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DIFFRACTION AT AN INHOMOGENEOUS PLANE: THE BOUNDARY INTEGRAL EQUATION METHOD

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1. INTRODUCTION

Propagation of sound above a plane of finite impedance has been the subject of many papers. Recent examples giving accurate expressions for the propagation of sound from harmonic point and line sources respectively are [1] and [2]. Meteorological effects, variation of ground height and impedance, limit the application of these results to environmental sound propagation. Various authors, attending to the third of these factors, have developed calculation methods for propagation from a point source over a two-impedance plane [3,4,5,6].

The boundary integral equation method is well known in application to acoustic scattering and its application to outdoor sound propagation has been discussed by Seznec [7] with reference to the effect of barriers. In this paper we apply the method to propagation above an inhomogeneous impedance plane. We consider in detail two-dimensional problems of propagation from a line source, in particular over a strip of one impedance embedded in a plane of different impedance. The calculation method is used to investigate the effect of varying the location between source and receiver of a strip of soft ground embedded in hard ground.

2. THE BOUNDARY INTEGRAL EQUATIONS

We are concerned with the propagation of sound from a harmonic point or line source ($e^{-i\omega t}$ time dependence) over normally reacting ground of variable admittance. We assume a homogeneous medium in which the wavenumber is k. The problem may be stated as the following boundary value problem where ϕ is the acoustic potential and $\underset{\sim}{r}$, $\underset{\sim}{r}_o$ are the receiver and source positions relative to some fixed point, D is the region above the ground and ∂D the ground surface:

$$(\nabla^2 + k^2)\ \phi(\underset{\sim}{r}) = \delta(\underset{\sim}{r}-\underset{\sim}{r}_o) \text{ for } \underset{\sim}{r} \text{ in D} \tag{1}$$

with boundary conditions

$$\frac{\partial\phi}{\partial n}(\underset{\sim}{r}) = ik\beta(\underset{\sim}{r})\phi(\underset{\sim}{r}) \text{ for } \underset{\sim}{r} \text{ in } \partial D \tag{2}$$

and, at infinity,

$$\lim_{\substack{r\to\infty \\ \underset{\sim}{r} \text{ in D}}} r^{\varepsilon}\left(\frac{\partial\phi}{\partial r} - ik\phi\right) = 0 \tag{3}$$

Here n is the normal to ∂D drawn out of D β is the ground admittance (β=0 (rigid ground) or Re(β)>0 (energy absorbing condition)), $r = |\underset{\sim}{r}|$ and $\varepsilon=\frac{1}{2}$, 1 in 2 or 3 dimensions respectively.

By an application of Green's theorem and the use of the equations above we obtain the following general integral equation [7] in which $G(\underset{\sim}{r}, \underset{\sim}{r}_o)$ is any solution of (1) (i.e. a Green's function).

$$\phi(\underset{\sim}{r}) = G(\underset{\sim}{r}, \underset{\sim}{r}_o) - \int_{\partial D}\phi(\underset{\sim}{r}_s)\left(ik\beta G(\underset{\sim}{r}, \underset{\sim}{r}_s) - \frac{\partial G}{\partial n(\underset{\sim}{r}_s)}(\underset{\sim}{r}, \underset{\sim}{r}_s)\right) dS\,(\underset{\sim}{r}_s) \tag{4}$$

for $\underset{\sim}{r}$ in D. In the particular case when ∂D is flat and $G(\underset{\sim}{r}, \underset{\sim}{r}_o)$ satisfies (2) with $\beta(\underset{\sim}{r}) = \beta_c$, a constant, (4) becomes

$$\phi(\underset{\sim}{r}) = G(\underset{\sim}{r}, \underset{\sim}{r}_o) - ik \int_{\partial D} G(\underset{\sim}{r}, \underset{\sim}{r}_s)\, \phi(\underset{\sim}{r}_s)\, (\beta(\underset{\sim}{r}_s)-\beta_c)\, dS\,(\underset{\sim}{r}_s) \tag{5}$$

which holds both for $\underset{\sim}{r}$ in D and in ∂D, and constitutes a singular Fredholm integral equation of the second kind for $\underset{\sim}{r}$ on ∂D. Numerical solution of this equation, obtaining values of $\phi(\underset{\sim}{r})$ for $\underset{\sim}{r}$ on ∂D, enables the calculation, by substitution in (5) of $\phi(\underset{\sim}{r})$ for $\underset{\sim}{r}$ in D.

