OPTIMIZATION OF SONAR TRANSDUCERS USING A COUPLED FE-BE METHOD

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1 INTRODUCTION

The use of software tools in the design of SONAR transducers is becoming widespread. In particular, numerical techniques, such as the finite-element method (FEM) and the boundary-element method (BEM) are being increasingly employed. The FE method is generally applied to the internal analysis of an object whilst the BE method is used to study the radiated field. A SONAR transducer can be modelled by the application of a coupled FE-BE method, so that its behaviour, when it is fluid loaded, can be obtained quantitatively.

The main aim of this paper is to demonstrate how optimization can be employed within the coupled FE-BE method. To illustrate this process the production of a hemispherical directivity pattern from an acoustic structure involving a flooded piezoelectric cylinder and a steel backing plate is discussed.

2 METHOD

The standard coupled FE-BE matrix equations for the dynamic flooded structural system are given by

$$[F_l] + [F_A] = [K_{uu}][\alpha] + [K_{u\phi}][\phi] - \omega^2[M][\alpha] + j\omega[R][\alpha] \qquad (1a)$$

$$-[Q] = [K_{\phi u}][\alpha] + [K_{\phi\phi}][\phi] \qquad (1b)$$

where $[F_l]$ represents the interaction forces generated by the acoustic fluid acting on fluid-solid boundaries, $[F_A]$ represents externally driven forces, $[Q]$ represents externally driven charges, and $[\alpha]$ and $[\phi]$ represents nodal displacements and nodal electric potentials. The element matrices of $[K_{uu}]$, $[K_{u\phi}]$, $[K_{\phi u}]$, $[K_{\phi\phi}]$, $[M]$, $[R]$ are defined to be elastic stiffness matrix, piezoelectric stiffness matrix, transpose piezoelectric stiffness matrix, dielectric stiffness matrix, mass matrix and dissipation matrix respectively. Details of isoparametric derivation of element matrices are presented by H.Allik et. al. [1,2]. The interaction force vector can be defined through a coupling matrix $[L]$ and fluid impedance matrices $[A]^{-1}[B']$ that is,

$$[F_l] = \omega^2\rho_f[L][A]^{-1}[B'][\alpha] \qquad (2)$$

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where $[\rho_f]$ is fluid density. Details of acoustic radiation formulation for the coupled finite element-boundary integral may be found in references [3,4]. The system matrix equations for a three dimensional structure can be formulated into a two dimensional expression if the structure is axis-symmetric.

The above equations are applied to a SONAR transducer which is shown diagrammatically by Fig. 1. It comprises a flooded radially-polarized piezoelectric cylinder mounted above a circular steel plate of diameter D and thickness T. The separation between them is S. The purpose of the optimization is to produce a hemispherical radiation pattern as would be required for a transponder. The parameters to be optimized are D, T and S, and the method employed is summarized in Appendix A.

3 EXPERIMENTAL MODEL

An experimental model based on a PZT4 piezoelectric cylinder with a nominal radial resonance of 12 KHz, was built so as to test predictions. The cylinder was waterproofed by coating it with a thin layer of varnish. The cylinder and the steel plate were suspended in a large tank (see Fig. 2). The mounting allowed the steel plate to be removed so that the cylinder could be tested alone. The admittance/frequency characteristics and the vertical directivity pattern were measured. Table 1 shows the dimensions of the SONAR transducer model

Ceramic	Density [Kg/m^3]	7517
cylinder	Inner radius [mm]	22.4
	Outer radius [mm]	28.3
	Thickness [mm]	5.9
steel	Density [Kg/m^3]	7860
disc	Separation from the cylinder [mm], S	0.5
	Diameter [mm], D	63.0
	Thickness [mm], T	30.4

Table 1 Mechanical defaults of ceramic cylinder and steel disc.

4 RESULTS AND COMPARISONS

Fig. 3 shows the admittance locus for the cylinder in water calculated using the coupled FE-BE method (left line), and the measured graph (right line). Table 2 provides a

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quantitative comparison of the frequencies at the points 1, 2, 3 and 4. Point 2 is the maximum conductance whilst 1 & 3 are the half conductance frequencies of the main radial mode. The resonance at point 4 is thought to be the lowest cavity mode.

