

## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

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### 1. INTRODUCTION

The effect of ground type on the propagation of sound has long been of interest; many authors have investigated propagation over homogeneous ground. More recently propagation over inhomogeneous ground has been considered and several approximate methods for the calculation of the propagation of sound from a harmonic point source over a two-impedance plane have been proposed [1-4].

One such method is described in this paper, a generalisation of the "compensation approach" method of Durnin and Bertoni [3]. This method is based on the boundary integral equation (BIE) method, which the authors have already discussed [5,6], and can be seen as an approximate form of the BIE method. The theory of the method in two and three dimensions is described and comparison made in two-dimensions with exact BIE calculations, and in three dimensions with the results of model experiments.

The application of the method to investigate traffic noise propagation over ground of mixed type is discussed.

### 2. THEORY

We are concerned with the propagation of sound from a harmonic point or line source over flat normally reacting ground of variable admittance. We assume a homogeneous medium in which the wave number is  $k$ . Let  $\underline{r}$ ,  $\underline{r}_0$  denote the receiver and source positions relative to some fixed point and  $G_M(\underline{r}, \underline{r}_0)$  the acoustic potential at  $\underline{r}$ .  $G_M(\underline{r}, \underline{r}_0)$  satisfies a boundary value problem consisting of a homogeneous Helmholtz equation for  $\underline{r} \neq \underline{r}_0$  in the region  $D$  above the ground, together with a normally reacting boundary condition at the ground surface  $\partial D$ , and a Sommerfeld radiation condition at infinity. It can be shown [5] that this boundary value problem is equivalent to the integral equation, for  $\underline{r}$  in  $D$  and  $\partial D$ ,

$$G_M(\underline{r}, \underline{r}_0) = G_{BC}(\underline{r}, \underline{r}_0) - ik \int_{\partial D} G_{BC}(\underline{r}, \underline{r}_s) G_M(\underline{r}_s, \underline{r}_0) (\beta(\underline{r}_s) - \beta_c) dS(\underline{r}_s) \quad (1)$$

In this equation  $\beta(\underline{r}_s)$  is the admittance at a point  $\underline{r}_s$  in the ground surface and  $\beta_c$  is an (arbitrary) constant value of admittance;  $G_{BC}(\underline{r}, \underline{r}_0)$  is the potential that would be observed at  $\underline{r}$  if the ground had constant admittance  $\beta_c$ . Equation (1) holds both in three dimensions with a point source, and for effectively two-dimensional problems when the source is a line source and  $\beta(\underline{r}_s)$  is constant in a direction parallel to it. The integral over  $\partial D$  is a surface or line integral in these two cases respectively. Figure 1 shows the two-dimensional case, the plane of the paper being perpendicular to the line source and  $\beta(\underline{r}_s)$  a function of  $x$  alone (independent of  $y$ ). We have also assumed that  $\beta(\underline{r}_s) = \beta_c$  outside a region  $\partial D_1$  of  $\partial D$ . When this is the case  $\partial D$  in (1) may be replaced by  $\partial D_1$ .

## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

The numerical solution of (1) has been discussed previously [5,6]. A set of linear equations is solved to determine values of  $G_M(\underline{r}, \underline{r}_0)$  for  $\underline{r}$  in  $\partial D_1$ . Subsequently these values are substituted in the right hand side of equation (1) to calculate by numerical integration  $G_M(\underline{r}, \underline{r}_0)$  at any point in  $D$ . For either of these steps to be possible we must be able to calculate accurately  $G_{\beta c}(\underline{r}, \underline{r}_0)$ , the solution for uniform surface admittance. This has been discussed in the appendix of [5].

A particularly simple case of considerable interest is when  $\partial D_1$  is a half-plane, the source lies above  $\partial D_1$ , and  $\beta(\underline{r}_s) \equiv \beta$ , a constant, for  $\underline{r}_s$  in  $\partial D_1$ .