3. NUMERICAL SOLUTION

We consider in this section the numerical solution of (5), restricting our attention for simplicity to the two-dimensional case of propagation from a line-source, the method extending to the three-dimensional case with a corresponding increase in computational time and storage. We assume that $\beta(\underset{\sim}{r})$ differs from β_c only within some bounded region of ∂D and that $\beta(\underset{\sim}{r})$ varies only in a direction perpendicular to that of the line source so that the problem is effectively two-dimensional (see Fig. 1). Equation (5) becomes, for $\underset{\sim}{r}$ in ∂D,

$$\phi(x) = G(x,0,z_o) - ik \int_a^b g(k|x-x_s|)\,\phi(x_s)\,(\beta(x_s)-\beta_c)\,dx_s \tag{6}$$

Here $G(x, x', z')$ is the potential at $(x,0)$ in a surface of admittance β_c due to a line source at (x', z'), and $g(k|x-x_s|) \equiv G(x,x_s,0)$.

We have used the simplest quadrature method of solution of this equation (Mayers [8]) in which we approximate the integral in (6) by the midpoint rule, modified slightly to account for a logarithmic singularity in $g(t)$ at $t = 0$. The right hand side of (6) then involves only values $\phi_n = \phi(x_n)$ of the potential, at N equally spaced points $x_n = a + (n-\frac{1}{2})h$ for $n = 1, 2, \ldots, N$ with $Nh = L=b-a$. Once these values are known, equation (6), or the corresponding version for $\underset{\sim}{r}$ in D, can be used to calculate values of ϕ elsewhere in D or ∂D.

Having approximated the right hand side of (6) by our quadrature rule so that it involves only the unknowns ϕ_n, we obtain N equations for these unknowns by setting $x=x_n$ for $n=1, 2, \ldots, N$ in (6) leading to

$$\sum_{n=1}^{N} a_{mn}\phi_n = b_m \text{ for } m=1,2,\ldots,N \text{ with } b_m = G(x_m,0,z_o) \tag{7,8}$$

$$\text{and } a_{mn} = \begin{cases} 1+ikh\bar{g}(0)\,(\beta(x_m)-\beta_c) & \text{if } m=n \\ ikh\bar{g}(k|x_m-x_n|)\,(\beta(x_n)-\beta_c) & \text{otherwise} \end{cases} \tag{9}$$

$$\text{with } \bar{g}\,(k|x_m-x_n|) = \frac{1}{h}\int_{-h/2}^{h/2} g(k|x_m-x_n+t|)\,dt$$

$$\approx \begin{cases} \frac{1}{\pi}\ln(hk) - {}^1/2 + \frac{1}{\pi}(\gamma-2\ln 2-1) + \frac{i\beta}{\pi\sqrt{(1-\beta^2)}}\ln\left\{\frac{(c+d)\,(\beta-i(1+cd))\,(1-cd)}{(c-d)\,(\beta-i(1-cd))\,(1+cd)}\right\} & \text{if } m=n \\ g(k|x_m-x_n|) & \text{otherwise,} \end{cases} \tag{10}$$

with $c = \sqrt{(1+\beta)}$, $d = \sqrt{(1-\beta)}$ and $-\frac{\pi}{2} < \arg\sqrt{(1-\beta^2)}$, $\arg(c)$, $\arg(d) \leqslant \frac{\pi}{2}$ and the principal value of the logarithm being taken. The detailed derivation of this result and the accurate evaluation of $G(x,x',z')$ and $g(t)$ are discussed in /2/. The equations (7) may be solved by Gaussian elimination. The Gauss-Seidl iterative method is also attractive when it converges.

