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# THIRD-ORDER ACOUSTIC FINITE ELEMENTS AND H-ADAPTIVITY FOR EFFICIENT ACOUSTIC SIMULATIONS

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The finite element method has emerged over the last decade as the reference simulation technique for modelling aero-vibro-acoustic phenomena in the low and mid frequency range. Its use is nevertheless constrained by the computational resources needed to solve large linear systems. Besides mesh adaptivity, also referred to as h-adaptivity, one way to improve the solution accuracy while controlling both the memory and time costs is to increase the interpolation order of the finite element discretization. This paper presents a strategy in which h-adaptivity is coupled to a cubic interpolation scheme to further improve computational performances, without any compromise on the accuracy. Complete third-order interpolation spaces are constructed to provide formulations for tetrahedron, hexahedron, wedge and pyramid elements. Finally, the new interpolation schemes are used on three configurations: the modal analysis of a parallelepipedic acoustic cavity, the vibro-acoustic response of a car cavity excited by a vibrating panel and the noise transmission through a car firewall panel. Computations are conducted with the Actran commercial software, which provides a full implementation of the proposed methodologies.

Keywords: cubic interpolation, h-adaptivity, acoustics, Actran

## 1. Introduction

Modelling aero-vibro-acoustic phenomena in the low and mid frequency range using finite element method has become the reference numerical method for industrial applications. The amount of computational resources required to solve large numerical models has driven software editors to develop efficient, fast and accurate methodologies. Two methodologies exist to improve solution accuracy while controlling memory and computing time : mesh refinement (including h-adaptivity) and increase of the discretization interpolation order.

A strategy coupling h-adaptivity and cubic interpolation scheme is presented in this paper. Computations are conducted with the Actran commercial software, which provides a full implementation of the proposed methodologies [1].

The scheme is based on the enrichment of conventional linear finite element in a hierarchical way. Such procedure enables to develop cubic interpolation for all conventional acoustic elements: edges, triangles, quadrangles, tetrahedra, pyramids, wedges and hexahedra.

Investigations are limited to third order polynomials as the advantage of higher order schemes might be limited when considering the relatively low precision requirered for typical industrial application. The study of the transmission loss of a car firewall panel with h-adaptivity illustrates those limitations and highlights scenarios where cubic interpolation schemes are more effective than conventional quadratic interpolation schemes.

The present work is divided in three parts: presentation of the numerical scheme, validation of the implementation by convergence error study and study of the acoustic transmission through a car firewall panel with h-adaptivity.

## 2. Pseudo–Hierarchical Finite Element Scheme

Pseudo–Hierarchical FEM, later called PHFEM, developed in this paper is applied on acoustic wave propagation equation in inhomogeneaous media in frequency domain :

$$\nabla \cdot \left( \frac{1}{\rho} \nabla p \right) + \frac{\omega^2}{\rho c^2} p = 0, \quad \text{supplemented with appropriate boundary conditions} \tag{1}$$

with $p$ the acoustic pressure, $\omega$ the angular frequency, $\rho$ the mean density of the propagating media and $c$ the speed of sound.

### 2.1 Conventional vs Hierarchic FEM

Let's consider $p$, a scalar variable, and $p^h$ its finite element approximation:

$$p(\mathbf{x}) \approx p^h(\mathbf{x}) = \sum_j^N \Phi_j(\mathbf{x}).a_j \tag{2}$$

with $\Phi_j(\mathbf{x})$ the shape function associated to unknown $a_j$.

Conventional finite element method assumes that unknowns $a_j$ are equal to element nodal values $p_j$ which leads to the following conditions :

$$\Phi_j(\mathbf{x_i}) = \begin{cases} 1 & \text{when } \mathbf{x_i} = \mathbf{x_j} \\ 0 & \text{when } \mathbf{x_i} \neq \mathbf{x_j} \end{cases} \quad \text{with } \mathbf{x_{i/j}} \text{ the element nodes positions} \tag{3}$$

$$\sum_j^N \Phi_j(\mathbf{x}) = 1, \quad \text{also called "Partition of Unity" property} \tag{4}$$

On the opposite, hierarchic finite element is based on functional enrichment of conventional linear element with arbitrary functions:

$$p^h(\mathbf{x}) = \sum_j^{N_\mathrm{L}} \Phi_j^\mathrm{L}(\mathbf{x}).p_j^\mathrm{L} + \sum_l^{N_\mathrm{H}} \Phi_l^\mathrm{H}(\mathbf{x}).a_l \tag{5}$$

where: $\Phi_j^\mathrm{L}(\mathbf{x})$ and $\Phi_l^\mathrm{H}(\mathbf{x})$ are respectively conventional linear and hierarchic shape functions.

