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APPLICATION OF GEOMETRIC CONSTRAINTS TO LOCATING SOURCES USING TDD INFORMATION

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INTRODUCTION

An important method of locating signal sources involves the estimation of Time Delay Differences (TDDs) for a signal received at pairs of spatially separated sites. Most related research has emphasized the characterization and optimization of TDD estimation in the context of two receivers. Locating a source in multidimensional space requires more than two receivers, and TDD estimates will be correlated whenever there is a receiver common to two or more receiver pairs. When characterizing the performance of such systems it is important to incorporate the covariance between the TDD estimates. This is especially true when the linear constraints implicit in the source/receiver geometry are used. Their explicit incorporation in TDD estimation involves the enforcement of the condition that the three TDDs obtained for each receiver triplet sum to zero [1, 2]. This procedure, aside from reducing the TDD variances, can simplify the geometrical portion of the problem.

This paper develops expressions for the covariance between TDD estimates and applies these to realize a more general model for characterizing performance.

TDD VARIANCES AND COVARIANCES

The concept of the generalized correlation method for TDD estimation was introduced by Knapp and Carter [3]. It was shown that many delay estimation algorithms can be thought of as different prefiltering strategies for the two signals to be cross-correlated. It was also shown that maximum likelihood (ML) prefiltering can be used if Gaussian statistics are assumed. By rearranging the result given in [3], the variance of a TDD estimate, $\tilde{D}_i$, corresponding to the i-th site pair (k,m), for stationary real signals is given by

$$\mathrm{Var}[\tilde{D}_i] = \frac{\int_0^\infty (2\pi f)^2\, W_i^2(f) U_i(f)\, df}{2T[\int_0^\infty (2\pi f)^2\, W_i(f) G_s(f)\, df]^2} \qquad (1)$$

where T is the observation time, $G_s(f)$ is the power spectrum of the signal, $W_i(f)$ is the real cross-spectrum weighting function for the i-th site pair, and

$$U_i(f) \triangleq G_s(f)G_k(f) + G_s(f)G_m(f) + G_k(f)G_m(f) \qquad (2)$$

where $G_k(f)$ and $G_m(f)$ are the noise power spectrums at sites k and m respectively. When ML cross-spectrum weighting, given by

$$W_i(f) = G_s(f)/U_i(f) \qquad (3)$$

is used, then (1) reduces to

$$\mathrm{Var}[\tilde{D}_i]_{ML} = \left[2T \int_0^\infty (2\pi f)^2\, \psi_i(f)\, df\right]^{-1} \qquad (4)$$

where

$$\psi_i(f) \triangleq G_s^2(f)/U_i(f) \qquad (5)$$

This $\psi_i(f)$ function can also be computed as

$$\psi_i(f) = C_i(f)/(1-C_i(f)) \quad (6)$$

where $C_i(f)$ is the magnitude squared coherence (MSC) function, and is easily estimated using standard FFT techniques [4]. It was also shown in [3] that (4) is the Cramer-Rao lower bound (CRLB) for the variance of a TDD estimator. Thus the generalized real cross-correlator can achieve optimum performance for real signals. In practice the performances indicated by (1) and (4) hold true for real baseband signals but not for real RF signals, because of peak ambiguities and decorrelation in the RF phase from site to site. If the RF signals are assumed to have fixed but unknown and uniformly random relative phases then the ML TDD estimator locates the peak of the magnitude of the complex cross-correlation between the complex baseband equivalent RF signals. Using a derivation very similar to that in the Appendix, it can be shown that a reasonable approximation to the variance of a TDD estimate involving RF signals is given by [5]

$$\mathrm{Var}[\tilde{D}_i] = \frac{\int_{-\infty}^{\infty} [2\pi(f-\bar{f}_i)]^2 W_i^2(f) U_i(f)\, df}{2T[\int_{-\infty}^{\infty} [2\pi(f-\bar{f}_i)]^2 W_i(f) G_s(f)\, df]^2} \quad (7)$$

with

$$\bar{f}_i = [\int_{-\infty}^{\infty} f\, W_i(f) G_s(f)\, df]/[\int_{-\infty}^{\infty} W_i(f) G_s(f)\, df] \quad (8)$$

where T is the observation time, $G_s(f)$ is the power spectrum of the complex baseband equivalent RF signal, $W_i(f)$ is the real cross-spectrum weighting function for the i-th site pair, and $U_i(f)$ is as defined previously in (2) with the noise power spectrums also being complex baseband equivalents.
With ML cross-spectrum weighting, given by (3), equation (7) reduces to

