EFFICIENT INTEGRAL EQUATION METHODS FOR THIN SCREEN NOISE BARRIERS ON ABSORBING GROUND

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1. INTRODUCTION

Boundary integral equation methods are now fairly well known as a tool for calculating outdoor sound propagation over noise barriers. Their advantage is in flexibility, in that barriers of arbitrary cross-section and surface acoustics properties can be accurately represented. Their disadvantage is computational expense which can be considerable for large barriers and high frequency calculations, and very considerable if a full three-dimensional calculation is made.

In this paper, two types of methods are presented to reduce the computational expense. The first one consists of solving the integral equation by a collocation method and taking advantage of the particular shape of the matrix. The second consists of solving the integral equation by using a Kirchhoff approximation and a least-square minimisation method.

These methods are both applied to the case of a thin screen situated on an absorbing, flat ground. For simplicity, only the two-dimensional problem is considered.

2. SOLUTION BY A COLLOCATION METHOD

The geometry of the problem is shown in Figure 1. A thin screen of height H is located on a plane characterised by a specific surface admittance β_c. It is assumed that the barrier surface is also locally reacting, $\beta^{\pm}(\tilde{r}_s)$ denotes the specific admittance at $\tilde{r}_s=(x_s,y_s)$ on each side $\Gamma^{\pm}$ of Γ.

Let $\tilde{r}_0=(x_0,y_0)$ denote the source position and $p(\tilde{r})$, the acoustic pressure at $\tilde{r}=(x,y)$. Let $G(\tilde{r},\tilde{r}_0)$ be the solution of the problem in the absence of the screen ; G denotes the acoustic pressure emitted above the plane of admittance β_c with no barrier present. Then, the pressure $p(\tilde{r})$ can be written as [1] :

$$p(\tilde{r})=G(\tilde{r}_0,\tilde{r})-\int_{\Gamma}\left\{\frac{\partial G}{\partial x_s}(\tilde{r}_s,\tilde{r})\ (p^+-p^-)(\tilde{r}_s)+\ ikG(\tilde{r}_s,\tilde{r})\ [\beta^+p^++\beta^-p^-](\tilde{r}_s)\right\}dy_s \quad (1)$$

where $\tilde{r}_s$ lies on Γ (i.e. $x_s=0$), k is the wavenumber, $p^{\pm}$ denotes the value of p on each side $\Gamma^{\pm}$.

Equation (1) expresses p at an arbitrary point in the region of propagation and can be used to compute the pressure throughout the region once $p^{\pm}$ are known. To determine $p^{\pm}$, a pair of coupled integral equations is obtained from (1) by letting $\tilde{r}$ approach Γ and using the boundary conditions and the properties of the layer potentials [2,3] on Γ. For $\tilde{r}$ on Γ,

$$\frac{(p^+ + p^-)(\tilde{r})}{2} = G(\tilde{r},\tilde{r}_0) - ik \int_\Gamma G(\tilde{r}_s,\tilde{r})\, [\beta^+ p^+ + \beta^- p^-](\tilde{r}_s)\, dy_s \qquad (2)$$

and :

$$-\frac{ik}{2} [\beta^+ p^+ - \beta^- p^-](\tilde{r}) = \frac{\partial G}{\partial x}(\tilde{r},\tilde{r}_0) - \lim_{x'\to 0} \int_\Gamma \frac{\partial^2 G(\tilde{r}',\tilde{r})}{\partial x\, \partial x'} [p^+ - p^-](\tilde{r}_s)\, dy_s \qquad (3)$$

where $\tilde{r}'=(x',y_s)$.

Equations (2) and (3) are now solved by a simple boundary element method. For brevity, we consider just the simpler case when the admittance is the same on each side of the barrier, i.e. $\beta^+=\beta^-=\beta$. Equations (2) and (3) are then decoupled. The unknowns are $[p] = (p^+-p^-)$ and $\bar{p} = (p^++p^-)/2$.
To obtain the solution, Γ is divided into N elements Γ_j of length h=H/N, i.e :

$\Gamma_j = \{(0,y_s) : (j-1)h \leqslant y_s \leqslant jh\}$, and $\tilde{r}_j=(0,y_j)$ is the midpoint of Γ_j.
A simple collocation method is applied, assuming that β and $p^{\pm}$ are approximately constant on each element and setting $\tilde{r}=\tilde{r}_j$, for j=1(1)N in equations (2) and (3). The unknowns $\bar{p}(\tilde{r}_j)$ and $[p](\tilde{r}_j)$ are the solutions of two systems of N equations which can be written :

$$\bar{p}(\tilde{r}_j) = G(\tilde{r}_j,\tilde{r}_0) - 2\, ik \sum_{m=1}^{N} s_{jm}\ \beta(\tilde{r}_m)\, \bar{p}(\tilde{r}_m) \qquad (4)$$