To maintain simple coupling conditions between elements and orders, hierarchic shape functions are designed to preserve nodal solution value at linear nodes. Many kind of polynomials can be chosen to build a complete function space : Lagrange, Legendre or Chebyshev polynomials. Legendre based polynomials (also referred as Lobatto method) have been shown to improve matrix conditioning [2] (mass matrix becomes diagonal for 1D cases).

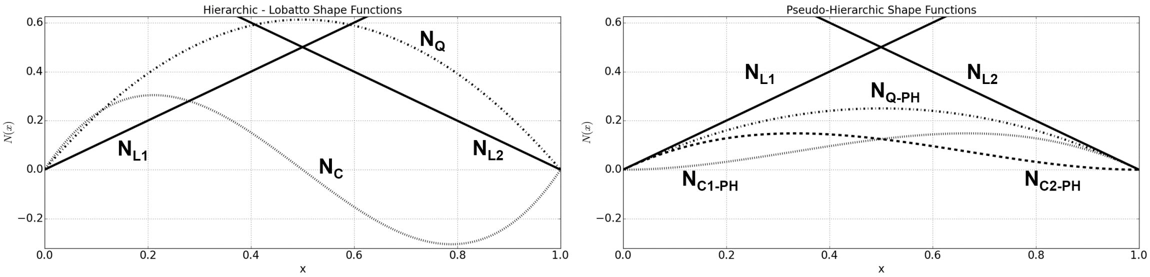
### 2.2 Hierarchic vs Pseudo–Hierarchic FEM

Despite the advantageous numerical properties of Legendre based approximations on matrix conditioning, those shape functions suffer from not being positive over the entire element with sometimes null integrals over the element. Such property prevents lumping or normalization by shape function integral. To satisfy positivity over the element while preserving simple coupling condition between orders, Lobatto functions are combined to construct polynomials used by Pseudo–Hierarchical method:

$$N_{C-PH}(\mathbf{x}) = a_1.N_Q(\mathbf{x}) \pm b_2.N_C(\mathbf{x}) \tag{6}$$

$$N_{Q-PH}(\mathbf{x}) = N_{C1-PH}(\mathbf{x}) + N_{C2-PH}(\mathbf{x}) \tag{7}$$

with $N_Q$ and $N_C$ the quadratic and cubic shape functions derived with Lobatto rule.




(a) Hierarchic–Lobatto (b) Pseudo–Hierarchic

Figure 1: Comparisons of Hierarchic and Pseudo-Hierarchic shape functions (arbitrary amplitude)

Functional basis cannot anymore be called hierarchic as increasing the interpolation order can modify previous order but its advantage of simple coupling conditions between orders and elements (Eq. 7) is preserved. Note that when extending this pseudo-hierarchical scheme to higher order, adding an even order do not modify previous shape functions. Adding an odd order duplicates the previous even shape functions into two new functions of the new max order. This scheme can be seen as pair-hierarchic.

Higher dimension elements are derived by polynomial products proposed by [3], using a topological based data structure.

## 3. Modal Extraction convergence Study

In order to characterize and assess the implementation of PHFEM into Actran, a convergence study is performed on an acoustic rectangular cavity. Numerical eigenfrequencies are comprared to analytical references to assess accuracy of the method with respect to dispersion errors (and compared with conventional quadratic FEM).

Modal extraction is computed for different mesh types to cover the range of acoustic finite elements available Actran (triangles, quadrangles, tetrahedra, pyramids, wedges and hexahedra) :

- 2D air cavity $[1.1m \times 0.3m]$, eigen-frequency of mode (12,0) ($\approx 2173$Hz)
- 3D air cavity $[1.1m \times 0.3m \times 0.1m]$, eigen-frequency of mode (5,3,0) ($\approx 1867$Hz)

Eigen-frequency of mode $i, j, l$ of rectangular cavity are given by:

$$f_e = \frac{c}{2}\sqrt{\frac{i^2}{a^2} + \frac{j^2}{b^2} + \frac{l^2}{d^2}} \; [4] \tag{8}$$

where $c$ is the speed of sound, $a$, $b$ and $d$ are the edge lengths of the cavity.