$$\mathrm{Var}[\tilde{D}_i]_{ML} = [2T \int_{-\infty}^{\infty} [2\pi(f-\bar{f}_i)]^2 \psi_i(f)\, df]^{-1} \quad (9)$$

with

$$\bar{f}_i = [\int_{-\infty}^{\infty} f\, \psi_i(f)\, df]/[\int_{-\infty}^{\infty} \psi_i(f)\, df] \quad (10)$$

where $\psi_i(f)$ is as defined in (5). Notice the similarities between results (7) and (9) for complex baseband equivalent RF signals, and results (1) and (4) for real baseband signals. The major differences in the formulas for RF signals are that the integration is now two-sided and the frequency weighting is shifted to the centroid of the signal power spectrum. The CRLB for RF signals is also known [6, 7], and is exactly that given in (9) and (10). Thus the generalized complex cross-correlator can also achieve optimum performance .

In order to properly model a TDD covariance matrix we must also know the covariance between TDD estimates. The covariance between two TDD estimates, $\tilde{D}_i$ and $\tilde{D}_j$, for site pairs (k,m) and (k,n) respectively, has been derived in the Appendix for RF signals, and is given by

$$\mathrm{Cov}[\tilde{D}_i,\tilde{D}_j] = \frac{\int_{-\infty}^{\infty} (2\pi)^2 (f-\bar{f}_i)(f-\bar{f}_j) W_i(f) W_j(f) G_S(f) G_k(f) df}{2T\{\int_{-\infty}^{\infty} [2\pi(f-\bar{f}_i)]^2 W_i(f) G_S(f) df\}\{\int_{-\infty}^{\infty} [2\pi(f-\bar{f}_j)]^2 W_j(f) G_S(f) df\}} \quad (11)$$

where $\bar{f}_i$ and $\bar{f}_j$ are as defined in (8), $G_s(f)$ is again the power spectrum of the complex baseband equivalent RF signal, $W_i(f)$ and $W_j(f)$ are the cross-spectrum weighting functions for the i and j site pairs respectively, and $G_k(f)$ is the power spectrum of the complex baseband equivalent RF noise at the k-th site. Note that the k-th site is the shared site and is the first site in both pairs. If the shared site is first in one pair and second in the other pair then the only effect is to make the covariance negative. It has been assumed that the noises at each site are uncorrelated, in which case, if there are no shared sites, the covariance is zero. This result assumes arbitrary cross-spectrum weighting and thus also holds when ML weighting is used for each TDD estimate. The covariance formula for real baseband signals is essentially the same with $\bar{f}_i$ and $\bar{f}_j$ equal to zero, and the integrals going from zero to infinity.

As an example, and also as a model for the results presented in the following sections, if we assume white signal and noise RF processes with ML cross-spectrum weighting (i.e. constant over the signal bandwidth) then the variances of, and covariance between, two delay estimates, $\tilde{D}_i$ and $\tilde{D}_j$, corresponding to site pairs (k,m) and (k,n) respectively, evaluate to

$$\mathrm{Var}[\tilde{D}_i] = \beta[\gamma_k + \gamma_m + \gamma_k\gamma_m] \triangleq \beta\gamma_{km} \tag{12}$$

$$\mathrm{Var}[\tilde{D}_j] = \beta[\gamma_k + \gamma_n + \gamma_k\gamma_n] \triangleq \beta\gamma_{kn} \tag{13}$$

$$\mathrm{Cov}[\tilde{D}_i,\tilde{D}_j] = \beta\gamma_k \tag{14}$$

where γ_k, γ_m, and γ_n are the noise-to-signal power ratios (i.e. 1/SNR) at sites k, m, and n respectively, and the scale factor, β, is given by

$$\beta = 3/(2\pi^2 TB^3) \tag{15}$$

where T is the observation time, and B is the RF signal bandwidth. Note that the variance expressions in (12) and (13) are equal to the CRLB for this model.

