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DIGITAL MODELLING OF ROOM ACOUSTIC FREQUENCY RESPONSE FUNCTIONS

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1. INTRODUCTION

The efficient representation of room acoustic frequency response functions can be difficult in the frequency range where too many modes are excited to be accounted for individually, but they are not excited in sufficient number to enable the application of statistical methods. However in this "mid frequency" range, there is a tendency for "bunches" of adjacent individual room resonances to form reasonably well defined "clumps" of modes, with each clump exhibiting the characteristics of an individual second-order system [1]. This paper describes a preliminary investigation of the possibility of exploiting this feature in providing a computationally efficient representation of room acoustic frequency response functions. Work has been undertaken in order to establish the extent to which a multi-degree of freedom system may be adequately described using a model of significantly lower order. This lower order model has thus been chosen in accordance with the number of "clumps" of modes in a given frequency range, rather than in accordance with the total number of individual room modes. The work presented below firstly describes the theoretical background to the approach used. This involves the representation of the reduced order model with an IIR (infinite impulse response) digital filter whose coefficients are determined to give a "best fit" (in the least squares sense) to a measured output time sequence. Experiments are described in which the frequency response function (relating "output" pressure to "input" source volume velocity) in a reverberant room is measured accurately using conventional transform methods. These results are compared to the frequency response of the reduced order model derived from a considerably smaller number of samples of the input and output time histories. The results are sufficiently encouraging to indicate that a computationally efficient representation of the room acoustic frequency response function may well be feasible. A practical means of "system identification" of this type will very often be a prerequisite for any attempt to control or influence the nature of an enclosed sound field using digitally based electronic techniques.

2. THEORETICAL BACKGROUND

The complex pressure $p(\underline{x},\omega)$ at a position $\underline{x}$ in a lightly damped enclosed sound field is produced by a point source of strength $q(\underline{y},\omega)e^{j\omega t}$ at a position $\underline{y}$. The solution for the pressure can be expressed in terms of a spatially dependent "transfer function" such that

$$p(\underline{x},\omega) = H(\underline{x},\underline{y},\omega)q(\underline{y},\omega) \tag{1}$$

where the theory presented by both Pierce [2] and Morse [3] shows that the form of this transfer function can be written as

$$H(\underline{x},\underline{y},\omega) = \sum_{n=1}^{N} \frac{A_n\omega}{2\zeta_n\omega_n\omega - j(\omega_n^2 - \omega)} \qquad (2)$$

where $A_n = (\rho_o c_o^2/V)\psi_n(\underline{x})\psi_n(\underline{y})$. Thus, ω_n, ζ_n are the natural frequency and damping of the second order system representing the response of each mode having a normalised characteristic shape function ψ_n. The density and sound speed of the medium are given by ρ_o, c_o and V is the volume of the enclosure. Thus the room response is considered to consist of a superposition of N second order responses. The corresponding impulse response can thus be written as

$$h(\underline{x},\underline{y},t) = \sum_{n=1}^{N} \left[c_n e^{-d_n t} + c_n^* e^{-d_n^* t}\right] \qquad (3)$$

where the complex coefficients c_n, d_n can be deduced in terms of ω_n, ζ_n and A_n for each mode in the series for given values of $\underline{x}$ and $\underline{y}$.

A digital representation of this impulse response can be deduced using the principle of the impulse invariant transformation (see Rabiner and Gold [4]). Thus if the equivalent continuous impulse response (3) is sampled at integer (k) multiples of the sampling period T, then one can write

$$h(\underline{x},\underline{y},kT) = \sum_{n=1}^{N} \left[c_n e^{-d_n kT} + c_n^* e^{-d_n^* kT}\right] \qquad (4)$$