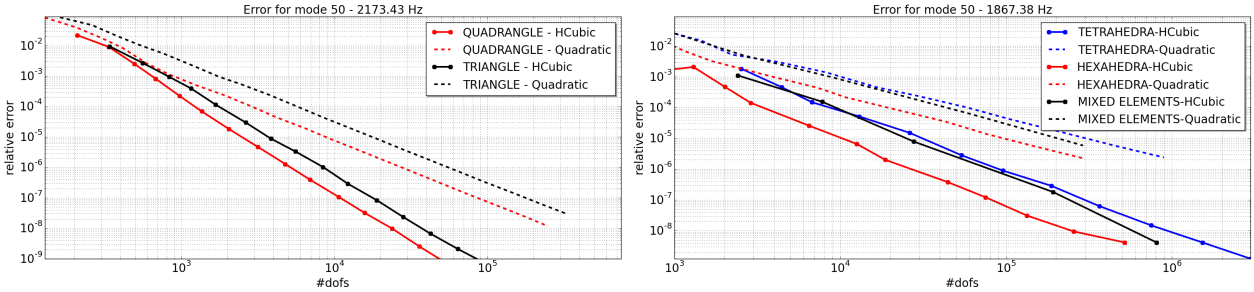
The two studied frequencies are categorized into relatively high $ka$ modes, where $k$ is the acoustic wave number and $a$ is a characteristic dimension of the problem. $ka$ is an indicator of the acoustic size of the model, giving the entension of the FE mesh measured in wavelengths.

Table 1: Order of convergence for conventional quadratic and pseudo–hierarchic cubic (HCubic) interpolation for different mesh types (Fig. 2)

| | Triangles | Quadrangles | Tetrahedra | Hexahedra | Mixed 3D elements |
|---|---|---|---|---|---|
| Quadratic | 3.728 | 3.983 | 3.944 | 4.070 | 3.803 |
| HCubic | 5.801 | 6.017 | 5.853 | 6.049 | 5.567 |

The analysis of the error on eigen-frequency computation is performed to validate the implementation of PHFEM :

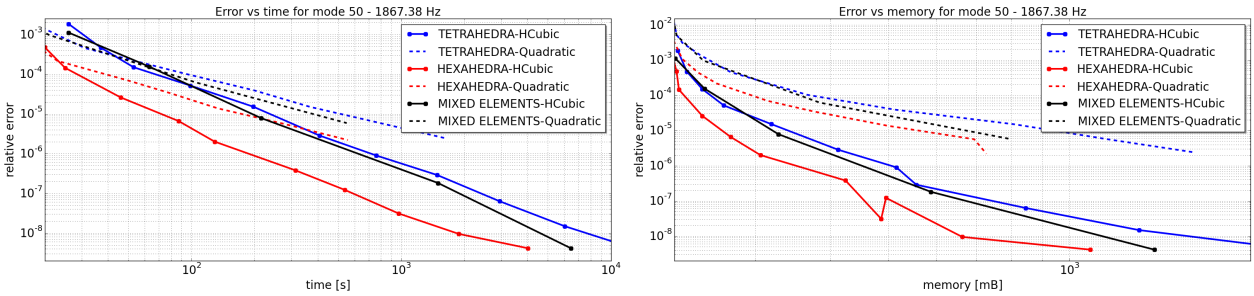
$$e_h <= C_1\theta^p + C_2(p)k\theta^{2p} \quad \text{with} \quad \theta = \frac{kh}{2p} \; [5] \tag{9}$$




(a) 2D – Mode 12,2 (2173Hz) – $ka = 44$ (b) 3D – Mode 5,3,0 (1867Hz) – $ka = 38$

Figure 2: Errors vs number of degrees of freedom on eigen frequency (log-log scales)

where $p$ is the polynomial order, $k$ is considered wave-number, $h$ is the element size, $C_1$ and $C_2$ are constants.
The first term is the dissipation error and the second term is the dispersion error that characterizes the numerical wave velocity error, often the main source of error for large $ka$ models. For a modal extraction, the first term is seen as an error on eigenmode shapes, while the dispersion term drives the eigenfrequencies errors. Asymptotic convergence order $2p$ is given by the opposite of the slope of convergence curves (Fig. 2). Values in Tab. 1 indicate that asymptotic convergence is reached for all elements/interpolations and therefore validate the implementation of the method.