With those restrictions equation (1) describes propagation above a plane consisting of two impedance half-planes. In this case we may adopt on intuitive physical grounds the approximation  $G_M(\underline{r}, \underline{r}_0) \approx G_{\beta}(\underline{r}, \underline{r}_0)$  for  $\underline{r}$  in  $\partial D_1$ , whereupon equation (1) becomes,

$$G_M(\underline{r}, \underline{r}_0) = G_{\beta c}(\underline{r}, \underline{r}_0) - ik(\beta - \beta_c) \int_{\partial D_1} G_{\beta c}(\underline{r}, \underline{r}_s) G_{\beta}(\underline{r}_s, \underline{r}_0) dS(\underline{r}_s) \quad (2)$$

While this approximation is not as accurate as the correct numerical solution, which can be made as accurate as desired with sufficient computing power, it is computationally preferable as the expensive exact solution of (1), for  $G_M(\underline{r}, \underline{r}_0)$ ,  $\underline{r}_s$  in  $\partial D_1$  has been avoided. The approximate equation (2) is a generalisation of the 'compensation approach' approximation of Durnin and Bertoni [3], who consider only the case where source and receiver are in the ground surface and either  $\beta_c$  or  $\beta$  is zero.

In the two-dimensional case, illustrated in figure 1 if we take  $a \rightarrow -\infty$  and  $\beta(x) \equiv \beta$  constant, equation (2) is explicitly;

$$G_M(\underline{r}, \underline{r}_0) = G_{\beta c}(\underline{r}, \underline{r}_0) + \frac{ik}{16} (\beta - \beta_c) \int_{-\infty}^b H_0^{(1)}(kR_1) (1+Q^{(2)}(\beta, z_0, x)) \cdot H_0^{(1)}(kR_2) (1+Q^{(2)}(\beta_c, z, r-x)) dx \quad (3)$$

Here we have written  $G_{\beta}(\underline{r}_s, \underline{r}_0) = -\frac{i}{4} H_0^{(1)}(kR_1) (1+Q^{(2)}(\beta, z_0, x))$ , and similarly  $G_{\beta c}(\underline{r}, \underline{r}_s) = -\frac{i}{4} H_0^{(1)}(kR_2) (1+Q^{(2)}(\beta_c, z, r-x))$ ; that is  $Q^{(2)}(\beta, z, x)$  is a cylindrical wave reflection coefficient at homogeneous ground of admittance  $\beta$ , with  $z$  the sum of source and receiver heights and  $x$  the horizontal separation of source and receiver.  $H_0^{(1)}$  is the Hankel function of the first kind of order zero and  $R_1 = \sqrt{x^2 + z_0^2}$ ,  $R_2 = \sqrt{(r-x)^2 + z^2}$ . Calculations have been made, comparing  $G_M(\underline{r}, \underline{r}_0)$  calculated by numerical integration of (3) with the value calculated by exact solution of equation (1). Calculations were made for the case in which one half-plane is rigid ( $\beta = 0$ ) and the other half-plane has admittance  $\beta$  given by equation (7) with flow resistance  $\sigma = 250000 \text{ Nsm}^{-4}$ . This equation and value of  $\sigma$  have been commonly used to model the admittance of grassland. Calculations were made with  $z = z_0 = 1.0 \text{ m}$ ,  $r = 1, 5, 10, 20, \dots, 50 \text{ m}$ ,  $b = 0, 5, \dots, 20 \text{ m}$  and for each  $1/3$  octave centre frequency  $f$  in the range  $100 \leq f \leq 2000 \text{ Hz}$ . A maximum deviation of 0.13 dB was noted between the approximate and the exact solution.

## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

In the corresponding three-dimensional case of propagation from a point source over a two-impedance plane with the line of discontinuity of impedance perpendicular to the line from source to receiver, equation (2) is explicitly

$$G_M(\underline{r}, \underline{r}_0) = G_{\beta_C}(\underline{r}, \underline{r}_0) - \frac{ik}{16\pi^2} (\beta - \beta_C) \int_{-\infty}^b dx \int_{-\infty}^{+\infty} \frac{e^{ik(R_A + R_B)}}{R_A R_B} (1 + Q^{(3)}(\beta, z_0, r_A)) (1 + Q^{(3)}(\beta_C, z, r_B)) dy \quad (4)$$

where  $Q^{(3)}(\beta, z, x)$  is a spherical wave reflection coefficient  $(\beta, z, x)$  defined as in  $Q^{(2)}(\beta, z, x)$  and

$$R_A = \sqrt{R_1^2 + y^2}, R_B = \sqrt{R_2^2 + y^2}, r_A = \sqrt{x^2 + y^2}, r_B = \sqrt{(x-x)^2 + y^2}$$

