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COMPARATIVE STUDY ON DETECTION OF INCLINED EDGE CRACK USING STATIC DEFLECTION MEASUREMENTS AND NATURAL FREQUENCIES

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This paper discusses both vibration and static deflection based methods for detection of an inclined edge crack. The crack is represented by a rotational spring of finite stiffness present at the location of crack tip. The crack is present in a beam of uniform cross section in cantilever configuration. An open crack, when modelled so, is characterised by its location, depth and rotational spring stiffness. In static deflection based detection, static deflection of the crack-free beam is treated as the basis. The methodology of crack detection uses the difference in measured static deflections in crack-free and cracked beams. Thus crack location and its severity i.e. depth is predicted. In vibration based crack detection also natural frequencies of the crack-free and cracked beam are used to locate the crack and its severity. In frequency based detection, first three natural frequencies for beam with and without crack are used to estimate the crack location and rotational spring stiffness. In static deflection based method, deflection is measured at two locations for crack-free and cracked beams to estimate crack location and rotational spring stiffness. In both cases, the crack depth/severity is measured indirectly in terms of rotational springs used to represent the crack. The results presented are experimental for static deflections and finite element method based for natural frequencies. Both crack inclinations i.e. oriented towards the support and away from it, were considered. M.S. rectangular beam specimens with various crack inclinations, locations and depths were used for deflection experiments. It has been observed both the methods predict the crack location with acceptable accuracy. However, prediction of rotational spring stiffness by both the approaches does not show good agreement. The paper discusses these facts on quantitative basis.

Keywords: Crack detection, rotational spring stiffness, static deflection measurements, frequency based detection, FEA

1. Introduction

Appearance of crack in a component results in change in stiffness at crack location. This change in local stiffness has direct effect on static and dynamic behaviour of the component. Parameters like natural frequencies, mode shapes, damping coefficient, static deflection, etc., undergo changes. These changes can be employed for developing non-destructive evaluation (NDE) methods for predicting location and severity of a crack.

Use of rotational spring to represent the flexibility induced in component due to presence of a crack is very popular choice to model transverse or bending vibrations [1-2]. Crack detection methods based on static deflection measurements are investigated for detection of edge normal open surface crack. Caddemi and Morassi [3-4] proposed a method using static deflection to identify single and multiple cracks in an elastic straight beam in bending for variety of boundary conditions. A relatively simple method for detection of a single edge normal crack in cantilever and simply supported pipes using measurements of static deflections at two points was proposed by Naik [5].

A vibration based method for detection of an inclined subsurface and edge crack is presented by Nandwana and Maiti [1]. They used changes in first three natural frequencies for detection of the crack which is modeled using a rotational spring. Naniwadekar et al. [2] used the rotational spring approach to model a crack in transverse cross section in various angular positions for slender beams.

In this paper, the static deflection measurement based method, which has been found useful for detection of edge normal crack, is employed for detection of an inclined edge crack. Usefulness of static deflection measurements and rotational spring model for detection of an inclined edge crack is demonstrated. The results are also obtained by frequency based method for the purpose of comparison.

2. Theory

Static deflection and Frequency based methods for predicting an edge normal crack are briefly presented here.

2.1 Two Point Static Deflection Method

This method is proposed by Naik [5] for detection of edge normal crack in pipes is used to predict the location and rotational spring stiffness in prismatic cantilever beam. In this method, two measurements of static deflections at two distinct locations (refer Fig. 1) are essential for detection of a crack. Measurements of static deflections are carried out for crack free and cracked beams in this manner.