TDD VARIANCE REDUCTION WITH CONSTRAINED ESTIMATION

When estimating the TDDs it is useful to incorporate the constraint, based on the distance difference geometry, that the 3 TDDs, for each set of 3 receivers, sum to zero (with appropriate signing). First, consider the simple case of only 3 receivers, 1, 2, and 3, with 3 receiver pairs (1,2), (1,3), and (2,3), which we will denote as pairs 1, 2, and 3 respectively. Then the following single constraint applies: $\tilde{D}_1 - \tilde{D}_2 + \tilde{D}_3 = 0$. This constraint can be expressed in matrix form as $R\tilde{D}=0$, where $\tilde{D}$ is the column vector of 3 delay estimates, and $R=[1\ -1\ 1]$. With 4 receivers, 1, 2, 3, and 4, there are 6 pairs, (1,2), (1,3), (1,4), (2,3), (2,4), and (3,4), which we will denote as pairs 1 through 6 respectively. For this case there are 3 independent constraint equations, which again can be expressed in matrix form as

$$R\tilde{D} = 0, \text{ with } R = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

Similarly, with 5 receivers there are 10 possible TDDs with 6 independent constraint equations.

The formulation and solution to constrained least squares estimation is well known for spherical (or uniformly orthogonal) perturbations [8]. These results can be readily extended to generalized constrained least squares estimation [2], and when the constraint is of the null form of (16), the vector of constrained TDDs and corresponding covariance matrix are given by

$$\tilde{D}^c = \tilde{D} - G\tilde{D} \qquad (17)$$

$$C^c = C - GC \qquad (18)$$

with

$$G \triangleq CR^t(RCR^t)^{-1} R \qquad (19)$$

where the superscripts "c", "t", and "-1" denote constrained estimates, matrix transpose, and matrix inverse respectively. Note that the constrained TDDs and corresponding covariance matrix are expressed in terms of the unconstrained TDDs and corresponding covariance matrix minus a correction term. Assuming white signal and noise spectrums over RF bandwidth B, the unconstrained covariance matrix can be easily constructed using equations (12) through (15). For the 3 receiver example above, the unconstrained covariance matrix is given by

$$C = \beta \begin{bmatrix} \gamma_{12} & \gamma_1 & -\gamma_2 \\ \gamma_1 & \gamma_{13} & \gamma_3 \\ -\gamma_2 & \gamma_3 & \gamma_{23} \end{bmatrix} \qquad (20)$$

The unconstrained covariance matrix can be similarly constructed for an arbitrary number, K, receivers. One measure of the improvement with constrained estimation is the reduction in the variance of the TDD estimates. If we assume that the signal-to-noise ratio (SNR) is the same at all K receivers (that is γ_k=1/SNR and $\gamma_{km}=2/SNR+1/SNR^2$ for all k and m, k≠m), then the variance reduction factor (VRF) is the same for all K-choose-2 TDDs, and is given by

$$VRF_K = \frac{2K\ SNR + 2}{2K\ SNR + K} \qquad (21)$$

This formula has been algebraically confirmed for K=2, 3, 4, and 5 receivers, and is conjectured to hold in general for K≥2 receivers.

Figure 1 plots the VRF for SNRs in the range -20 dB to +20 dB, and for K=2, 3, 4, 5, and 6 receivers. As expected the variance reduction is seen to be quite small for good SNRs, but can be significant at low SNRs. A lower bound on the VRF is 2/K and is obtained only for independent TDD estimates.

GEOMETRY IMPLICATIONS

A reduction in the TDD variances does not translate directly into a proportional reduction in the location error since the entire covariance matrix is not reduced by the same proportion as the diagonal elements. Further, in practice the SNRs will depend on the transmitter location relative to each intercept site. Thus, the improvement with constrained estimation is strongly geometry dependent.

In [9] it is shown that the NxN location covariance matrix, C_x, for a given location estimate in N-space, can be linearly approximated by

$$C_x = (G^t C^{-1} G)^{-1}, \quad \text{with } G \triangleq \frac{1}{c} HF \qquad (22)$$

where C is the IxI TDD covariance matrix for the subset of I site pairs selected, and c is the signal propagation speed. The IxK H matrix defines the site pairs used, where K is the number of sites. The i-th row of H, corresponding to the i-th site pair (k,m), contains a 1 in the k-th position, a -1 in the m-th position, and zeros elsewhere. The k-th row of the KxN F matrix is a unit vector pointing in the direction of the transmitter from the k-th site. The root-mean-square (RMS) location error is then given by

$$\sigma_e = \left(\mathrm{trace}(C_x)\right)^{\frac{1}{2}} \tag{23}$$

The noise-to-signal ratio at site k, γ_k, required for constructing the TDD covariance matrix, C, can be modelled using

$$\gamma_k = (d_k)^4 / SNR_o \tag{24}$$

where d_k is the distance from the transmitter to the k-th site, and SNR_o is the reference SNR for a reference distance of d_k=1. This 4-th power propagation model is often a good approximation for VHF ground communications.