It can be shown [5] that the approximations for  $G_M(\underline{r}, \underline{r}_0)$  of equations (3) and (4) satisfy reciprocity. The integration with respect to  $y$  in equation (4) may be performed approximately by the method of stationary phase, giving,

$$G_M(\underline{r}, \underline{r}_0) = G_{\beta_C}(\underline{r}, \underline{r}_0) + \frac{k}{16\pi} \sqrt{\frac{\pi k}{2}} e^{i\pi/4} (\beta - \beta_C) \int_{-\infty}^b (R_1 + R_2)^{-1/2} H_0^{(1)}(kR_1) (1 + Q^{(3)}(\beta, z_0, x)) \cdot H_0^{(1)}(kR_2) (1 + Q^{(3)}(\beta_C, z, r-x)) dx \quad (5)$$

Using equation (5) the potential may be calculated as rapidly in the three-dimensional as in the two-dimensional case.

### 3. COMPARISON WITH EXPERIMENTAL RESULTS

Propagation over a two impedance plane has been investigated by experiment in an anechoic chamber. The source was the open end of a stout brass tube of internal diameter 4.5mm, the other end of which was connected to a speaker mounted inside an insulated box. The receiver was a  $\frac{1}{4}$ " microphone. The two surfaces were a hard surface of melamine-covered chipboard, and a soft surface produced by covering this with a 10mm thick foam backed carpet. The heights of source and receiver above the hard surface were 85 and 63mm, respectively, and their horizontal separation was 1.0m. Narrow band (50Hz) spectra between 1 and 20 kHz were measured for propagation over hard, soft, and mixed surfaces with a quarter, half, and three-quarters of the surface soft. In all cases the boundary was perpendicular to the direction from source to receiver with the soft ground below the receiver. The free-field spectrum was also measured to enable the excess attenuation spectrum to be calculated for propagation over each surface type.

## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

Theoretical curves for the experimental conditions are shown in figure 2.

Excess attenuation, defined by,

$$EA = -20 \log_{10} |G_M(\underline{r}, \underline{r}_O) / G(\underline{r}, \underline{r}_O)|, \quad (6)$$

where  $G_M, G$  are the total potential and the potential of the direct wave, respectively, is plotted against frequency. The curves are calculated by numerical integration of equation (5) in the two-impedance ground cases. The hard ground was assumed perfectly rigid, and the soft ground assumed homogeneous with admittance  $\beta_G$ , given by,

$$1/\beta_G = 1 + 9.08(1000f/\sigma)^{-0.75} + 111.9(1000f/\sigma)^{-0.73} \quad (7)$$

where  $f$  and  $\sigma$  are frequency and flow resistance respectively in SI units.

This equation, due to Delany and Bazley [7], has been used successfully to model the admittance of grassland [8]. Assuming further that the soft ground is normally reacting, has thickness  $T$ , and complex wave number  $k_G$  satisfying Delany and Bazley's equation [7],

$$k_G = k(1 + 10.8(1000f/\sigma)^{-0.70} + i 10.3(1000f/\sigma)^{-0.59}), \quad (8)$$

the normal surface admittance is [9],

$$\beta = -\tan(Tk_G)\beta_G \quad (9)$$

This equation is used in calculating figure 2, with  $T=10\text{mm}$ , the experimental value, and  $\sigma$  chosen as  $300\,000\text{ Nsm}^{-2}$  to give the best fit to the experimental results.