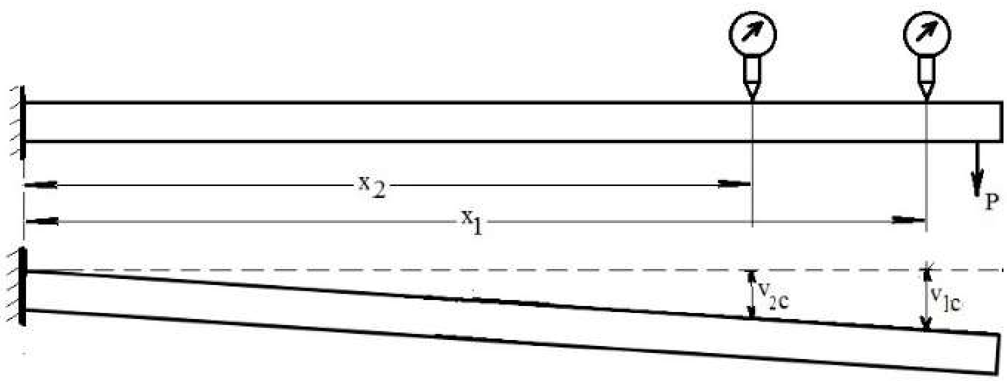

Figure 1: Two point static deflection measurements on edge crack-free beam [5]

Static deflections of the beam in segment 1 (Refer Fig. 2) can be written as

$$v_{Ic} = -\frac{Px_c^2}{6EI}(3L - x_c) \tag{1}$$

A rotational spring of stiffness K is used to represent the flexibility due to the crack and it is located at the crack tip. This spring connects segments I and II. Slope in segment II of the beam includes jump in slope due to the crack in addition to slope of segment I, therefore

$$\frac{dv_{IIc}}{dx} = \frac{dv_{Ic}}{dx} + \frac{M}{K} \tag{2}$$

where, M is the moment at the crack section. Subscripts I and II refer to the segments I and II respectively. At crack location, the displacements corresponding to the two segments are identical. Hence, the deflection in segment II will be,

$$v_{IIc} = -\frac{PLx^2}{2EI} + \frac{Px^3}{6EI} - \frac{M}{K}(x - x_c) \qquad for\ x_c \le x \le L \tag{3}$$

The additional deflection due to presence of crack in segment II can be written as

$$\delta v = v_{IIc} - v_{nc} \qquad for\ x_c \le x \le L \tag{4}$$

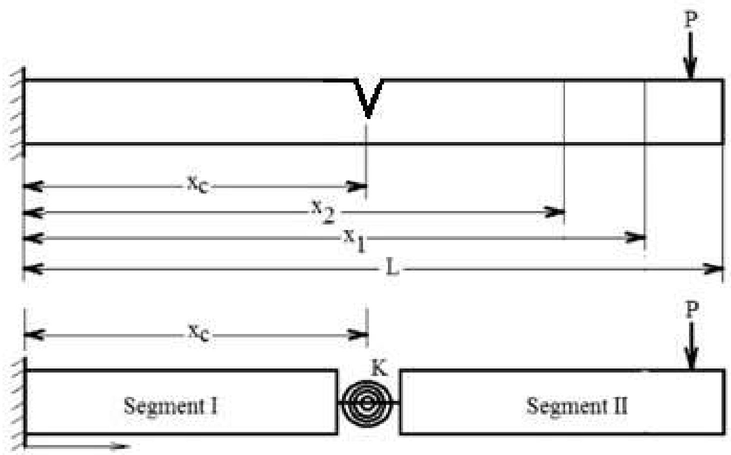

Figure 2: Representation of edge normal crack using rotational spring

Equation (4) for the two locations will give,

$$\delta v_1 = v_{IIc1} - v_{1nc} \tag{5}$$

and

$$\delta v_2 = v_{IIc2} - v_{2nc} \tag{6}$$

where, v_{IIc} and $v_{IIc2}$are measured experimentally within segment II at axial locations x_1 and x_2 respectively. Therefore, change in deflections at locations x_1 and x_2 can be written.

The values of δv_1 and δv_2 are the change in static deflections at known locations x_1 and x_2 respectively. The predicted value of the crack location x_c can be found out from Eqs. (5) and (6) as

$$\frac{\delta v_1}{\delta v_2} = \frac{(x_1 - x_c)}{(x_2 - x_c)} = r(say) \tag{7}$$

which gives, crack location as

$$x_c = \frac{(x_1 - r\, x_2)}{(1 - r)} \tag{8}$$

and, rotational stiffness as

$$K = \frac{-P(L - x_c)(x_1 - x_c)}{\delta v_1} \tag{9}$$

or

$$K = \frac{-P(L - x_c)(x_2 - x_c)}{\delta v_2} \tag{10}$$

Equations (8), (9) and (10) can be employed for detection of an inclined edge crack and the rotational spring stiffness due to the crack.