(a) Error vs Time (log-log scales) (b) Error vs Memory (log-log scales)

Figure 3: Performance on error for Mode 5,3,0 (1867Hz) – $ka = 38$

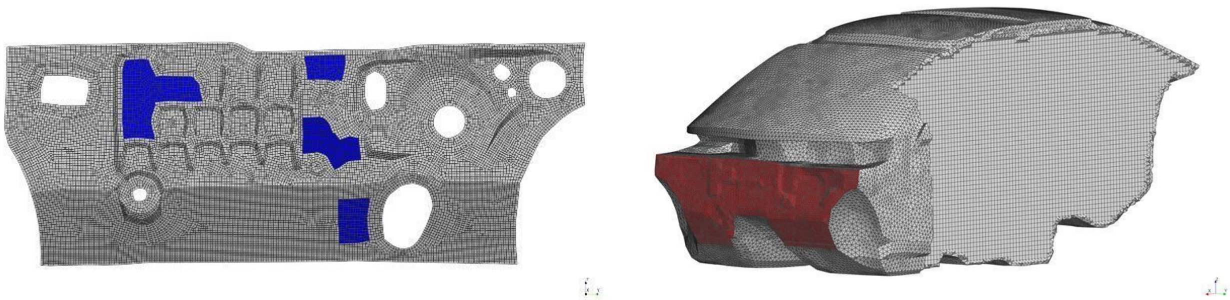
Performance of PHFEM method is compared in terms of CPU time and memory comsumption to conventional FEM on Fig. 3. At high accuracy, HCubic is always faster and requires fewer memory than conventional FEM. At lower precision (error $\approx 1e^{-3} - 1e^{-4}$), time convergence curves crosses, reducing slightly the performance advantage of using HCubic. This suggests that higher order PH-FEM might not be to most efficient solution for all analysis scenarios with respect to conventional quadratic interpolations.

## 4. Application on Industrial cases

The convergence study done on the parallelepipedic cavity highlights the benefit of using higher order PHFEM to improve the efficiency of a simple modal extraction calculation. In this section, performance of PHFEM method is investigated for two cases corresponding to typical vibro-acoustic calculations run in the industry.

### 4.1 Car Cavity excited by a Vibrating Panel

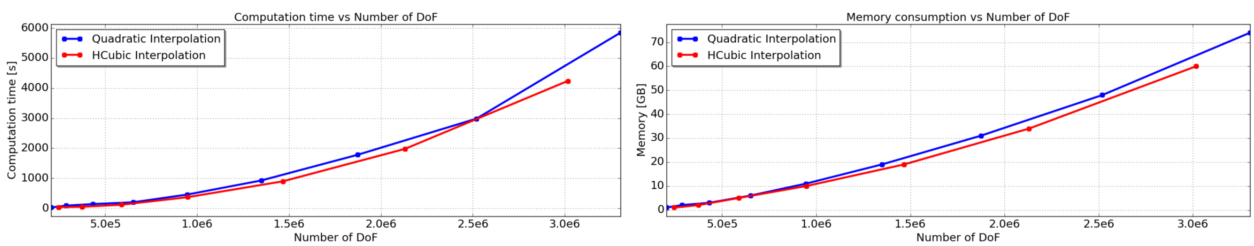
The first studied model is shown in Fig. 4. It consists of a car firewall panel excited by a diffuse sound field and allowed to radiate noise in a car cavity. The firewall panel is modelled using a linear thin shell formulation, meshed with triangle and quadrangle elements, and includes damping pads modelled using a linear thick shell formulation and meshed with Pentahedra and Hexahedra elements. The acoustic cavity is meshed with Tetrahedra, Pyramids and a majority of Hexahedra elements, its size in term of wavelengths is $ka$=185 at 4000Hz. The firewall panel is interfaced with the cavity through an incompatible mesh interface. The calculation is run for 50 frequencies evenly distributed from 3500Hz to 4000Hz. The mean pressure in the cavity and the pressure at driver's ear are output. The parametric study is performed by changing the $n\lambda_{4000}$ of the cavity mesh. $n\lambda_{freq}$ is an indicator of the acoustic mesh refinement which describes the number of finite elements $n$ per acoustic wavelength $\lambda$ at frequency $freq$. The surface mesh used for the incompatible interface is not remeshed ensure no bias is introduced by the incompatible coupling in relative error results.



