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STABILITY OF ADAPTIVE ACTIVE NOISE CONTROL SYSTEMS

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1. INTRODUCTION

An adaptive active noise control system may go unstable in any of several ways. If the electrical controller has a finite impulse response and there is no acoustic feedback path the only type of instability that can arise is that in which the adaptive algorithm itself becomes unstable. If, however, the electrical controller has an infinite impulse response (i.e. it incorporates an electrical feedback path) then it is also possible for instability to arise as a result of the parameters of the controller being changed by the adaptive algorithm to values that would, were the system time-invariant, be unstable in the classic "howling" fashion. A potential instability of this type results in a complex interaction between the rate of growth of the output of the controller due to the "classic" instability and the efforts of the adaptive algorithm to reduce this output and return the system to a stable state. Finally, whether the electrical controller is finite- or infinite-impulse-response (FIR or IIR), there is a similar potential for instability if there is acoustic feedback present in the controller structure. In this case the acoustic conditions play a major part in defining the stability properties of the controller.

The purpose of this paper is to demonstrate some of the effects that the presence of feedback has upon the stability region of an active noise control (ANC) system. These effects have important implications for the convergence performance of adaptive ANC controllers. The particular case that is concentrated on is that of an IIR controller in a configuration where there is acoustic feedback from secondary source to detector, so the effects that are seen result from the interplay of two distinct feedback paths. It is shown how changes in the acoustic feedback path, such as an increase in reverberation, alter the environment in which the adaptive algorithm is operating, even when the interplay between the acoustic feedforward and feedback paths is such that the parameters of the ideal controller remain unchanged.

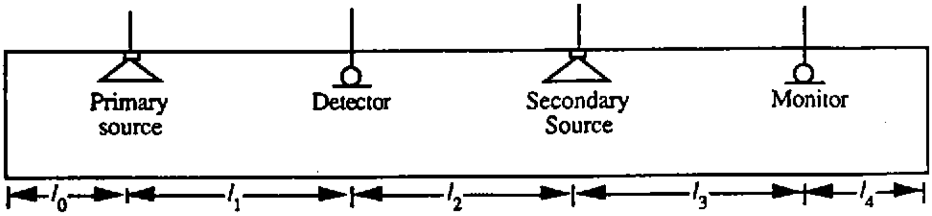

Figure 1. Schematic diagram of the acoustic and electroacoustic parts of a single channel active noise control system with acoustic feedback between the secondary source and the detector

2. THE SINGLE CHANNEL IIR ACTIVE NOISE CONTROL SYSTEM

Figure 1 shows a schematic diagram of the acoustic and electroacoustic parts of a single channel active noise control system with acoustic feedback between the secondary source and the detector. A block diagram representation of the complete system is shown in Figure 2. H_{03} represents the transfer function from the input of the primary source to the output of the monitor, H_{01} from the input of the primary source to the input of the digital controller, A and B the transfer functions of the feedforward and feedback parts of the IIR controller, and H_{23} the transfer function from the output of the controller to the output of the monitor. H_{21} represents the transfer function from the output of the digital controller back to its input via the (in general reverberant) acoustic path between secondary source and detector.

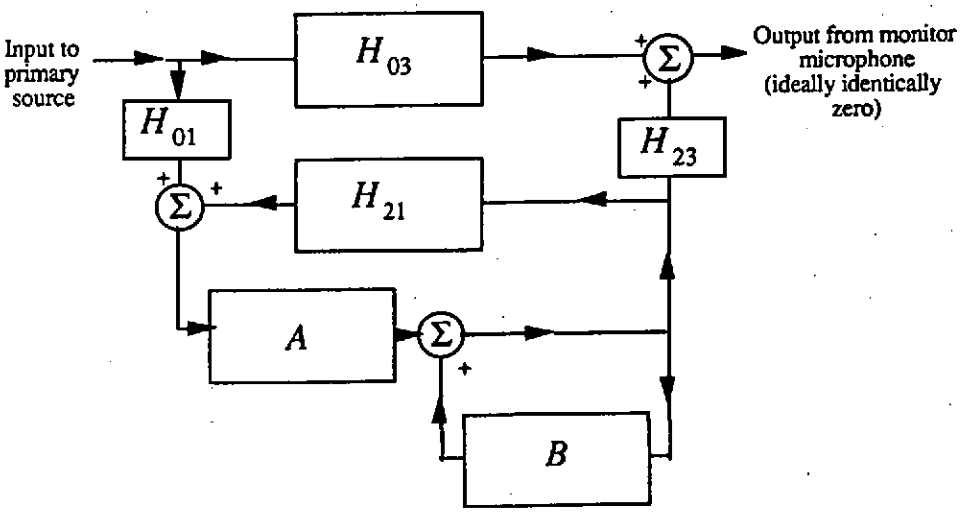

Figure 2. Block diagram representation of a single channel IIR active noise control system in a situation where there is acoustic feedback from the secondary source to the detector

