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NORMAL MODES OF CHURCH BELLS

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Introduction

The traditional ad hoc method for classifying bell partials [1,2] is to quote the number of nodal meridians 2m and the number and position of nodal circles n (e.g. one circle near the rim) for the component modes, where n, m = 0, 1, 2, ... This has never been satisfactory because of the anomalous position of the Hum, being the only partial with no circles. We have found that there are also problems with numerous high partials, probably unknown to earlier workers, with a variety of numbers and positions of rings. Clearly a better understanding of the mechanisms responsible for the various modes is required in order to replace the ad hoc scheme by one with a sound physical basis.

We have taken a good quality modern English church bell and measured the frequencies and nodal patterns for all the partials up to about 9 kHz as accurately as our equipment would allow. Accurate measurements of the geometry of the bell were then made and used as the basis for a finite element calculation of the normal modes. The hope was that we would be able to match up experimental and theoretical modes and so decide the physical nature of each experimentally found partial by looking at the finite element solution for its form. Families of partials should then be identifiable on a sound physical basis. At the time of writing we have successfully classified all partials with four or more meridians in this way. Those with zero or two pose a special problem because the effective boundary conditions on crown motion are different for them and we shall not consider them further here, especially as they are usually reckoned to be of little acoustical importance.

The Comparison

Since the number of nodal meridians 2m is confidently known both experimentally and theoretically one might expect to be able to write down two lists of partials for fixed 2m and simply pair them off in sequence. In practice it is not so simple because: (1) a given partial may appear more than once in the experimental list because it had not been realised that the doublet had become split, (2) some partials may be missing from the experimental list for any one of several possible reasons, (3) theoretical frequencies are expected to get increasingly too high as one goes up the sequence, (4) experimental ring positions and numbers may be unreliable, especially above the shoulder and when n is large, (5) the sequences should be roughly the same in the two lists but two consecutive theoretical modes may come out in the wrong order, especially if their frequencies are close and they are of different physical types. In the table we list results for the case of 8 meridians as a typical example. Theoretical mode numbers 1-5 and 7-9 are matched well.

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Mode Number	Frequency (Hz) Theory	Frequency (Hz) Experiment	Class (R)	Mode Number	Frequency (Hz) Theory	Frequency (Hz) Experiment	Class (R)
1	1166	1172	RIR (-1)	8	6067	5769	6
2	1465	1473	1	9	6858	6516	7
3	1962	1949	2	10	7133	-	RER(α)
4	2840	2832	3	11	7989	7824	β
5	3909	3867	4	12	8201	7683	8
6	4367	4368 4401	RA (0)	13	9008	-	γ
7	5081	4997	5	14	9732	8968	9

The experimental partial corresponding to mode 6 has been quite badly split, while numbers 10 and 13 are missing. Numbers 11 and 12 have come out the wrong way around, a fact which can be ascertained by making a careful comparison of theoretical and experimental ring positions.

The Classification

Examination of the finite element solutions shows that there are two broad categories of mode: (1) primarily in-azimuthal-plane but some some out-of-plane motion (apart from $m = 0$ cases which are pure breathing modes), (2) primarily out-of-azimuthal plane but with some in-plane motion (apart from $m = 0$ cases which are pure twisting modes).

The primarily in-plane modes divide into "ring" driven and "shell" driven. For a given value of $2m$ there is always one partial which corresponds to the heavy ring on the rim of the bell going into its inextensional radial mode [3], and driving the shell along with it. Consequently the number of nodal rings varies with $2m$ depending upon the details of the nearest "shell alone" mode. This family, which includes the notorious Hum, we designate RIR (or alternatively $R = -1$ for reasons to be explained later). A second "ring" driven mode for each value of $2m$ corresponds to the heavy ring going into its axial mode. These we designate RA (or alternatively $R = 0$) and again they do not correspond to a fixed number of nodal rings, and for the same reason.