(a) Firewall panel with damping pads (in blue) (b) Car Cavity

Figure 4: Vibro-Acoustic model 1 - Damped Firewall Panel and Acoustic Cavity

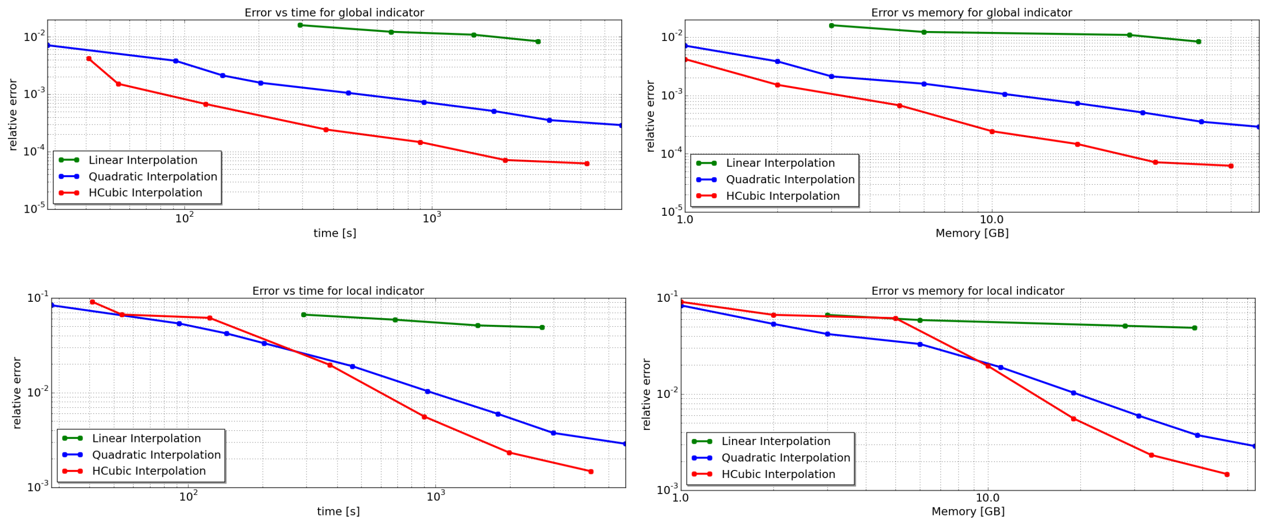
The evolution of the computation time and memory consumption with respect to the number of degrees of freedom in the system (Fig 5) shows that the use of Quadratic or HCubic interpolation for the acoustic elements of the cavity leads to similar efficiency for the solving of the finite element problem. For all acoustic meshes, the computation time and required memory are equivalent or lower when using elements of interpolation HCubic.




(a) Computation time vs Number of DoF (b) Memory consumption vs Number of DoF

Figure 5: Performance of the HCubic elements implementation

The mean relative error over the complete frequency range of interest is computed for both local and global indicators using a reference configuration where acoustic mesh is refined (HCubic interpolation with $n\lambda_{4000Hz} = 4$). Results for the mean square pressure in the cavity (global indicator) and the pressure at driver's ear (local indicator) are presented in Fig 6.




(a) Error vs Time (log-log scales) (b) Error vs Memory (log-log scales)

Figure 6: Performance on error for vibro-acoustic case

The performance gain brought by the HCubic interpolation depends on the indicator and the size of the problem. Looking at the global indicator for a constant accuracy level of 0.1%, a 84% computation time gain is observed whereas only a 35% time reduction is observed when looking at the local indicator for a constant accuracy level of 1%. For the local indicator, the quadratic and cubic curves are intersecting which indicates that there is not necessarily a benefit of using HCubic interpolation elements depending on the required accuracy. One can also observe that the gain obtained by switching from Quadratic to Cubic interpolation is interesting in most regions but far less important than the gain obtained by switching from Linear to Quadratic interpolation.