The overall transfer function from primary source input to monitor output of this entire system, $H_{overall}$, is given by

$$H_{overall} = H_{03} + H_{01} \frac{A}{1 - B - H_{21} A} H_{23} \quad (1)$$

The rest of this paper is devoted to studying the consequences of this equation for the simplest possible non-trivial example of such a system. By taking a very simple system the mathematics of the transfer functions remains tractable (though perhaps surprisingly complicated in view of the apparent simplicity of the system studied).

STABILITY OF ADAPTIVE ACTIVE NOISE CONTROL SYSTEMS

3. ANALYSIS OF A SIMPLE REVERBERANT SYSTEM

3.1 Description of the transfer functions

The system discussed in detail here is that in which the lengths l_0, l_1, l_2, l_3, l_4 are equivalent to 0, 0, 1, 0, and 0 clock cycles of the digital controller. The transducers are all assumed to have ideal responses (i.e. their transfer functions are assumed equal to 1) and the sound propagation in the acoustic system is assumed to be perfectly one-dimensional (so all sound propagating within the system is taking place in left-to-right and right-to-left directions only). Given these assumptions, (z-) transfer functions corresponding to H_{03}, H_{01}, H_{21}, and H_{21} can be substituted into equation (1) to give

$$H_{\text{overall}}(z) = \frac{-(r_1+1)(r_2+1)(Az^{-2}-(1-B)z^{-1}-A)}{(1-B)r_1r_2z^{-2}+A(1+r_1)(1+r_2)z^{-1}-(1-B)} \qquad (2)$$

where r_1 and r_2 are the reflection coefficients at the left and right hand ends of the acoustic enclosure and z^{-1} is the unit delay operator.

For the system described the ideal values for A and B are $-z^{-1}$ and z^{-2} respectively, i.e. for perfect cancellation of the primary source at the monitor position all the coefficients of A should be zero except a_1 and all the coefficients of B should be zero except b_2, and these two coefficients should have values of -1 and $+1$ respectively. Note that this result does not depend on the reflections in the acoustic system; the transfer function for perfect cancellation in the one-dimensional system considered here is the same whether or not the conditions are anechoic. Hence it is of particular interest to look at how H_{overall} behaves as a function of a_1, b_2, r_1 and r_2. Substituting $A = a_1 z^{-1}$ and $B = b_2 z^{-2}$ into (2) and taking the denominator gives

$$\text{denominator}(H_{\text{overall}}(z)) = r_1 r_2 b_2 z^{-4} - (a_1((r_1+1)(r_2+1)+r_1 r_2 + b_2) z^{-2} + 1\,. \qquad (3)$$

3.2 Stability boundaries of the system

The stability of the system depends on whether or not all the poles of the overall transfer function lie inside the unit circle. Equation (3) is a quadratic in z^{-2} so we can find the stability boundary in an analogous way to the well-known stability triangle for second order systems.

The conditions that must be satisfied for a system of the form $\dfrac{1}{p z^{-2} + q z^{-1} + 1}$ to be stable are: $p < 1$, $p > (1-q)$, and $p > -(1-q)$. Substituting from (3) into these inequalities gives the following set of conditions for the poles of z^2 (and hence of z) to lie within the unit circle, and thus for the active noise control system to be stable:

$$b_2 < \frac{1}{r_1 r_2}, \quad b_2 > 1 + \frac{(r_1+1)(r_2+1)a_1}{1 - r_1 r_2}, \quad \text{and } b_2 > -1 - \frac{(r_1+1)(r_2+1)a_1}{1 + r_1 r_2}\,. \qquad (4)$$

The region in the a_1-b_2 plane delineated by these inequalities is shown in Figure 3 for three pairs of values of r_1 and r_2, namely r_1 or $r_2 = 0$, $r_1 = r_2 = 0.5$, and $r_1 = r_2 \to \infty$.