and

$$-\frac{ik}{2}\beta(\tilde{r}_j)[p](\tilde{r}_j) = \frac{\partial G(\tilde{r}_j,\tilde{r}_0)}{\partial x} - \sum_{m=1}^{N} t_{jm}\, [p](\tilde{r}_m) \qquad (5)$$

where :

$$s_{jm} := \int_{y_m-h/2}^{y_m+h/2} G(\tilde{r}_j,\tilde{r}_m)\, dy_m \quad (6) \text{ and } \quad t_{jm} := \lim_{x'\to 0} \int_{y_m-h/2}^{y_m+h/2} \frac{\partial^2 G(\tilde{r}',\tilde{r}_j)}{\partial x \partial x'}\, dy_m \qquad (7)$$

The computation of G and its derivatives is discussed in [4].

Once (4) and (5) have been solved, p elsewhere in the region of propagation can be calculated by approximating (1) by the composite midpoint rule to give:

$$p(\tilde{r}) \cong G((\tilde{r}_0,\tilde{r}) - \sum_{j=1}^{N} \left\{ \frac{\partial G(\tilde{r}_j,\tilde{r})}{\partial x_a} [p](\tilde{r}_j) + 2\ ik\ G(\tilde{r}_j,\tilde{r})\ \beta(\tilde{r}_j)\ \bar{p}(\tilde{r}_j) \right\} \qquad (8)$$

The calculations of the integrals s_{jm} and t_{jm} are greatly simplified by noting than the Green's function G can be written as the sum of a direct and reflected wave , i.e. :

$G(\tilde{r},\tilde{r}_0) = G_{dir}(\tilde{r},\tilde{r}_0) + G_{ref}(\tilde{r},\tilde{r}_0)$ where $G_{dir}(\tilde{r},\tilde{r}_0) = -i\ H_0^{(1)}(k|\tilde{r}-\tilde{r}_0|)/4$

and that the value of $G_{ref}(\tilde{r},\tilde{r}_0)$ depends only on $(x-x_0)$ and $(y+y_0)$.

Writing $s_{jm} = s^d_{jm} + s^r_{jm}$, where s^d_{jm} and s^r_{jm} are defined by (6) with G replaced by G_{dir} and G_{ref} respectively, and using the same notations for t_{jm}, it follows that :

$$s^d_{j+1,m+1} = s^d_{jm} \quad \text{and} \quad t^d_{j+1,m+1} = t^d_{jm} \quad \text{for } j \text{ and } m = 1(1)N-1 \qquad (9)$$

and : $s^r_{j+1,m-1} = s^r_{jm}$ and $t^r_{j+1,m-1} = t^r_{jm}$ for $j=1(1)N-1$ and $m=2(1)N-1$. (10)

Then $S^d \equiv [s^d_{jm}]$ and $T^d \equiv [t^d_{jm}]$ are Toeplitz matrices and $S^r \equiv [s^r_{jm}]$ and $T^r \equiv [t^r_{jm}]$ are Hankel matrices. The whole matrices can then be obtained once a single row and column of each has been explicitly calculated.

The usual main costs of the boundary element method are a cost proportional to N^2 in evaluating the coefficient matrices for the linear equations (4) and (5) - this is reduced to a cost proportional to N by the use of equations (9) and (10) - and a cost proportional to N^3 (but with a much lower constant of proportionality) for solution of the linear equations by e.g. Gaussian elimination.

This latter cost may also be reduced very substantially by use of the pattern of the matrices S and T. For example, in the simplest case, when β is constant on Γ, the coefficient matrices of the equations (4) and (5) are $I+2ik\beta(S^d+S^r)$ and $-ik\beta I/2+T^d+T^r$ where I is the order N identity matrix. Each matrix is a sum of a Hankel and a Toeplitz matrix which can be solved efficiently in $O(N^2)$ operations using the algorithm given in [5].

This method has been applied to the configuration shown in Figure 2. The impedance of the ground surface is given by the Delany-Bazley formula with flow resistance σ=25000Nsm^{-4}, a value typical of grassland. Figure 3 is a plot of barrier insertion loss against frequency at the receiver position (x,y).