Examining figure 2 we see that as the proportion of soft ground increases the interference minimum in the EA spectrum associated with geometrical reflection effects disappears, and a minimum of lower frequency characteristic of soft ground reflection appears. An interesting point is that at low frequencies the curves for mixed ground converge to a level between and distinct from the levels for homogeneous soft and hard ground. This suggests that the ground type in the proximity of source or receiver is particularly important in determining EA level, a point which is discussed further in section 4.

The experimental and theoretical results are compared in figure 3, in which the experimental values have been averaged over  $1/12\text{th}$  octave intervals. While the agreement is generally good, there is a certain amount of scatter particularly at low frequencies where few points are included in each  $1/12\text{th}$  octave interval. This scatter is thought to be associated with imperfect absorption and background noise in the anechoic chamber; the similarity in pattern at low frequencies in each figure due to the same free-field spectrum being subtracted in each case. There are also systematic errors at the higher frequencies in the mixed ground cases, thought to be due to the  $10\text{mm}$  step, between the hard and soft ground in the experiment, having been neglected in calculating the theoretical curves.

### 4. APPLICATION TO ROAD TRAFFIC NOISE

The method of section 2 has an obvious application to the calculation of the propagation of road traffic noise over mixed terrain. Using experimentally derived free-field vehicle frequency spectra and equation (5), the SPL due to

## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

a single vehicle, represented by an omni-directional point source, can be calculated as a function,  $SPL(R)$ , of horizontal distance,  $R$ , for given receiver height. For an infinite straight traffic stream, consisting of a single vehicle type, the  $L_{eq}$  can be shown to be,

$$L_{eq} = 10 \log_{10} \left\{ 2D \int_0^{\infty} 10^{SPL(\sqrt{y^2+r^2})/10} dy \right\} \quad (10)$$

where  $r$  is the horizontal distance of the receiver from the traffic stream and  $D$  is the traffic density (vehicles per unit length of road).

In deriving this equation in the two-impedance half-plane case, the dependence of  $SPL$  on the angle between the impedance boundary and the source-receiver direction is neglected; the angle is assumed to be  $90^\circ$ .

Examples of the use of this method are shown in figure 4. On comparing the curves it can be seen that the introduction of a small proportion of soft ground below the propagation path adjacent to the receiver produces a significant reduction in level at grazing incidence.

### 5 CONCLUSION

An approximate method for the calculation of the propagation of sound from a harmonic source over a two-impedance plane has been presented. This method is based on the boundary integral equation method and calculations comparing the two methods have shown agreement within 0.13dB in the two-dimensional case. The corresponding three-dimensional equation (5) for the case when the impedance discontinuity is perpendicular to the direction from source to receiver has been compared with experiment showing generally good agreement.

The computational time involved in evaluating the integral in equation (5) numerically is large when  $kb$  is large, but the use of the equation for the calculation of road traffic noise is feasible with a large main frame computer.

The application of equation (5) to the calculation of  $L_{eq}$  values has been briefly presented, and illustrated with two contour maps which suggest that even a small proportion of soft ground in the propagation path is significant at grazing incidence. Further work is in progress to investigate the effect of a variety of ground conditions.

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## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

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### FIGURES

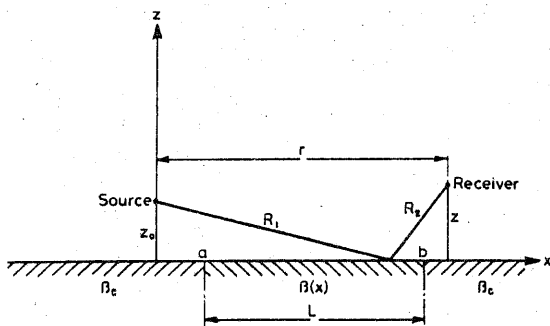


Figure 1. Geometry of source and receiver above an inhomogeneous plane.