The corresponding z-transform of this sequence is given by

$$H(\underline{x},\underline{y},z) = \sum_{k=o}^{\infty} \sum_{n=1}^{N} \left[c_n e^{-d_n kT} + c_n^* e^{-d_n^* kT}\right] z^{-k} \qquad (5)$$

and since the order of the summations can be interchanged and assuming a stable impulse response, evaluation of the z-transform (see reference [4]) results in

$$H(\underline{x},\underline{y},z) = \sum_{n=1}^{N} \frac{b_{no} + b_{n1} z^{-1}}{1 + a_{n1} z^{-1} + a_{n2} z^{-2}} \qquad (6)$$

where the coefficients b_{no}, b_{n1}, a_{n1}, a_{n2} are purely real numbers which can again be expressed in terms of ω_n, ζ_n and A_n for each mode in the series for given values of $\underline{x}$ and $\underline{y}$. The summation (6) can be expressed in the still more general form

$$H(\underline{x},\underline{y},z) = \frac{b_o + b_1 z^{-1} + b_2 z^{-2} \ \ldots \ b(2N-1) z^{-(2N-1)}}{1 + a_1 z^{-1} + a_2 z^{-2} \ \ldots \ a_{2N} z^{-2N}} \qquad (7)$$

where the real coefficients ($b_o \ldots b_{2N-1}, a_1 \ldots a_{2N}$), are now given in terms of combinations of ω_n, ζ_n, A_n of the N modes in the series. These coefficients will thus again also depend on $\underline{x}$ and $\underline{y}$. This therefore constitutes a (2N,2N-1) order ARMA (autoregressive moving average) model which is the natural counterpart to the continuous time model consisting of N parallel second order systems. The equivalent sampled pressure signal p(kT) and volume velocity signal q(kT) are related by the corresponding difference equation.

3. THE REDUCED ORDER MODEL

The purpose of this work is to achieve a good representation of the actual sampled pressure signal $p_a(kT)$ when these values and the actual sampled volume velocity signal $q_a(kT)$ are used to deduce the "output" $p_m(kT)$ from an ARMA model of the actual system. Furthermore the order, 2M say, of this model is to be determined by the number M of "clumps" of modes rather than the total number N of individual modes. Thus the model output sequence $p_m(kT)$ is deduced from the actual output and input sequences via the difference equation

$$p_m(kT) = -a_1 p_a((k-1)T) - a_2 p_a((k-2)T) \ldots a_{2M} p_a((k-2M)T)$$
$$+ b_1 q_a((k-1)T) + b_2 q_a((k-2)T) \ldots b_{2M-1} q_a((k-2M+1)T) \qquad (8)$$

where the coefficients of this ARMA model can be specified by the parameter vector

$$\underline{a} = \left[a_1, a_2, \ldots a_{2M}, b_1, b_2 \ldots b_{2M-1}\right]^T$$

Now note that the error $\varepsilon(kT)$ between the actual sampled pressure signal and the model output can be written as

$$\varepsilon(kT) = p_a(kT) - p_m(kT) = p_a(kT) - \underline{z}^T(kT)\underline{a} \qquad (9)$$

where the vector $\underline{z}(kT)$ is given by

$$\underline{z}(kT) = \left[-p_a((k-1)T) \ldots -p_a((k-2M)T),\ q_a((k-1)T) \ldots q_a((k-2M)T)\right]^T$$

This error can be evaluated over L samples of the actual and model output pressure signal such that the L'th order error vector is given by

$$\underline{\varepsilon} = \underline{p}_a - \underline{Z}\underline{a} \qquad (10)$$

where $\quad \varepsilon = \left[\varepsilon(kT), \varepsilon((k+1)T) \ldots \varepsilon((k+L)T)\right]^T,$

$$\underline{Z} = \left[\underline{z}(kT), \underline{z}((k+1)T) \ldots \underline{z}((k+L)T)\right]^T,$$

and $\quad \underline{p}_a = \left[p_a(kT), p_a((k+1)T) \ldots p_a((k+L)T)\right]^T.$

We now choose $\underline{a}$ in order to minimise the sum of the squared errors given by $\underline{\varepsilon}^T\underline{\varepsilon}$. The solution which minimises this quadratic function of the vector $\underline{a}$

follows from equation (10) and is given by

$$\underline{a}_o = [\underline{Z}^T\underline{Z}]^{-1}\underline{Z}^T\underline{p}_a \qquad (11)$$