Any variation of admittance in $a < x < b$ can be accommodated in this method. Perhaps of particular interest however is the case when $\beta(x) \equiv \beta$ is constant, i.e. we are considering a strip of admittance β embedded in a plane of admittance β_c. In this case equations (7) are highly structured, the coefficient matrix $/a_{mn}/$ being symmetric and Toeplitz. (A matrix $/z_{mn}/$ is Toeplitz if the elements down each diagonal are the same, i.e. the value of z_{mn} depends only on m-n). The application of the Gauss-Seidl method to (7) in this case, including conditions for convergence has been discussed in /2/. Somewhat more efficient however, and equally effective in cases where Gauss-Seidl iteration fails to converge, is the algorithm of Trench /9/ which solves (7) with $3N^2$ multiplications and storage 4N. By contrast, even making use of the symmetry of $/a_{mn}/$, Gaussian elimination requires $^1/_6\ N^3$ multiplications and storage of $\frac{1}{2}N^2$ numbers. The accuracy of the values of ϕ calculated on ∂D and in D depends on the quadrature step-size h and is discussed in /2/ in which a value of h=0.2 wavelengths is found to give results accurate to within 0.4dB in a range of cases.

3. CALCULATIONS - THE EFFECTS OF SOFT GROUND LOCATION

In this section we illustrate the calculation method of the previous section. The situation is that of Fig. 1 with β_c=0 so that the ground is rigid except in $a < x < b$ where the ground has finite impedance with admittance β given as a function of frequency by equations of Delany and Bazley /10/ with flow resistance σ = 250000 Nsm^{-4}. These equations and value of flow resistance have been found to model well the admittance of grassland /11/.

Graphs of excess attenuation (EA) at a receiver point, defined by,

$$EA = -20 \log_{10} |\phi/\phi_d| \tag{11}$$

where ϕ,ϕ_d are the total potential and the potential of the direct wave respectively are shown in Figs. 2 and 3. In Fig. 2 are shown the cases of rigid ground (L=0), finite impedance ground (L=∞), and the variation of EA with a when L=10m and all other variables are fixed. Fig. 3 shows, for various values of a, the variation of EA with r. The calculation method is that of the appendix of /2/ in the homogeneous case, and that of the previous section with h=0.2 wavelengths in the case of inhomogeneous ground.

Figs. 2 show that the received sound pressure level can depend on the location of soft ground between source and receiver and not merely the proportion. In particular in Fig. 2(a) where source and receiver are close to the ground, it is clear that at frequencies where the soft ground has the effect of reducing the noise level, the reduction is most significant when the soft ground is immediately adjacent to the source (or by reciprocity the reciver). In Fig. 2(b) where source and receiver are lifted off the ground so that the path difference between direct and geometrically reflected rays becomes significant, the situation is more complex. We note that the EA values for half soft ground between source and receiver do not, in this figure, at certain frequencies, lie between those for all soft and all hard ground. In Fig. 2(a) however, it is a reasonable approximation, particularly when a = 0 or 10m, to estimate the EA by the average of the EA's with all soft and all hard ground.

Fig. 3 gives some idea of the global effects of a strip of soft ground parallel to the source embedded in a rigid plane. In this case source and receiver are both at height 0.5m, the frequency is 800 Hz, and the strip width 25m. It can be seen that as distance (r) from the source increases, initially the curve for all hard ground is followed. As the soft strip is traversed a diffraction pattern gives way to an increasing excess attenuation. Another diffraction pattern is observed at the end of the soft ground and the EA then decreases to a final asymptotic value. In Fig. 3 the noise level at large distances from the source is lower the nearer the soft ground is to the source. It is seen that placing the soft ground strip 5m away from rather than adjacent to the source increases the sound pressure level at the receiver (for r > 50m) by over 3dB. This suggests that soft ground near the source (and by reciprocity the reciver) may be particularly important in reducing noise. As a result of this an accurate calculation of the propagation of environmental noise over mixed ground may have to include specifically the ground cover in the proximity of the source and receiver as well as the proportion of ground types beneath the propagation path.

4. CONCLUSION

In this paper the boundary integral equation method has been applied producing integral equations modelling in particular sound propagation above an inhomogeneous impedance plane. The numerical solution of the integral equation modelling propagation from a line source over flat ground of impedance varying in a direction perpendicular to that of the line source has been discussed in detail. Results calculated using this method in the particular case of a strip of finite impedance embedded in rigid ground suggest that the location as well as the proportion of soft ground between source and receiver may be significant in the calculation of the propagation of environmental noise over ground of mixed type.

REFERENCES

[1] T. Kawai, T. Hidaka, and T. Nakajima, 'Sound propagation above an impedance boundary', J.S.V., Vol. 83, 125-138 (1982).