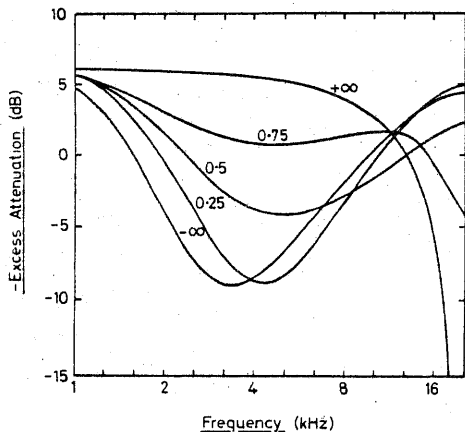


Figure 2. Theoretical excess attenuation of a spherical wave over homogeneous and inhomogeneous ground. Situation is figure 1 with  $a=-\infty$ ,  $r=1.0m$ ,  $z=85mm$ ,  $z=53mm$ ,  $\beta(x)=0$ ,  $\beta_c$  and  $\beta_c$  given by equation (7)-(9) with  $T=10mm$ ,  $\sigma=300\ 000\ Nsm^{-4}$ . Parameter is  $b$  in metres.

## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

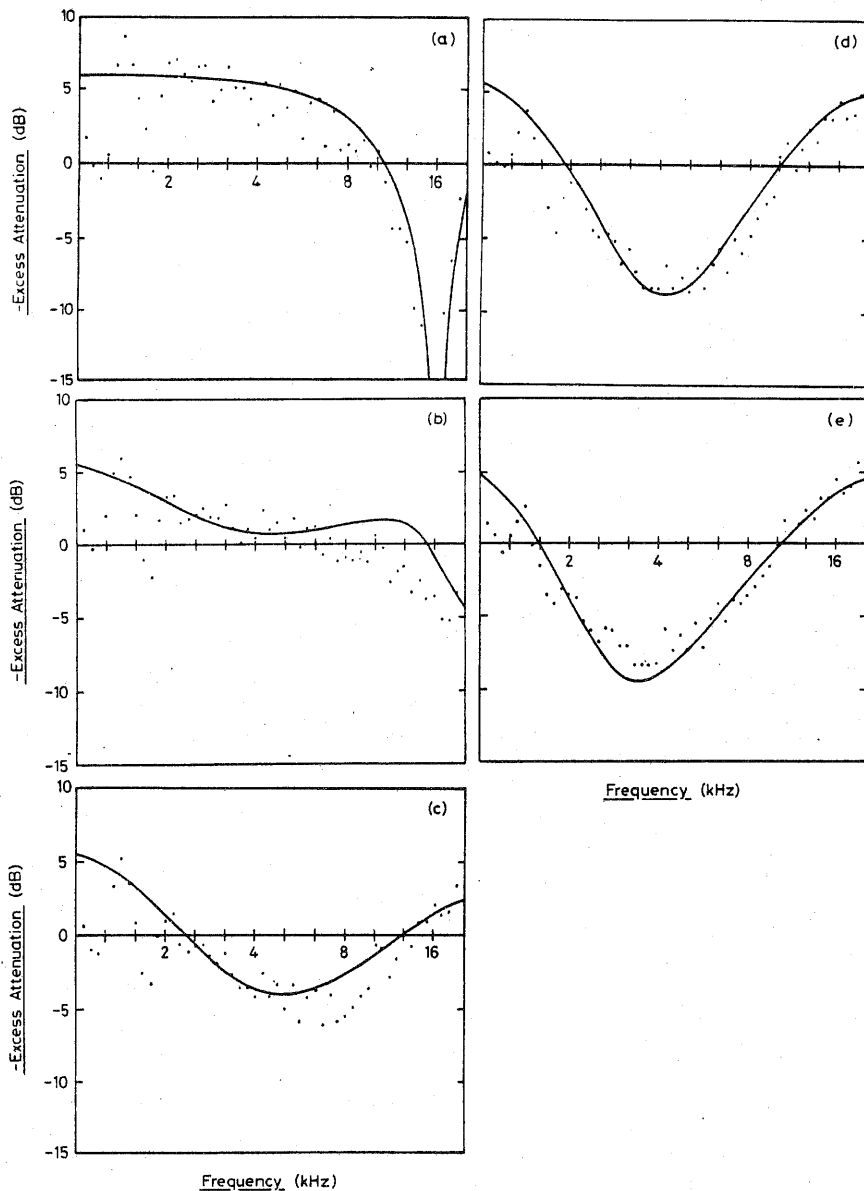


Figure 3. Excess attenuation of a spherical wave over homogeneous and inhomogeneous surfaces. As figure 1 with  $a = -\infty$ ,  $r = 1.0\text{m}$ ,  $z = 85\text{mm}$  (75mm (e)),  $z = 53\text{mm}$  (63mm (a)),  $\beta(x) \equiv 0$ , and  $\beta_c$  calculated as in figure 2. — theoretical; • experimental. (a)  $b = +\infty$  (rigid surface); (b)  $b = 0.75\text{m}$ ; (c)  $b = 0.5\text{m}$ ; (d)  $b = 0.25\text{m}$ ; (e)  $b = -\infty$  (soft surface).

## PROPAGATION OF ROAD TRAFFIC NOISE OVER GROUND OF MIXED TYPE

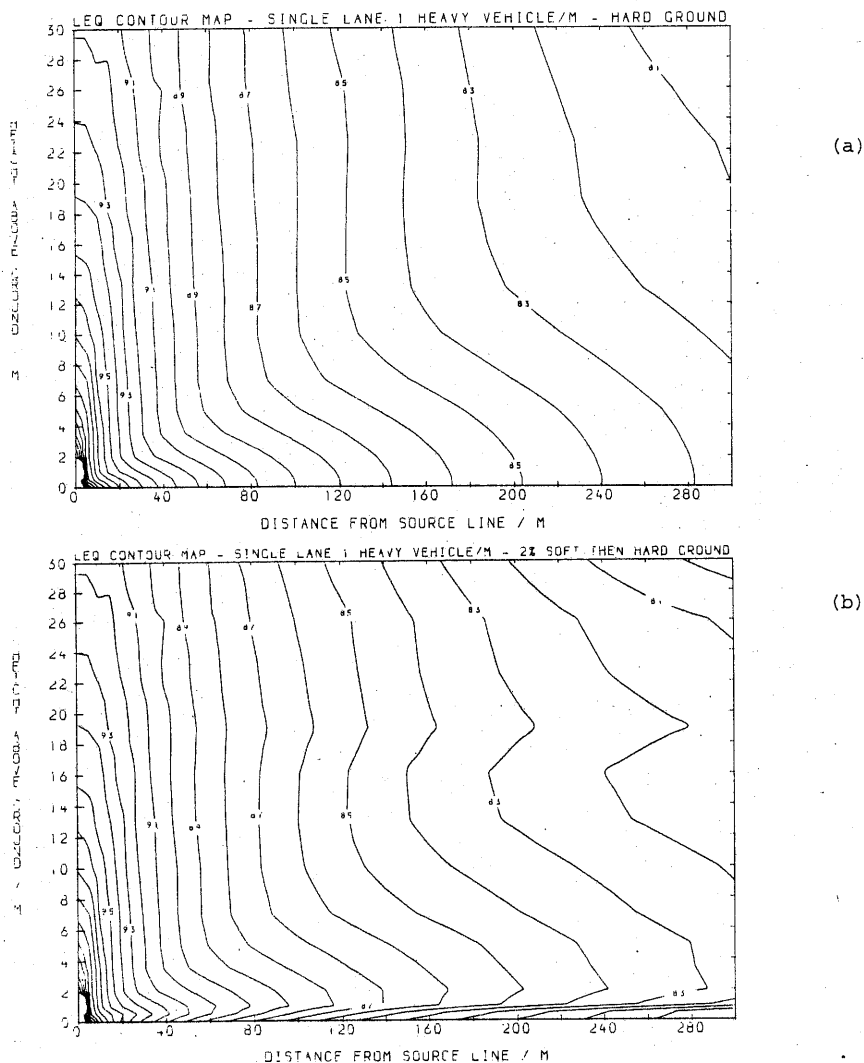


Figure 4.  $L_{eq}$  contour maps for propagation of traffic noise over level terrain. (a) hard ground, (b) half hard ground, half grassland with dividing line parallel to the road, and the grassland under the receiver and under the last 2% of the propagation path.