Clearly the deduction of $\underline{a}_o$ requires the inversion of a matrix of potentially very high order. However, the form of the matrix $\underline{Z}^T\underline{Z}$ is such that the optimal vector $\underline{a}_o$ can be deduced recursively (at successive time samples) using the "Recursive least squares" (RLS) algorithm [5]. This approach was taken in the work described below using sampled data derived from signals proportional to the pressure and volume velocity when a reverberant room was excited by a loudspeaker driven by a white noise input. The equivalent frequency response function (in modulus and phase) associated with the model described by the parameter vector $\underline{a}_o$ was then compared with the "actual" frequency response function measured using conventional transform techniques on a considerably larger number of data samples.

4. EXPERIMENTAL RESULTS

The initial choice of model order was found to be very important in this work, as two model coefficients are necessary and sufficient to describe one second order system (e.g. mode, or hypothetically, clump). For the experiment, a model order of M=8 was chosen as being sufficiently high to have a chance of matching system behaviour yet sufficiently low to be readily computed.

The input signal was the output of a laser vibrometer which measured the surface velocity of an eight inch speaker excited with white noise. The room response was measured with a microphone near the opposite corner of the room. Both signals were passed through a digital antialiasing filter before being sampled, initially at 2000 Hz. The reference transfer function used was that found by averaging 500 840-point Discrete Fourier Transforms. As shown in Figure 1, for low frequencies and modal densities, the model can be a reasonably good match, only failing noticeably in the noise below 60 Hz. This model was produced by filtering to cut off frequencies above 200 Hz and resampling at 400 Hz. Note that this model was produced using the RLS algorithm operating on only 1600 data samples (a factor of over 250 smaller than the number of samples used in deducing the reference transfer function). There is certain to be aliasing at such a low sample rate, but where model poles and zeros are so few it is better to ensure that they are all in the range of interest.

Of more note is Figure 2, which shows results in the middle frequency range containing distinct mode "clumps". Despite containing more resonances, the 400-450 Hz band shown here appears smoother and is better matched by the ARMA model than in the 0-200 Hz band. The phase response does not at first seem as promising as that of the modulus, but if the considerable accumulated 0-400 Hz system phase trend is compensated (as shown in the figure) a reasonably good match results. The progressive phase trend of the model over this frequency range "falls behind" the actual system phase by 4π radians. This however, has resulted from a model having 8 modes being used to match a system having 52 modes over this frequency range. Of importance is that the system contains roughly 10 to 12 "clumps" of modes. Other preliminary results have indicated that provided the model order matches the number of clumps, then the phase trend of the model over a given frequency range is very similar to that of the actual system. To achieve the results of Figure 2, some re-sampling of the data was required. Faced with the same need (as in the lowpass case) to concentrate the model on the range of interest, the small undersampling used before is ineffective in the 0-400 Hz region. Sampling at only 100 Hz, however, produces a simple spectrum mapping (reference [6] discusses theory and details) which overlaps the range of interest and the (0-½ sampling frequency) model range using multiple aliases. The resulting model used 400 data points, over a 1000-fold reduction from the reference transfer function.

5. CONCLUSIONS

The work to date indicates that a model of significantly lower order can produce a reasonable approximation of system behaviour. The explanation of why clumps behave much like individual resonances is not altogether clear, though Lyon's work [7] on phase trends in multi degree of freedom systems seems relevant in view of the slower phase progression at clumps, where the individual modes appear to be reinforcing their neighbours. Certainly for the purposes of implementing active noise control for example, using digital filters, the prospect of using lower-order filters whose coefficients may be determined quickly and from a small number of data points looks reasonably promising. Work planned for the near future includes incorporation of a progressive phase delay into the model, in an attempt to model more nearly the rapid system phase progression, particularly at low frequencies. Whether the problems of using what is known to be an approximate method can be overcome sufficiently to allow practical application remains to be shown.

