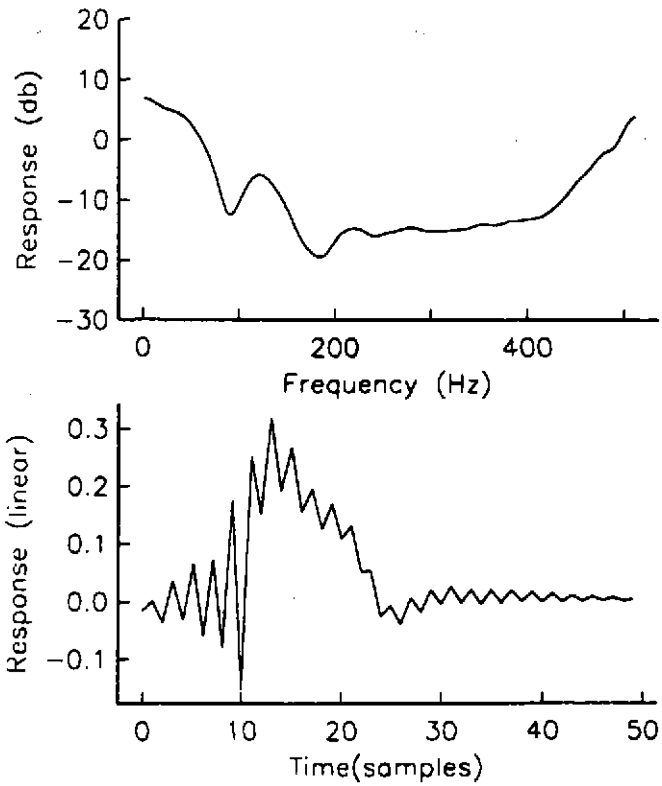


Figure 5:

The frequency and impulse response of the equalisation filter which minimises the sum of the mean squared modelling error at all four microphones.

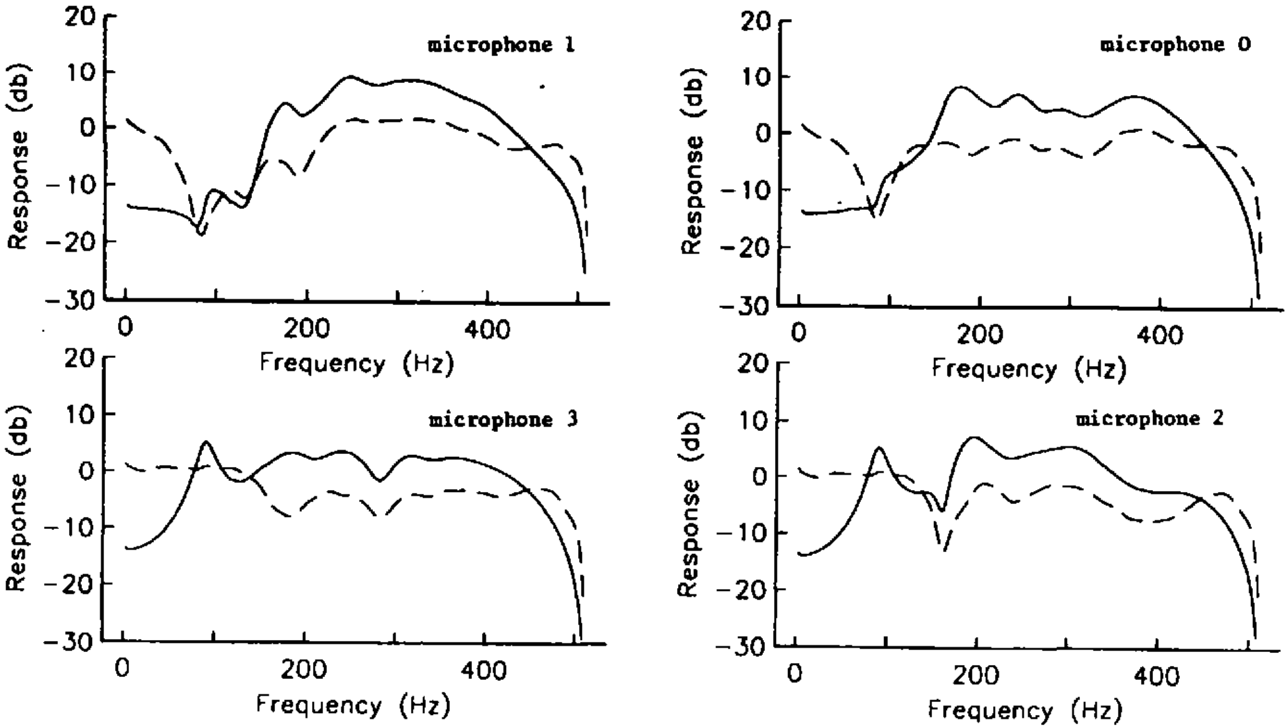


Figure 6 : The original response from the loudspeaker to four microphone positions in the enclosure (solid line) and the response after the introduction of an equalisation filter which minimises the mean square modelling error at all four microphones (dashed line).