2.2 Frequency Based Detection

The governing equation of transverse vibration of two segments (Refer Fig. 2) is given by,

$$\frac{d^2}{dx^2}\left[EI\ \frac{d^2U}{dx^2}\right] - \omega^2 \rho A U = 0 \tag{11}$$

where, U is the transverse displacement, ω is the natural frequency of the vibration of the beam, E is the Young's modulus, I is the second moment of full area of cross-section, ρ is the mass density, A is the cross-sectional area and the x-axis is aligned with the axis of the beam. Introducing non-dimensional frequency parameter $\lambda^4 = (\omega^2 \rho A L^4)/EI$ and $\xi = x/L$ where x = distance from the fixed support and L = length of the beam, the equations for the two segments are

$$\frac{d^4U_1}{d\xi^4} - \lambda^4 U_1 = 0 \tag{12}$$

and,

$$\frac{d^4U_2}{d\xi^4} - \lambda^4 U_2 = 0 \tag{13}$$

The boundary conditions, at the fixed and free ends, are

$$\text{at } \xi=0,\ U_1 = U_1' = 0 \text{ and at } \xi=1,\ U_2'' = U_2''' = 0 \tag{14}$$

$$\text{at } \xi = \beta = x_c/\text{L, continuity conditions are, } U_1 = U_2,\ U_1' = U_2' \text{ and } U_1'' = U_2'' \tag{15}$$

and the compatibility condition due to increased flexibility at the crack location is given by,

$$U_1' + \left(\frac{E\,I}{K}\right) U_1'' = U_2'' \tag{16}$$

After incorporating all the conditions using Eqs. (14), (15) and (16), we get a characteristic equation for the cracked beam as,

$$\frac{K_t}{\lambda}|\Delta_1| + |\Delta_2| = 0 \tag{17}$$

Here, $|\Delta_1|$ and $|\Delta_2|$ are determinants of size 8x8 and K_t is non-dimensional stiffness of the rotational spring = KL/EI. Solution of this equation can be obtained in the form of K_t versus β plot which predicts the crack location and rotational spring stiffness [1].

3. Implementation

Procedure for crack detection using static deflection and frequency based methods is explained as follows.

3.1 Static Deflection Method

Twenty one mild steel specimens of rectangular cross-section are used for the experimentation. Crack depths varying from 3 mm to 10 mm are considered for three relative crack locations of 0.2, 0.4 and 0.6. Crack inclinations considered are $\pm 45^0$, $\pm 30^0$, $\pm 15^0$ and 0^0. Cracks are made in the specimen using jeweller's saw. Specimens are firmly clamped using special grips to obtain cantilever configuration (refer Fig. 3). The specimen details are as follows: length (L) =400 mm; depth (h) =20 mm; width (w) =12 mm; modulus of elasticity (E) =210 GPa and density (ρ) = 7860 kg/m^3.

For acquiring the baseline information, deflection measurements at two locations for the crack-free cantilever beam are taken. Accordingly, first point is at a distance of 270 *mm* and the other is at 36*0 mm* from the support. Both the locations are within segment II of the cracked specimen. Load is applied at a location *10 mm* from the free end. The load is gradually applied in steps of 1 kg up to 4 kg and the deflection is measured using two dial indicators (Mitutoyo Make–with least count of 1 micron) for each load step. For each specimen 15 to 20 readings are taken and the average value of these reading is reported. In this manner, static deflections in the crack free and the cracked beam at the two locations are measured.

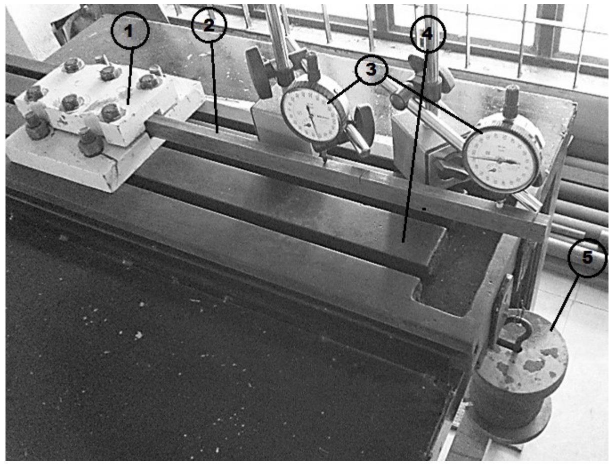

Figure 3: Photograph of the experimental setup: (a) 1. Fixture, 2. Specimen, 3. Dial indicator, 4. Rigid table, 5. Weight pan