The RMS location error performance is presented in Figures 2 and 3 for two different geometries and SNR_o = 5 dB. The approach taken is to plot contours of fixed relative RMS location errors in the x-y plane. The relative RMS location error is defined as $\sigma_r = \sigma_e/\sigma_o$, where σ_o is the fundamental ranging error for the reference distance d_k=1, and is given by $\sigma_o^2 = c^2\beta/SNR_o$. Figure 2 presents the results for 3 sites, 1, 2, and 3, located at (0,1), (-.866,-0.5), and (.866,-0.5) respectively. In both (a) and (b) only site pairs (1,2) and (1,3) were employed. Figure 3 presents the results for 4 sites, 1, 2, 3, and 4, located at (0,1), (-1,0), (0,-1), and (1,0) respectively. In both (a) and (b) only site pairs (1,2), (1,3), and (1,4) were employed. These figures demonstrate a reduction in the RMS location error as shown by the expanded contours when constrained estimation, using all the TDD s, is performed prior to solving for the transmitter location. For many locations the RMS location error is reduced by more than a factor of 2, which is equivalent to increasing the observation time by more than a factor of 4. It should be noted that the performance in (a), without constrained estimation, could have been improved by employing more than just K-1 site pairs. However, this would increase the dimension of the matrices involved in the geometry portion of the solution, and could introduce a singular TDD covariance matrix, C. This is especially true if some of the sites have high SNRs. The constrained estimation approach does not require that the C matrix be non-singular, and thus does not suffer this problem.