3. SOLUTION BY APPROXIMATION AND MINIMISATION

The geometry of the problem is the one shown in Figure 1, where the screen is now assumed to be infinitely thin and perfectly reflecting, i.e. β=0. For this particular case, the sound pressure p can be expressed as a double layer potential [2] :

$$p(\tilde{r}) = G(\tilde{r},\tilde{r}_0) + \int_\Gamma \mu(\tilde{r}_s) \frac{\partial G(\tilde{r},\tilde{r}_s)}{\partial x_s}\, dy_s \qquad \text{with } \tilde{r}_s=(x_s=0,y_s) \tag{11}$$

where G is, as previously, the Green's function for the plane characterised by the admittance β_c . μ represents the jump p^+-p^- on the screen and is now the unknown of the problem. When μ is known, the pressure p can be computed anywhere above the ground. By applying the boundary condition ($\partial p/\partial x=0$) on Γ, it is shown that μ is the solution of the integral equation :

$$\lim_{x\to 0} \frac{\partial}{\partial x} \int_\Gamma \mu(\tilde{r}_s) \frac{\partial G(\tilde{r}_s,\tilde{r})}{\partial x_s}\, dy_s = -\frac{\partial G(\tilde{r}_0,\tilde{r})}{\partial x} \tag{12}$$

Instead of solving this equation by a collocation method, we first approximate μ by the classical Kirchhoff approximation [6] : $\mu(\tilde{r}_s) \cong 2G(\tilde{r}_s,\tilde{r}_0)$, i.e. twice the incident pressure (above the absorbing ground, with no screen). The use of this approximation in (11) to obtain the sound levels behind the screen leads to correct results in the far field, i.e. at large distances from the screen. To improve this simple approximation and obtain correct sound levels even in the near-field, we chose to approximate μ by :

$$\mu(\tilde{r}_s) \cong 2G(\tilde{r}_0,\tilde{r}_s) + A(y_s)\cos ky_s + B(y_s)\sin ky_s$$

where A and B are complex functions. Let us assume first that A and B are constant on Γ. Then these constants can be obtained by minimising the following function :

$$\mathcal{F}(A,B) = \sum_{i=1}^{M} |F_i + Af_i + Bg_i| \tag{13}$$

$$\text{where : } F_i = \frac{\partial G(\tilde{r}_0,\tilde{r}_i)}{\partial x} + \lim_{x \to 0} \frac{\partial}{\partial x} \int_\Gamma 2G(\tilde{r}_0,\tilde{r}_s) \frac{\partial G(\tilde{r}_s,\tilde{r}_i)}{\partial x_s} \, dy_s$$

$$f_i = \lim_{x \to 0} \frac{\partial}{\partial x} \int_\Gamma \cos ky_s \frac{\partial G(\tilde{r}_s,\tilde{r}_i)}{\partial x_s} \, dy_s \quad \text{and} \quad g_i = \lim_{x \to 0} \frac{\partial}{\partial x} \int_\Gamma \sin ky_s \frac{\partial G(\tilde{r}_s,\tilde{r}_i)}{\partial x_s} \, dy_s$$

A and B are then obtained by solving a linear system of order 2. $\mathfrak{F}$ corresponds to the integral equation (12) and the points $\tilde{r}_i=(x=0,y_i)$ lie on Γ. The interest of the method is to save a large amount of computation time, compared with a collocation method. Indeed, in the classical collocation method, the distance between two collocation points (where the integral equation is written) must be equal or smaller than a sixth of the wavelength. In the method presented here, since the oscillating behaviour of μ is already taken into account with the Kirchhoff approximations and the sine and cosine functions, the number of points $\tilde{r}_i$ can be much smaller. This property is of course very important, especially at high frequency.
From several numerical examples, it seems reasonable to divide Γ into 2 or 4 subintervals on which A and B can be approximated by constants. Figure 4 shows an example of sound levels against distance computed by using the expression (14). The continuous line is obtained by evaluating μ by the classical collocation method ; the broken line is obtained by using the minimisation method ; the crosses are the levels obtained by replacing μ by the Kirchhoff approximation 2G.

4. CONCLUSION

The aim of the methods presented here is to reduce the computation time and the storage needed when using boundary integral equations.
They are presented on the simple example of a barrier on a locally reacting ground. It is obvious that the numerical treatment presented in section 2 can be used for many types of propagation problems, in two and three dimensions. The method presented in section 3 has only been applied to the screen problem. The next two steps will be to study more closely the interest of the method versus frequency and its applications to other types of propagation problems. If the first results are confirmed, this could be a very efficient way to solve boundary integral equations.

5. REFERENCES

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ACNOWLEDGEMENT

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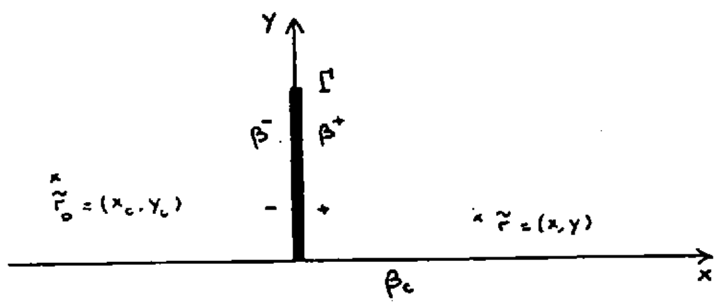

Figure 1. The two-dimensional noise barrier configuration.