3.2 Frequency Based Detection

Frequency analysis was performed on crack-free and cracked cantilever beams with an inclined edge crack using ANSYS software. This requires first three natural frequencies of the cracked and crack-free beam. Accordingly, FEA was carried out to obtain the natural frequencies. Cases with same crack inclinations, depth and normalised location as used in static deflection method are considered. Using the natural frequencies for each case, the crack location, β and rotational spring stiffness, K are predicted with the help of Eq. (17).

4. Results and Discussion

The measured static deflections are employed for detection of location of inclined crack and its rotational spring stiffness using Eqs. (8), (9) and (10). Figure 4 shows the error in prediction of crack location for all specimens tested. It is seen that the maximum error in prediction is less than 14%. This error in prediction could be attributed to the limitations in measurements of the static deflection accurately. Dial indicator with least count smaller than 1 micron could improve the accuracy of prediction of normalised crack location.

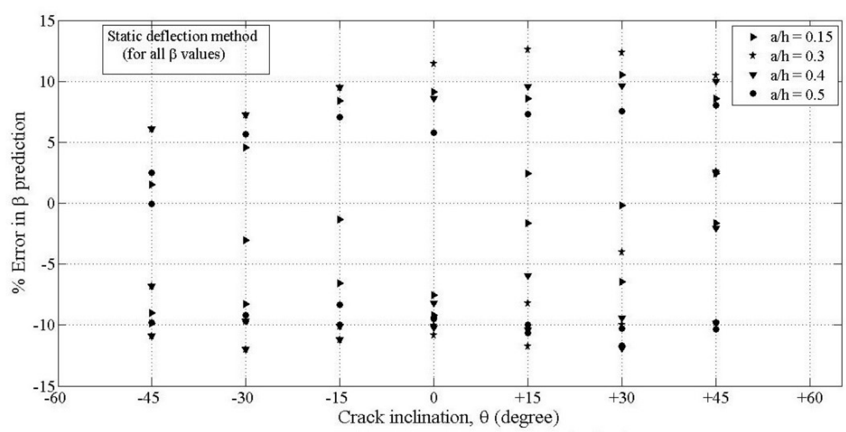

Figure 4 - % Error in predicting normalized crack location, β using static deflection method

In frequency based detection, first three natural frequencies of vibration of the beam with an inclined edge crack are obtained using FEA. These frequencies are employed for prediction of crack location and rotational spring stiffness using Eq. (17). The errors in prediction of crack location are shown in Fig. 5. Crack is located quite accurately by this method; the maximum error is less than 6%.

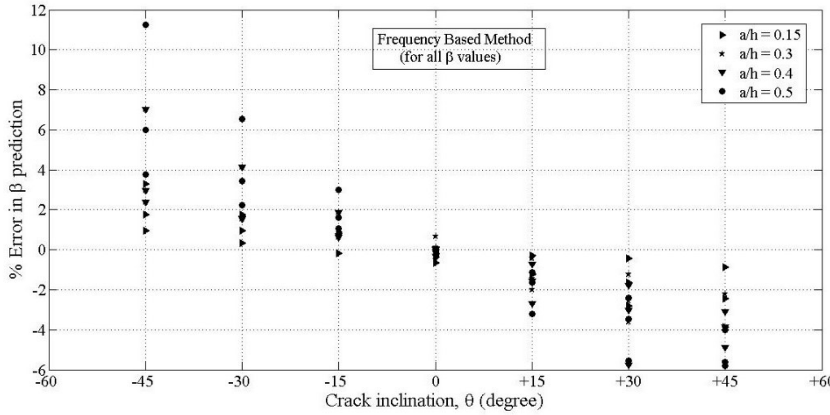

Figure 5 - % Error in predicting normalized crack location, β using frequency method

The comparison of predictions on crack location is shown with crack inclination in Fig. 6 for know crack location of 0.4. It is seen that the two predictions are comparable with the actual crack location and also with each other with acceptable accuracy.