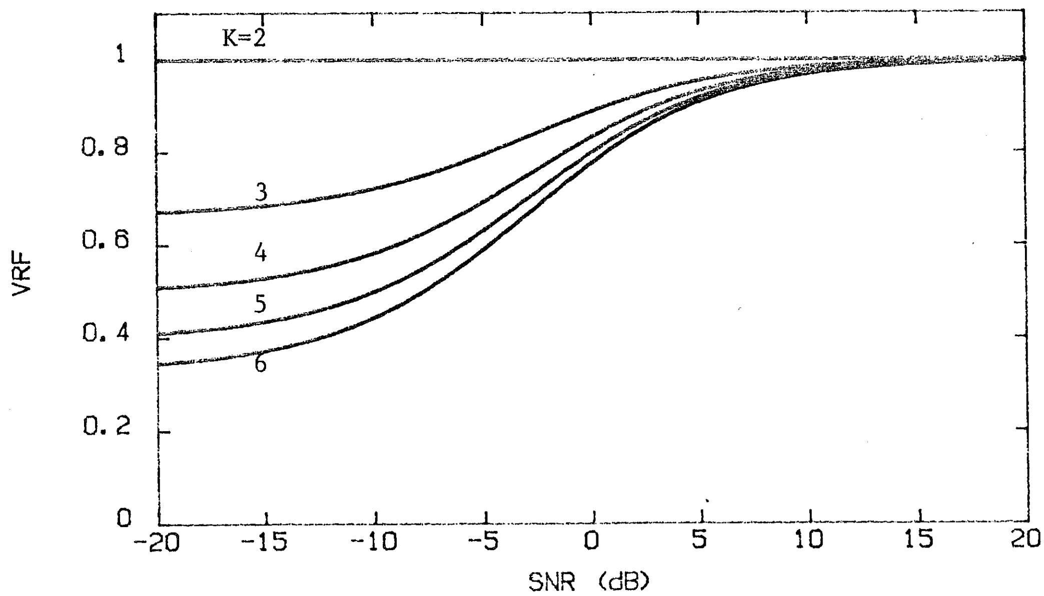
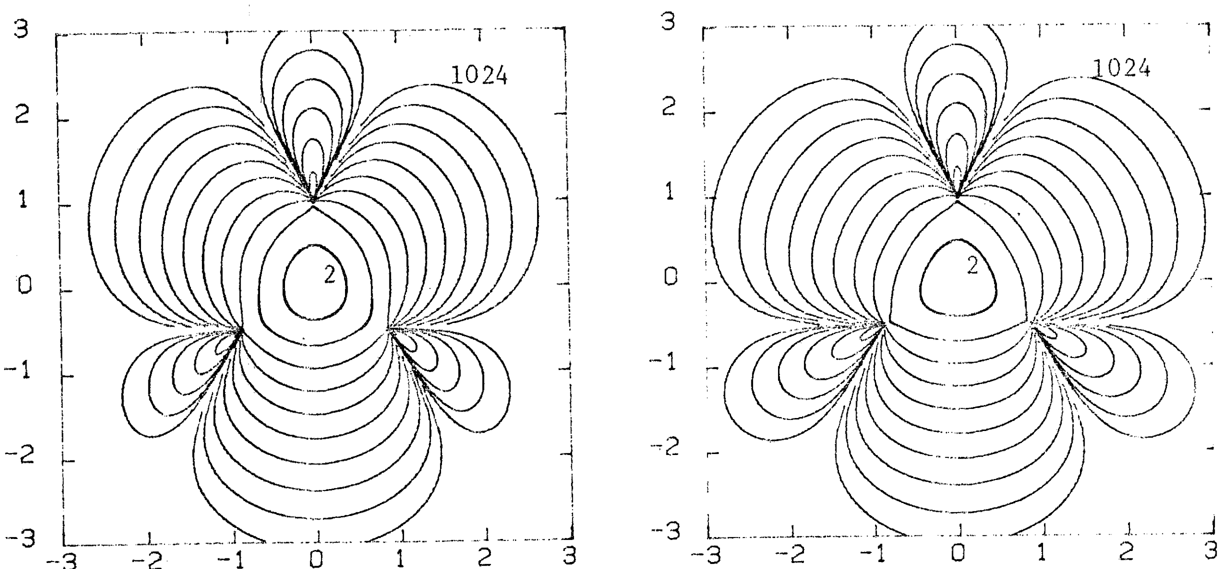
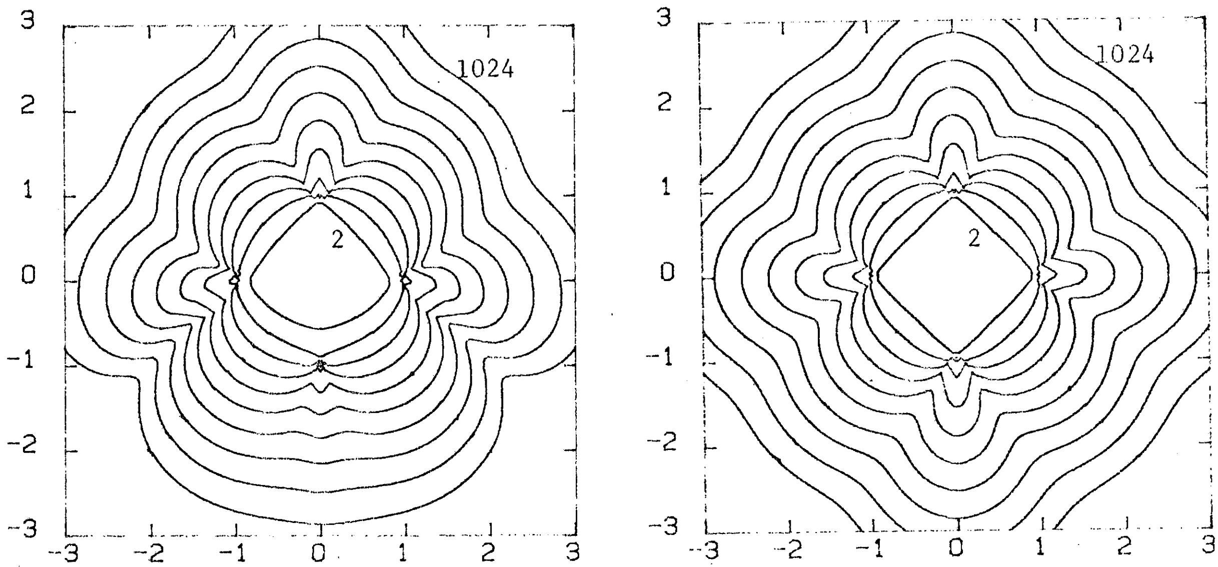


Figure 1: TDD Variance Reduction Factor Versus SNR for K Sites.