	1	2	3	4
Theoretical results	10964.5 Hz	11521 Hz	12410.5 Hz	19000 Hz
Experimental results	11608 Hz	12249 Hz	13189 Hz	22450 Hz
Percentage difference	5.9 %	6.3 %	6.3 %	18.2 %

Table 2 Frequencies corresponding to the numbers denoted in Fig. 3.

The nominal directivity patterns are compared in Fig. 4 for the isolated cylinder, where the thin line is the theory and the thick line is the experiment. There is reasonable agreement between the two curves, the slight differences are probably attributable to the effect of the mount. There are considerable variations in the directivity patterns over the hemisphere. The maxima in the horizontal direction is in the region of 20-25 dB above the minima in the vertical direction. Table 3 gives more detailed comparisons of this variation between the horizontal and vertical directions.

	Frequency [Hz]					
	10964.5	11521	12410.5	11608	12249	13189
Theoretical results	25.8 dB	24.0 dB	21.7 dB	*	*	*
Experimental results	*	*	*	19.8 dB	20.9 dB	17.3 dB

Table 3 Variations between maxima and minima in directivity patterns.

The optimum size of disc and its optimum separation were deduced using the methods outlined in section 2. These defaults are summarized in Table 1. The vertical directivity patterns obtained are compared in Fig. 5; the thin line is theory and the thick line is the experiment. The directivity patterns are very nearly hemispherical. Detailed comparisons of the variations over the full hemisphere at different frequencies are shown in Table 4.

	Frequency [Hz]					
	10964.5	11521	12410.5	11608	12249	13189
Theoretical results	0.81 dB	0.78 dB	0.81 dB	*	*	*
Experimental results	*	*	*	1.07 dB	0.92 dB	1.08 dB

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Table 4 Variations in directivity patterns over the hemisphere for the optimized structure.

In general, the main effects of fluid loading on the SONAR transducer are to introduce fluid damping and to increase the inertia of the transducer, therefore, lowering its resonant frequency [5]. When the steel mounting disc is placed near the cylinder, the inertia of the transducer is increased, thereby, lowering the resonant frequency still further. Fig. 6 shows the admittance locus calculated from the in-water cylinder mounted on the disc (left line), and the measured graph (right line). Table 5 provides a quantitative comparison of the frequencies at the points 1 and 2 which correspond to the radial and the cavity modes respectively.

	without mounting disc		with mounting disc	
	1	2	1	2
Theoretical results	11521 Hz	19000 Hz	7500 Hz	18600 Hz
Experimental results	12249 Hz	22450 Hz	9752 Hz	19000 Hz
Percentage difference	6.3 %	18.2 %	30.0%	2.2 %

Table 5 Frequencies corresponding to the numbers denoted in Fig. 6.

Fig. 7 shows the theoretical pressure response of the cylinder mounted with the disc for a constant driving voltage plotted against frequency. The numbers beside each line indicate different angular positions in the far field. 1,2,3 and 4 correspond to 0°, 30°, 60° and 90° relative to the vertical axis respectively. These graphs show that the vertical directivity patterns are consistently hemispherical at frequencies between 6.25 KHz and 17.5 KHz.

For the present paper, the optimization of the hemispherical beam pattern has been carried out for a particular frequency. Only three optimizing variables are considered; D, T and S. Further optimization can be developed to maintain the pressure response more nearly constant with frequency

Appendix A

The basis of the structural optimization is a minimizing technique for multiple variables, in this case S (spacing between cylinder and disc), D (diameter of disc) and T (thickness of disc). The aim of the minimization is to produce as near as possible a hemispherical directivity pattern for a particular frequency. The method involves fitting a polynomial to function values at equi-spaced points for each of the variables. For example, whilst D and T are constant, five values of S are considered; $S_1 = S_0 - 2dS$, $S_2 = S_0 - dS$, $S_3 = S_0$, $S_4 = S_0 + dS$ and $S_5 = S_0 + 2dS$ where S_0 is an initial choice of spacing between the