The remaining "primarily in-plane" modes, which form a large majority in the region up to 9 kHz, are all essentially due to the shell going into its inextensional radial modes with the heavy ring at the rim being almost at rest and supplying a nodal ring close to the rim. For a given value of $2m$ there is a sequence of these modes having 1, 2, 3, ... nodal circles which we designate as $R = 1, 2, 3, \ldots$ For low values of R the crown is essentially at rest, playing a similar role at the top of the bell to that played by the heavy ring at the bottom. However, when R has increased to the point where the wavelength of the standing waves has become comparable with the crown dimensions then the crown behaves as an integral part of the "shell", subject only to the constraint (for 2m > 2) of a node at the very top. The value of R at which the crown "takes off" increases as 2m increases and nodal circles arise above the shoulder only after this point has been passed. Further evidence for the existence of a family relationship amongst these modes, over and above the "theoretical" evidence of the finite element calculations, comes from three different ways of looking at the experimental data: (1) plotting the positions of nodal rings l_R v 2m for fixed R shows R smooth curves in each case, (2) plotting l_R v R for fixed 2m shows a characteristic pattern which varies very little with 2m, (3) plotting f v R for fixed values of 2m yields a smooth curve which is better than can be achieved with any other ordering or combination of modes with that value of 2m, with a kink at $R_{k,m}$ in the region corresponding to the crown "take-off". If one plots log f v R then the curve for fixed 2m is replaced by two straight lines which intersect at the point previously occupied by the kink: the analogy with phase changes in thermodynamics is obvious. If the value of 2m is changed a new pair of straight lines is produced parallel to the originals but with the point of intersection moving to a value of R which increases from its previous value by unity each time m increases by unity, i.e. $R_{k,m'} = R_{k,m} + (m' - m)$.

The modes which are primarily out-of-plane are relatively high in frequency and difficult to excite, the latter fact being responsible for a relatively high proportion of them being missing from the experimental list. For a fixed value of 2m there is a sequence of these modes having an increasing number of nodal rings for out-of-plane motion. We designate these as α, β, γ, ... The first of these is due to the heavy ring at the rim of the bell going into its extensional radial mode and driving the shell with it, so we give it also the alternative designation of RER. The others are due to various other ring components of the bell going into the extensional radial modes in a similar way but the picture is extremely complicated.

To understand why we have given the alternative designations of R = -1 and 0 to the RIR and RA modes it is necessary to reconsider the graphs of f v R for fixed 2m. What we find is that if the curves are extrapolated back below R = 1 then the RIR modes lie neatly on these curves for the R = -1 position. Clearly R = -1 has no physical meaning. Likewise for the RA modes using R = 0 but this time only if we use the mode with (2m - 8) meridians!

Discussion

We have produced a classification scheme which accounts for every observed partial for 2m > 2. Of the five musically important partials (i.e. those normally tuned by English founders [4]), three are of type R = -1 (the Hum, the Tierce and the Nominal) and two are of type R = 1 (the Fundamental and the Quint). There is no family connection between the R = -1 (or RIR) and the R = 1 modes so the fact that the former fit smoothly onto the f v R curves for fixed 2m is almost certainly a "deliberate" consequence of bell design. This is reasonable since one of the chief considerations of bell design is to get the relative frequencies of the five "musical" partials correct. Having "fixed up" the lower members of the R = -1 and R = 1 families in order to achieve this it is not surprising that the "smooth" relationship thus built in for 2m = 4, 6 and 8 carries on up through the spectrum for higher values of 2m.

The smooth fit of the R = O modes is almost certainly fortuitous. It may be that it is needed to reduce discord from some higher partials but this seems unlikely. For a given mechanical ring there is a definite relationship between the inextensional radial and the axial modes. Thus once the bell shape has been picked to fix the RIR modes (R = -1) the R = O will be fixed automatically.

Conclusions

By combining accurate experimental measurements with finite element calculations we have been able to produce a scheme of classification which works well for all cases with more than two meridians. The physical nature of the partials belonging to the various families has been established and some insight gained into the process of bell design. Work is proceeding on the extension of our scheme to cover the cases of zero and two meridians.

Acknowledgements

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References

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