(a) Without Constrained Estimation (b) With Constrained Estimation

Figure 2: Constant RMS Location Error Contours for a 3-Site System (SNR_0=5 dB)



(a) Without Constrained Estimation (b) With Constrained Estimation

Figure 3: Constant RMS Location Error Contours for a 4-Site System (SNR_0=5 dB)

APPENDIX - COVARIANCE BETWEEN TWO TDD ESTIMATES

For complex signals with fixed and uniformly random relative phases, the TDD is estimated by locating the peak of the magnitude of the generalized complex cross-correlator output, $\tilde{R}(\tau)$, where the ~ denotes a noisy estimate. Assume that we have estimators 1 and 2 which correspond to site pairs (k,m) and (k,n) respectively. That is, the 2 estimators share the signal from receiver k. Using the discrete Fourier coefficient representation,

$$\tilde{R}_i(\tau) = \sum_{n=-N}^{N} W_i(n)\tilde{G}_i(n)\exp(j2\pi n\Delta\tau)\,\Delta, \quad i=1,2 \qquad (A.1)$$

where $\Delta = 1/T$ is the fundamental frequency, T is the observation time, $W_i(n)$ is the real cross-spectrum weighting for the i-th receiver pair, and $\tilde{G}_i(n)$ is the computed cross-spectrum for the signals received at the i-th receiver pair. Locating the peak of $|\tilde{R}(\tau)|$ is equivalent to locating the root of the first derivative of $|\tilde{R}(\tau)|$ near the peak. If excursions of $\frac{d}{d\tau}|\tilde{R}(\tau)|$ from $\frac{d}{d\tau}E[|\tilde{R}(\tau)|]$ at $\tau = \tilde{D}$ are almost entirely confined to the linear region of $\frac{d}{d\tau}E[|\tilde{R}(\tau)|]$ near $\tau = D$, then the covariance between TDD estimates 1 and 2 can be approximated by

$$\mathrm{Cov}[\tilde{D}_1,\tilde{D}_2] = \left.\frac{E[(\frac{d}{d\tau_1}|\tilde{R}_1(\tau_1)| - \frac{d}{d\tau_1}E[|\tilde{R}_1(\tau_1)|])(\frac{d}{d\tau_2}|\tilde{R}_2(\tau_2)| - \frac{d}{d\tau_2}E[|\tilde{R}_2(\tau_2)|])]}{(\frac{d^2}{d\tau_1^2}E[|\tilde{R}_1(\tau_1)|])(\frac{d^2}{d\tau_2^2}E[|\tilde{R}_2(\tau_2)|])}\right|_{\substack{\tau_1=D_1\\ \tau_2=D_2}} \quad (A.2)$$

Here we introduce the approximation

$$|\tilde{R}_i(\tau)| \simeq \mathrm{Re}[\tilde{R}_i(\tau)\exp(-j\phi_i(\tau))] \triangleq A_i, \quad i = 1,2 \quad (A.3)$$

Equation (A.3) is accurate to within a factor of $\cos(\tilde{\phi}_i(\tau) - \phi_i(\tau))$ where $\tilde{\phi}_i(\tau)$ and $\phi_i(\tau)$ are the phases of $\tilde{R}_i(\tau)$ and $R_i(\tau)$ respectively. Clearly, for good SNRs and/or long observation times, $\tilde{\phi}_i(\tau)$ will be close to $\phi_i(\tau)$ and approximation (A.3) should be quite valid. Employing approximation (A.3), (A.2) becomes

$$\mathrm{Cov}[\tilde{D}_1,\tilde{D}_2] = \left.\frac{E[(\frac{d}{d\tau_1}A_1 - \frac{d}{d\tau_1}E[A_1])(\frac{d}{d\tau_2}A_2 - \frac{d}{d\tau_2}E[A_2])]}{(\frac{d^2}{d\tau_1^2}E[A_1])(\frac{d^2}{d\tau_2^2}E[A_2])}\right|_{\substack{\tau_1=D_1\\ \tau_2=D_2}} \quad (A.4)$$

Letting superscript (n) denote the n-th derivative we have

$$\frac{d}{d\tau}A_i = \mathrm{Re}[\tilde{R}_i^{(1)}(\tau)\exp(-j\phi_i(\tau)) + \tilde{R}_i(\tau)\exp(-j\phi_i(\tau))(-j)\phi_i^{(1)}(\tau)] \quad (A.5)$$

$$\frac{d}{d\tau}E[A_i] = \mathrm{Re}[R_i^{(1)}(\tau)\exp(-j\phi_i(\tau)) + R_i(\tau)\exp(-j\phi_i(\tau))(-j)\phi_i^{(1)}(\tau)] \quad (A.6)$$