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cylinder and the disc. For each S_k, the sound pressure amplitudes P_i are calculated at a set of n field points distributed around an arc starting on the axis of the transducer and finishing at 90° to the axis. The following function is evaluated:

$$F(S_k,D,T)=\sum_{i=1}^{n}(20\times\log_{10}P_m-20\times\log_{10}P_i)$$

where P_m is the maximum of the P_i's. From the values of S_k and their corresponding values of F, a polynomial function of S is derived:

$$f(S) = a_1\times S^4 + a_2\times S^3 + a_3\times S^2 + a_4\times S^1 + a_5$$

where

$$\begin{bmatrix}a_1\\a_2\\a_3\\a_4\\a_5\end{bmatrix}=\begin{bmatrix}F(S_1,D,T)^4 & F(S_1,D,T)^3 & F(S_1,D,T)^2 & F(S_1,D,T) & 1.0\\F(S_2,D,T)^4 & F(S_2,D,T)^3 & F(S_2,D,T)^2 & F(S_2,D,T) & 1.0\\F(S_3,D,T)^4 & F(S_3,D,T)^3 & F(S_3,D,T)^2 & F(S_3,D,T) & 1.0\\F(S_4,D,T)^4 & F(S_4,D,T)^3 & F(S_4,D,T)^2 & F(S_4,D,T) & 1.0\\F(S_5,D,T)^4 & F(S_5,D,T)^3 & F(S_5,D,T)^2 & F(S_5,D,T) & 1.0\end{bmatrix}^{-1}\begin{bmatrix}S_1\\S_2\\S_3\\S_4\\S_5\end{bmatrix}$$

The value S_m which minimizes the polynomial function provides an approximation to the value of S within the range S_1 to S_5 which minimizes the function F. A similar process leads to optimized values for D and T.

5 REFERENCES

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[4] Francis D.T.I. (1990):A boundary-element method for the analysis of the acoustic field in three-dimensional fluid-structure interaction problems, in SONAR transducers for the nineties, Proceedings of the Institute of Acoustics

[5] Macey P.C. (1988): Fluid loading and piezoelectric elements, Proc. Inst. of Acoust., vol. 10, part 9, pp:79-104.

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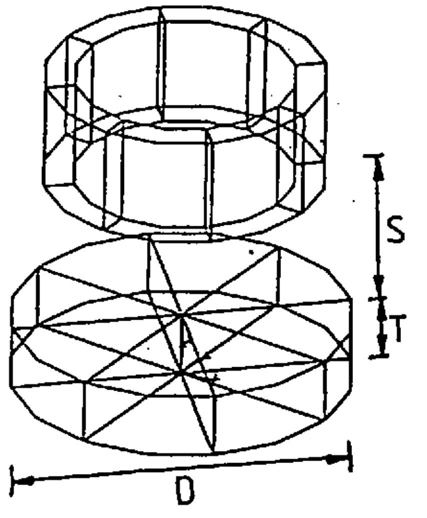

Fig. 1 The finite element model of a PZT4 ceramic cylinder and a steel disc (see text)