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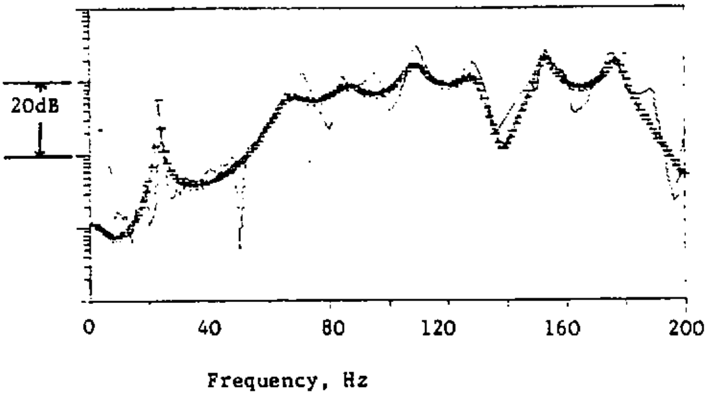

Fig 1(a)
Moduli of moder (+++) and reference (—) transfer functions.

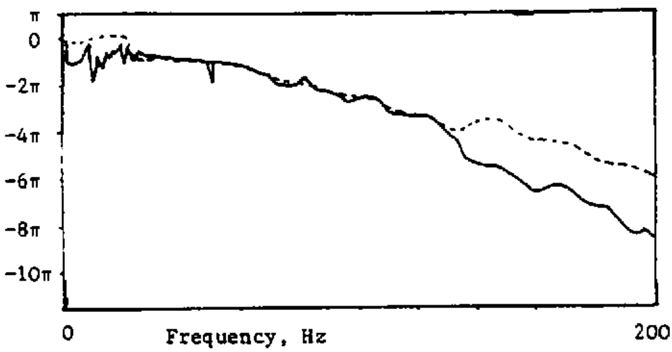

Fig 1(b)
Phase of model (---) and reference (—) transfer functions.

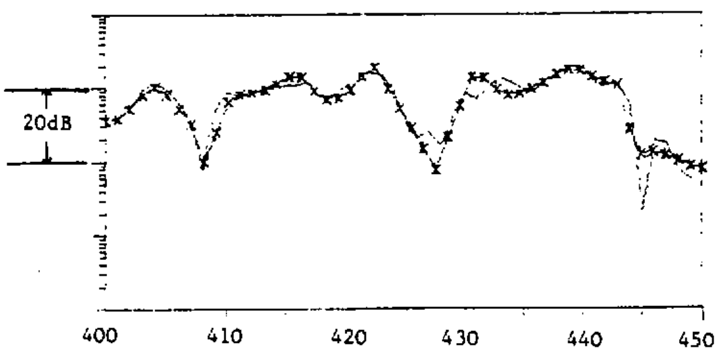

Fig 2(a)
Moduli of model (***) and reference (—) transfer functions.

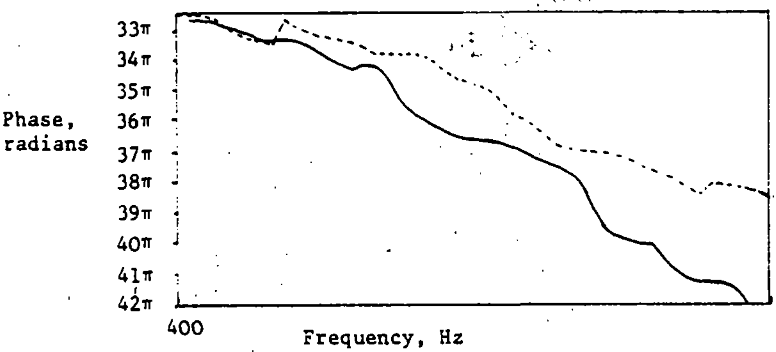

Fig 2(b)

Phase of model-32π radians (---)
and reference (—) transfer functions.