[2] S.N. Chandler-Wilde and D.C. Hothersall, 'Sound propagation above an inhomogeneous impedance plane', J.S.V., Vol. 98, No. 4, (1985), (to be published).

[3] M. Naghieh and S.I. Hayek, 'Diffraction of a point source by two impedance covered half-planes', J.A.S.A., Vol. 69, 629-637, (1981).

[4] B.A. de Jong, A. Moerkerken, and J.D. Van der Toorn, 'Propagation of sound over grassland and over an earth barrier', J.S.V., Vol. 86, 23-46, (1983).

[5] J. Durnin and H.L. Bertoni, 'Acoustic propagation over ground having inhomogeneous surface impedance', J.A.S.A., Vol. 70, 852-859, (1981).

[6] K.B. Rasmussen, 'Propagation of road traffic noise over level terrain', J.S.V., Vol. 82, 51-61, (1982).

[7] R. Seznec, 'Diffraction of sound around barriers: use of the boundary elements technique', J.S.V., Vol. 73, 195-209, (1980).

[8] D.F. Mayers, 'Quadrature methods for Fredholm equations of the second kind', in Numerical Solution of Integral Equations (L.M. Delves and J. Walsh, editors), Oxford, Clarendon Press, (1974).

[9] W.F. Trench, 'An algorithm for the inversion of finite Toeplitz matrices,' J. Soc. Indust. Appl. Math., Vol. 12, 515-522, (1964).
[10] M.E. Delany and E.N. Bazley, 'Acoustical properties of fibrous absorbent materials', Applied Acoustics, Vol. 3, 105-116, (1970).
[11] C.I. Chessell, 'Propagation of noise along a finite impedance boundary', J.A.S.A., Vol. 62, 825-834, (1977).

FIGURES

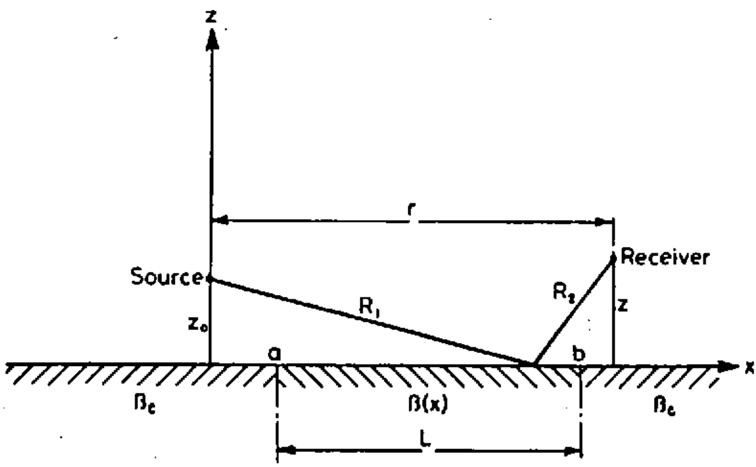

Fig. 1: Geometry of source and receiver above an inhomogeneous plane

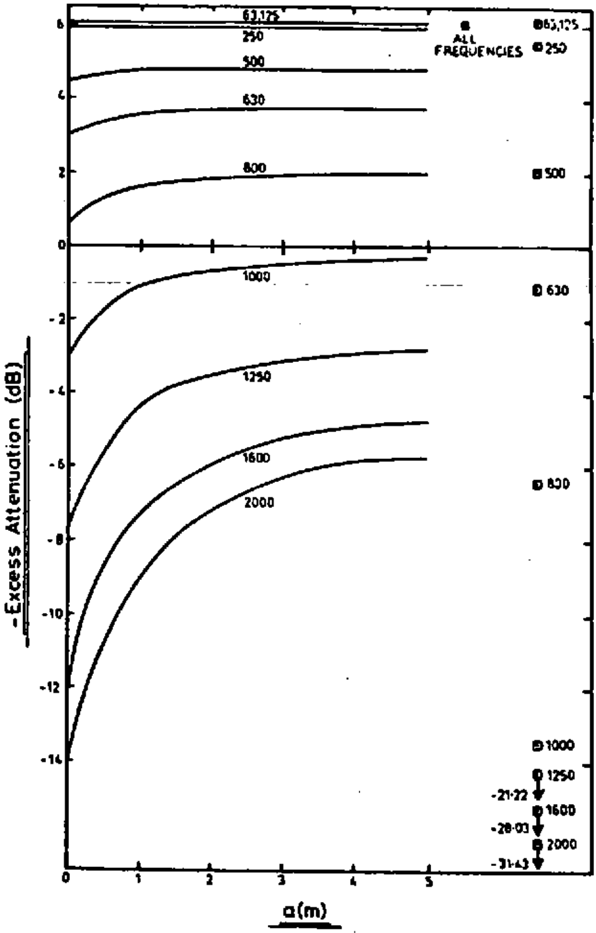

Fig. 2(a)