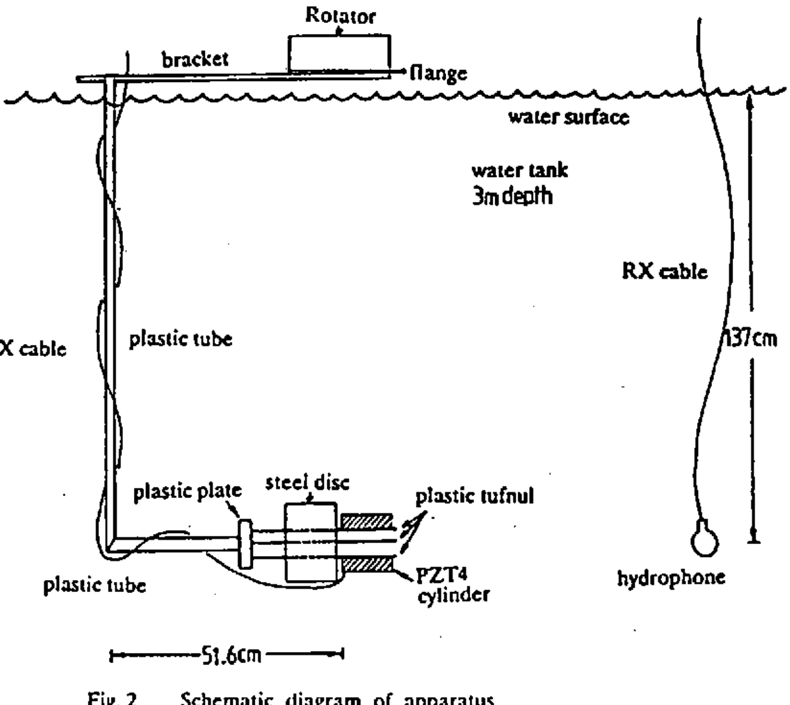

Fig. 2 Schematic diagram of apparatus used in experiment

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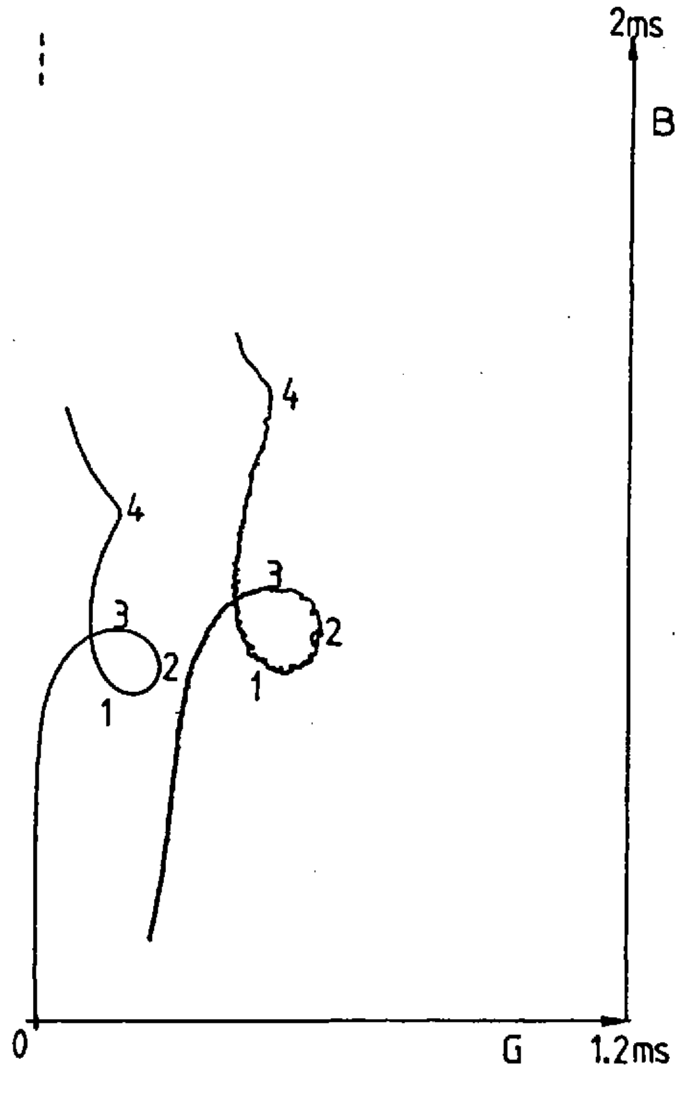

Fig. 3

The conductance response of the cylinder plotted against the susceptance
left line: Theory
right line: Experiment