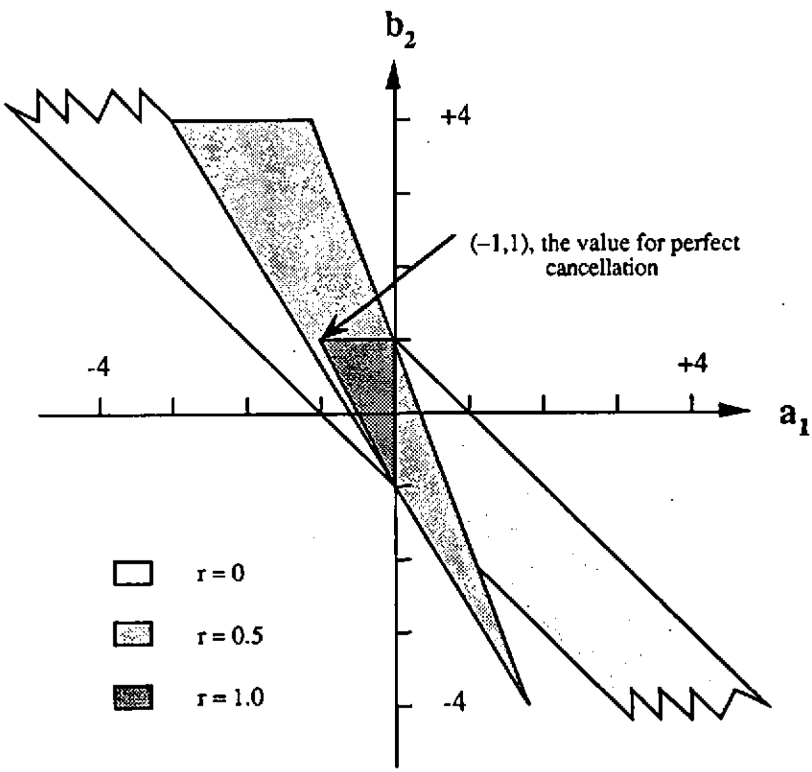

Figure 3. Diagram showing the stable region in the a_1-b_2 plane for a two-coefficient IIR filter controlling the noise in a reverberant one-dimensional system with length equivalent to one clock period of the digital controller. The three shaded regions correspond to a completely anechoic system, a totally reverberant system, and a system in which the reflection coefficients at the ends of the system are both equal to 0.5.

4. CONCLUSIONS

If there is no reflection at all from one of the ends of the acoustic system (i.e. r_1 or $r_2 = 0$) then the set of inequalities (4) reduce to $b_2 < \infty$, $b_2 > 1 + a_1$, and $b_2 > -1 - a_1$. The region that satisfies these constraints is an infinite length strip in the a_1-b_2 plane running diagonally from top left to bottom right and with the origin in its centre. The ideal values for a_1 and b_2, −1 and +1, lie centrally within this strip and any reasonable path from the origin to this point, such as might be followed by the coefficients of an adaptive IIR controller initialised with the coefficient values (0,0) and adapted by a gradient descent method , will lie entirely within the stable region.

The other limiting case that it is particularly instructive to look at is when both the ends are perfectly reflective, i.e. $r_1 = r_2 = 1$. The inequalities then become: $b_2 < 1$, $a_1 < 0$, and

$b_2 > -1 - 2a_1$, so the stable region has shrunk from an infinite strip to a triangle with the ideal values for a_1 and b_2 lying at its upper left vertex and the origin at the centre of its right-hand side. Hence particularly near the start and the end of adaption the coefficient trajectory will necessarily lie very close to the stability boundary and the random element inherent in any coefficient update scheme may push it into the unstable region. Any such excursion, however short, will have particularly deleterious effects on the behaviour of the the system if it happens when the filter has already adapted a significant way towards its final value as it will cause a rapid increase in error signal from the monitor which will result in a large contribution to the coefficient updates. The coefficient values will therefore suddenly be altered in an unpredictable way, but in a way that is likely to take them further away from the aiming point, and this will greatly increase the time required to achieve a steady state condition near the optimum. This type of behaviour has been seen in simulations and to prevent it the convergence coefficient has to be so greatly reduced that the convergence becomes impractically slow.

Between the two extreme cases of one end anechoic and of both ends perfectly reflective the stable region contracts rapidly as the reflection coefficients move away from zero so that even with them equal to only 0.5 the left-hand boundary is not very far from that for the limiting case of perfect reflection. As the gradient of the error surface at the origin is directed along the negative a_1-axis any gradient-descent based adaptive algorithm will cause the coefficients to head straight for this boundary, with the potential to overshoot unless the convergence coefficient is kept very small.

5. SUMMARY

It has been shown that the size and shape of the region in coefficient space within which a particular simple configuration of active noise control scheme using an IIR filter depends strongly on the reverberation properties of the acoustic system in which the controller is working even though the coefficient values that are required for perfect control are independent of the reverberation. The greater the amount of reverberation in the acoustic system, the greater is the probability of an adaptive control system becoming unstable at some time during adaption of the coefficients. The work reported here demonstrates analytically for a special case the reason for the often-observed phenomenon that the greater the reverberation within an acoustic system, the more likely is an active control system to go unstable, even when the reverberation would not be expected to alter the transfer function that is required for perfect control.