Substituting (A.5) and (A.6) in (A.4) we obtain the following expressions for the numerator

$$\mathrm{num} = E[\mathrm{Re}[B_1+C_1] \times \mathrm{Re}[B_2+C_2]] = \frac{1}{4}E[(B_1+B_1^*+C_1+C_1^*)(B_2+B_2^*+C_2+C_2^*)] \quad (A.7)$$

with

$$B_i \triangleq (\tilde{R}_i^{(1)}(\tau) - R_i^{(1)}(\tau))\exp(-j\phi_i(\tau)), \quad i=1,2 \quad (A.8)$$

$$C_i \triangleq (\tilde{R}_i(\tau) - R_i(\tau))\exp(-j\phi_i(\tau))(-j)\phi_i^{(1)}(\tau)), \quad i=1,2 \quad (A.9)$$

Multiplying out the cross terms in (A.7), substituting (A.1) in (A.8) and (A.9), and taking expected values, we find that $E[B_1B_2]$, $E[B_1C_2]$, $E[B_1^*B_2^*]$, $E[B_1^*C_2^*]$, $E[C_1B_2]$, $E[C_1C_2]$, $E[C_1^*B_2^*]$, and $E[C_1^*C_2^*]$ are all zero. For $E[B_1B_2^*]$ we obtain, after considerable algebra,

$$E[B_1B_2^*] = \sum_n W_1(n)W_2(n)(2\pi\Delta n)^2 G_s(n)G_k(n)\frac{1}{T^2} \quad (A.10)$$

where $G_k(n)$ is the noise power spectrum for site k. For T large ($\Delta = 1/T$ small), equation (A.10) becomes

$$E[B_1B_2^*] = \frac{1}{T}\int_{-\infty}^{\infty}(2\pi f)^2 W_1(f)W_2(f)G_s(f)G_k(f)\,df \quad (A.11)$$

with the obvious discrete to continuous correspondences. Similarly, we obtain (evaluated at $\tau_1 = D_1$ and $\tau_2 = D_2$)

$$E[C_1C_2^*] = \phi_1^{(1)}(D_1)\phi_2^{(1)}(D_2)\,\frac{1}{T}\int_{-\infty}^{\infty} W_1(f)W_2(f)G_s(f)G_k(f)\,df \qquad \text{(A.12)}$$

$$E[B_1C_2^*] = E[B_1^*C_2] = -\phi_2^{(1)}(D_2)\,\frac{1}{T}\int_{-\infty}^{\infty} (2\pi f)W_1(f)W_2(f)G_s(f)G_k(f)\,df \qquad \text{(A.13)}$$

and

$$E[C_1B_2^*] = E[C_1^*B_2] = -\phi_1^{(1)}(D_1)\,\frac{1}{T}\int_{-\infty}^{\infty} (2\pi f)W_1(f)W_2(f)G_s(f)G_k(f)\,df \qquad \text{(A.14)}$$

It is easily shown that [5]

$$\phi_i^{(1)}(D_i) \triangleq \frac{d}{d\tau}\phi_i(\tau)\Big|_{\tau=D_i} = 2\pi\bar{f}_i \quad , \; i=1,2 \qquad \text{(A.15)}$$

where $\bar{f}_i$ is the average frequency of the i-th weighted signal power spectrum, as defined in equation (8) of the main text. Substituting these results in (A.7) gives the following expression for the numerator of (A.4).

$$\text{num} = \frac{1}{2T}\int_{-\infty}^{\infty} (2\pi)^2(f-\bar{f}_1)(f-\bar{f}_2)W_1(f)W_2(f)G_s(f)G_k(f)\,df \qquad \text{(A.16)}$$

The denominator of (A.4) is straightforward. The required derivative formula is

$$\frac{d^2}{d\tau_i^2}E[A_i]\Big|_{\tau_i=D_i} = -\int_{-\infty}^{\infty}[2\pi(f-\bar{f}_i)]^2 W_i(f)G_s(f)\,df \quad , \; i=1,2 \qquad \text{(A.17)}$$

Substituting (A.16) and (A.17) into (A.4) yields the covariance as given in equation (11) of the main text.

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