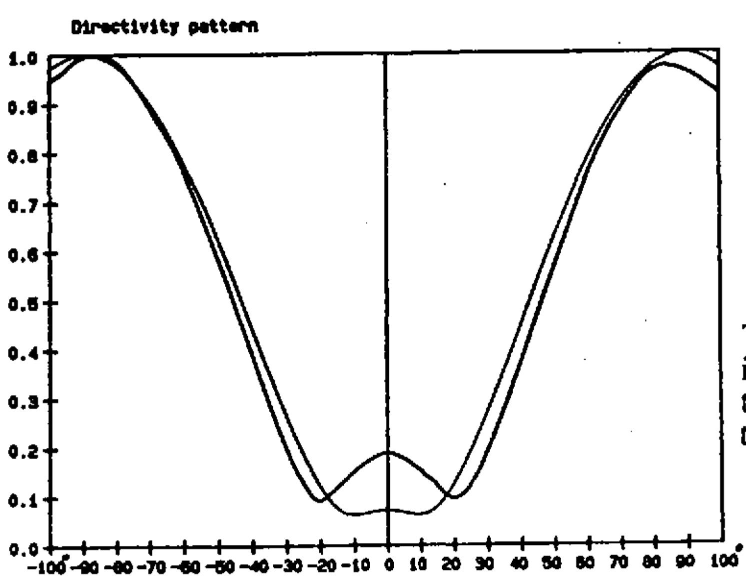

Fig. 4

The directivity pattern of the cylinder
thin line: Theory
thick line: Experiment

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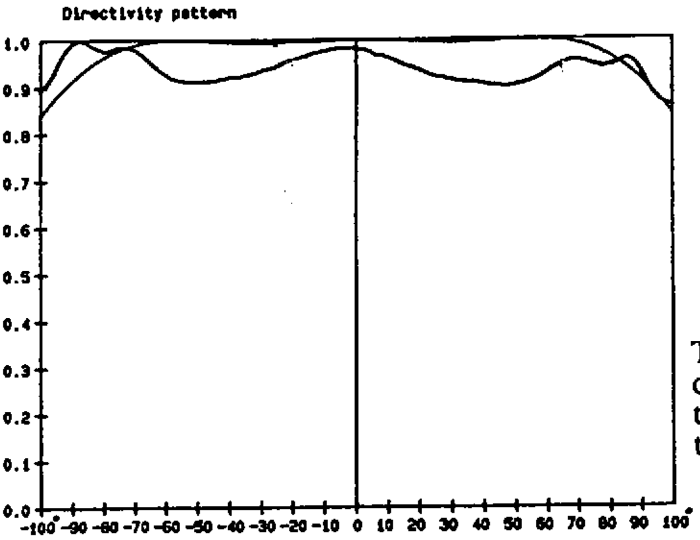

Fig. 5

The directivity pattern of the cylinder with the disc
thin line: Theory
thick line: Experiment

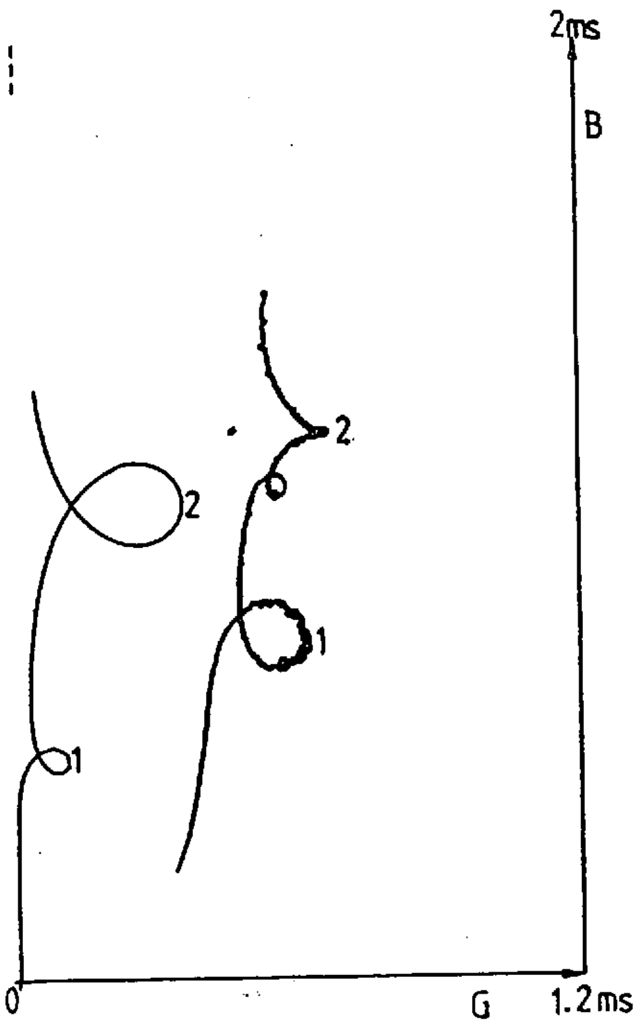

Fig. 6

The conductance response of the cylinder with the disc plotted against the susceptance
left line: Theory
right line: Experiment