In both static deflection and frequency based crack detection methodologies, crack severity is represented by the rotational spring stiffness. The stiffness predictions by both the methods with various crack inclination angles considered is shown in Fig. 7. It is obvious from the plot that stiffness predictions by both the methods do not show any agreement except for the value of crack -30^0 inclination. It appears that stiffness prediction by both the methods is also dependent on the accuracy of static deflection measurements. As it is, the values of static deflections are small for the applied loads and when they are needed at two close-by locations they are affected more by the least count of the dial gauges used. Therefore, it is likely that stiffness predictions by frequency based method could be more accurate than static deflection method. However it requires further investigations to arrive at this conclusion firmly.

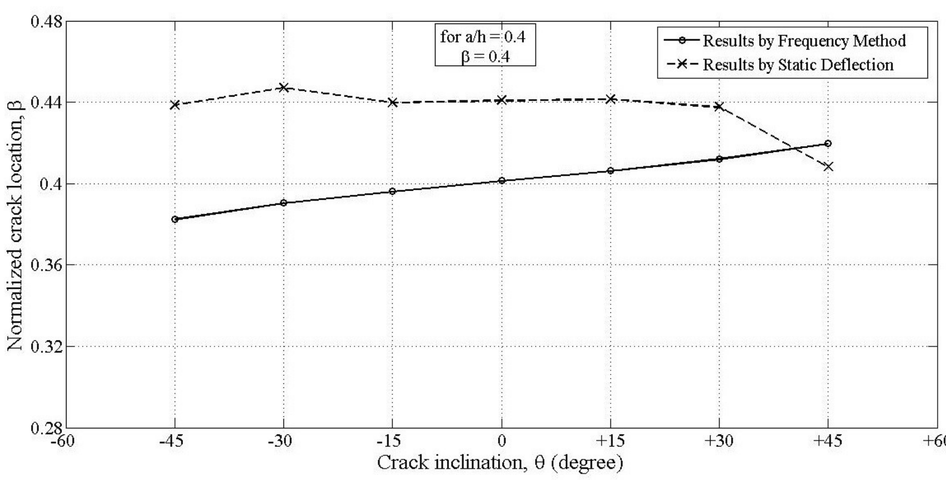

Figure 6 – Comparison of predicted values of Normalized crack location, β

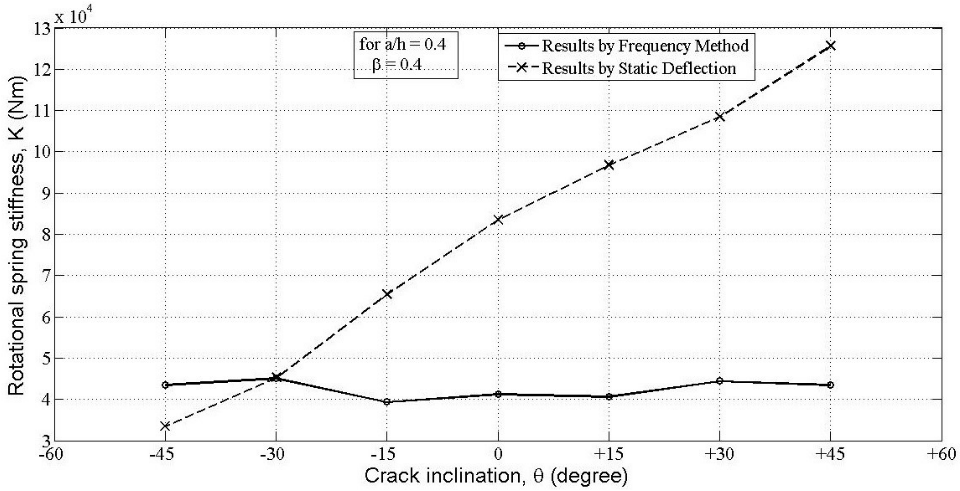

Figure 7 – Comparison of predicted values of Rotational spring stiffness, K

5. Conclusions

Both static deflection and frequency based methods predict crack location with good accuracy. The maximum error in prediction by static deflection method is 12.61% and that by frequency method is 11.25%. However the predictions of rotational spring stiffness by both the methods do not show acceptable agreement.

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