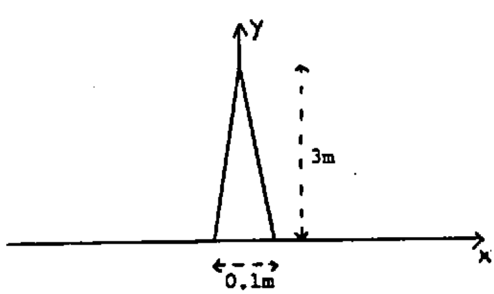

Figure 2. The wedge barrier approximation to the thin screen.

$$\text{where : } F_1 = \frac{\partial G(\tilde{r}_0,\tilde{r}_1)}{\partial x} + \lim_{x\to 0} \frac{\partial}{\partial x} \int_\Gamma 2G(\tilde{r}_0,\tilde{r}_s) \frac{\partial G(\tilde{r}_s,\tilde{r}_1)}{\partial x_s} \, dy_s$$

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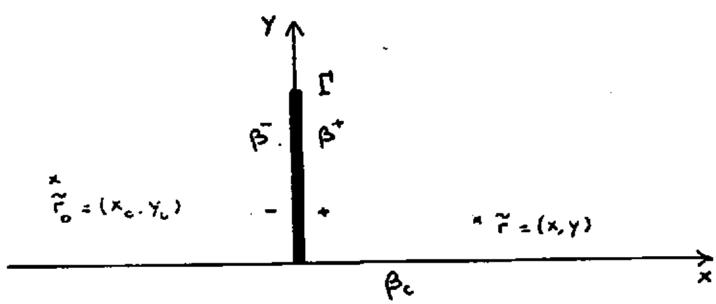

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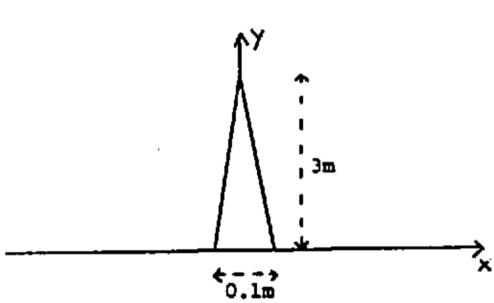

Figure 2. The wedge barrier approximation to the thin screen.

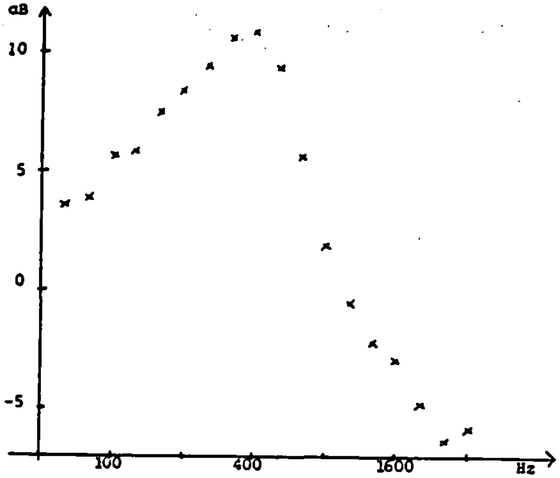

Figure 3. Insertion loss against frequency. H=3m, $\tilde{r}_0$=(-15m,0), $\tilde{r}$=(50,0).

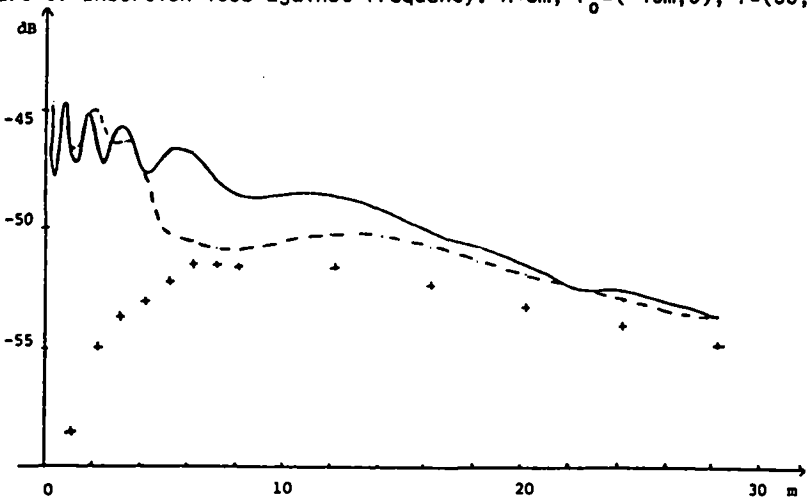

Figure 4. Sound levels emitted on the ground, against distance.
f=500Hz, H=4m, $\tilde{r}_0$=(-8m,0), ζ=(8,4.5).