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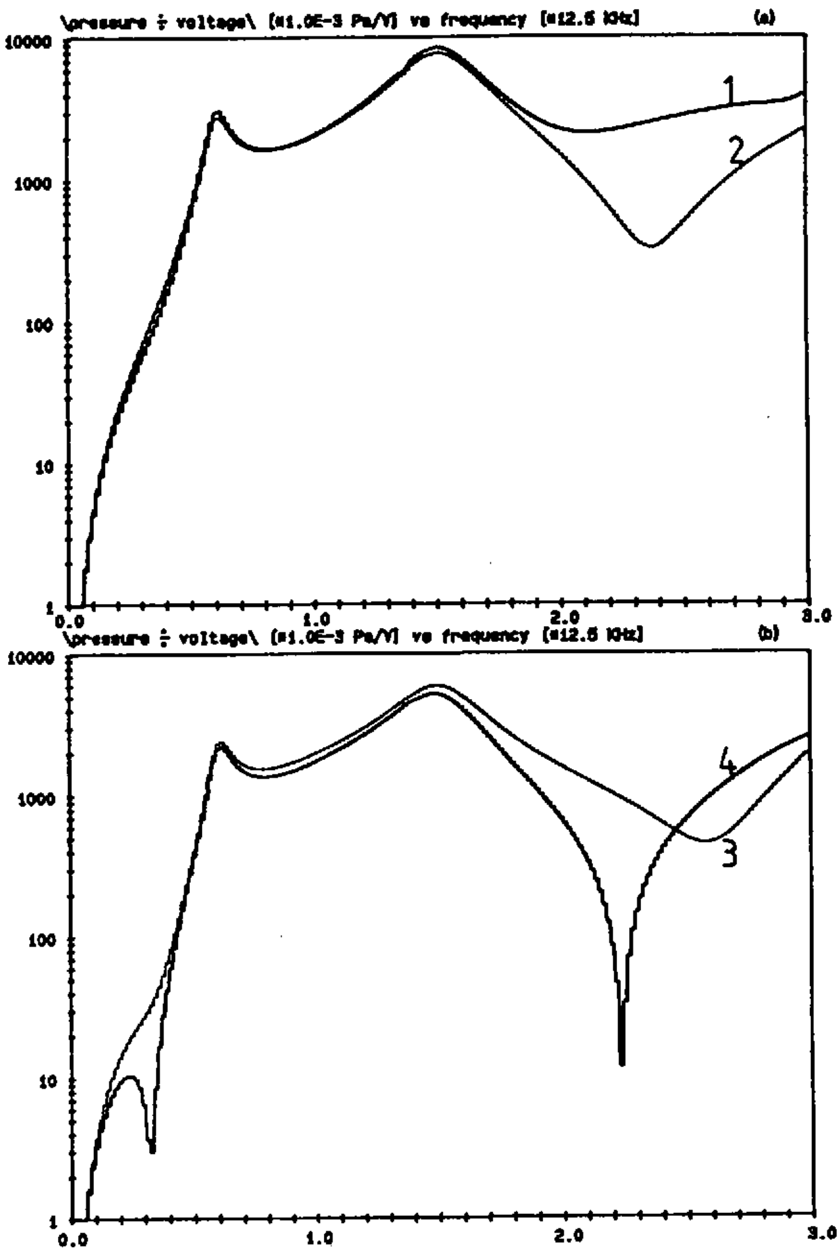

Fig. 7 The theoretical pressure response of the cylinder mounted axially near to the steel disc of optimum dimensions