Fig. 2 : Excess attenuation of a cylindrical wave. Situation is Fig. 1 with r=20m: (a) $z_o = z = 0.1$m; (b) $z_o = z = 1.0$m. Parameter is frequency in Hz.—inhomogeneous surface: L = 10m, $\beta_c = 0$, $\beta(x)$ as in text, a is varied. ● rigid surface. ▣ homogeneous impedance surface, $\beta(x) \equiv \beta_c$, calculated as in text. The curves are symmetric about a=5m by reciprocity.

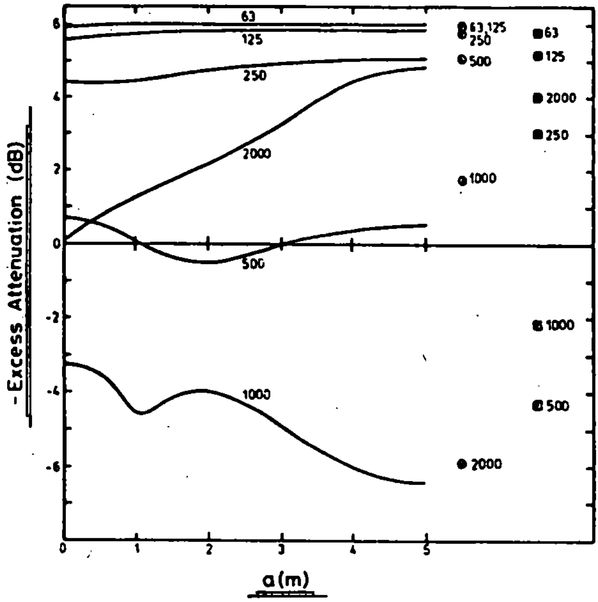

Fig. 2(b)

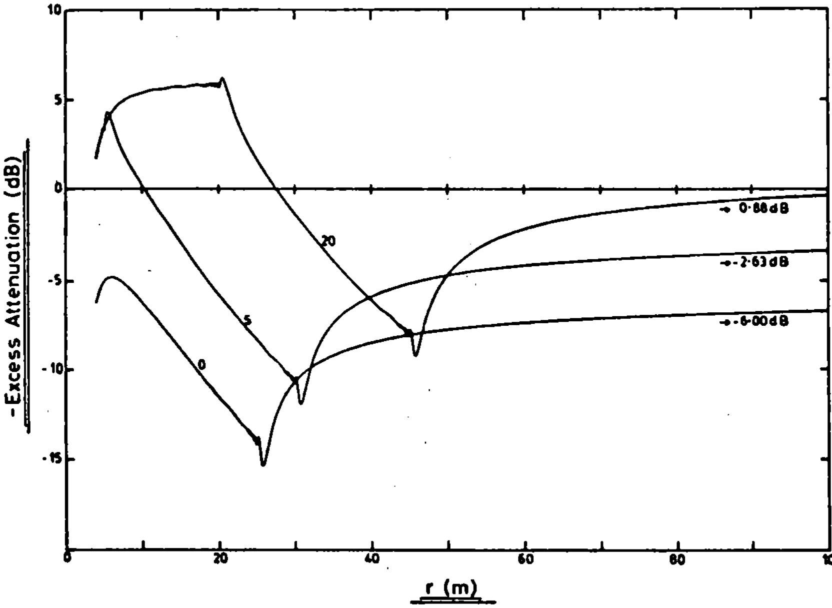

Fig. 3: Excess attenuation of a cylindrical wave as a function of distance from source. Situation is Fig. 1 with frequency = 800Hz, z_o=z=0.5m, L = 25m, β_c=0, $\beta(x)$ calculated as in text. Parameter is a in metres. The curves intersect at r = 30, 45, 50m by reciprocity.