## 4.2 Transmission Loss of a Car Firewall Panel

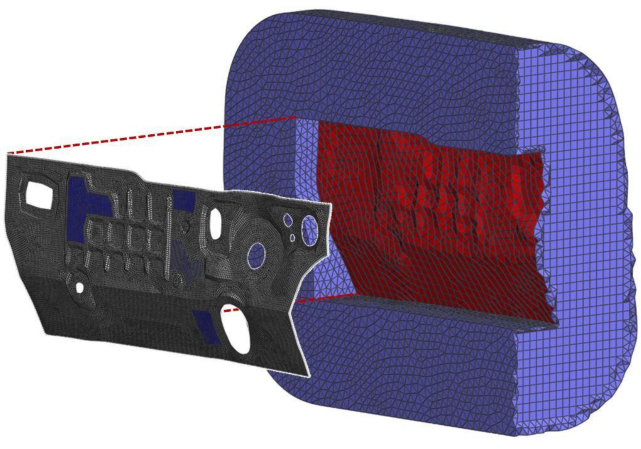
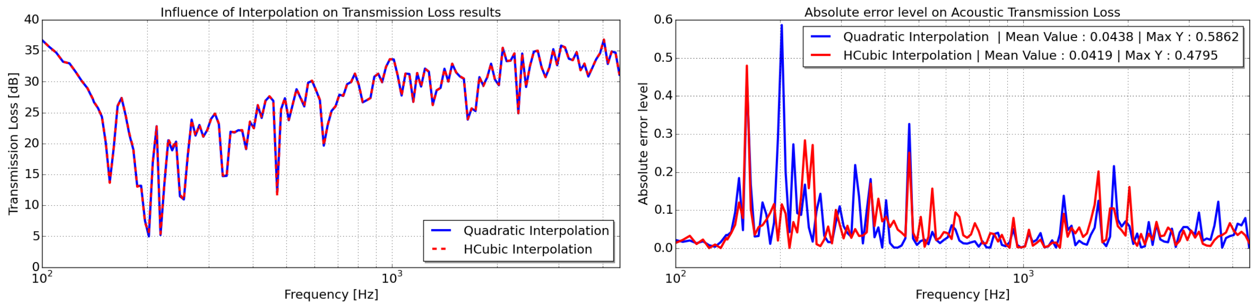


Figure 7: Vibro-Acoustic model 2 - Transmission Loss of a Car Firewall Panel

The second studied model is shown in Fig. 7. It consists of the same car firewall panel as in the first vibro-acoustic model. It is excited by a diffuse sound field excitation and allowed to radiate noise in a semi-free field environment in such a way that the radiated noise is evaluated and a Transmission Loss (TL) indicator is computed. The semi free field condition can be ensured through the use of infinite elements (IFE) [6] or Perfectly Matched Layers (PML) [7]. When IFE are used, a radial interpolation of order 10 and a tangential interpolation order consistent with the interpolation order of the acoustic mesh are used. If PML are used, the interpolation order of the PML is consistent with the interpolation order of the acoustic mesh.

Two vibro-acoustic calculations are run from 10Hz to 4500Hz computing 10 frequencies per 1/3 octave band. Quadratic and HCubic interpolations are used for the acoustic mesh and H-adaptivity is used to adapt the mesh size to the computed frequency. Seven different meshes are used to compute the response for the complete frequency range. Calculations are run in sequential on a machine equipped with Ivybridge Intel® E3 processors (3.4GHz) and 32GB of RAM.

For vibro-acoustic studies in an industrial context, a quadratic mesh with $n\lambda = 4$ is often used and considered as a good compromise between accuracy and finite element model size. Results computed with this mesh refinement will therefore be considered as target accuracy for this application. The $n\lambda$ of the HCubic interpolation mesh is determined so the accuracy level corresponds to this target accuracy level.




(a) Acoustic Transmission Loss of firewall panel

(b) Absolute error in the Transmission Loss

Figure 8: Influence of interpolation order on Transmission Loss of firewall panel

As shown on Fig 8, the acoustic transmission loss results is evaluated with less than 0.6dB absolute deviation compared to a reference case where a refined acoustic mesh with HCubic interpolation and $n\lambda = 4$ is used. The mean absolute error for the TL indicator is around 0.04dB for both mesh interpolation

Table 2: Computation time for the acoustic Transmission Loss of the firewall panel (sequential run)

| | Computation time per frequency | | | | | | | Total time |
|---|---|---|---|---|---|---|---|---|
| | 2050Hz | 3158Hz | 3760Hz | 4087Hz | 4265Hz | 4362Hz | 4467Hz | h-adaptivity |
| Quadratic | 42s | 02m39s | 04m52s | 06m57s | 08m03s | 09m02s | 09m44s | 03h54m |
| HCubic | 42s | 02m02s | 03m38s | 04m36s | 05m38s | 05m40s | 06m20s | 03h21m |
| $ka$ | 9.47 | 14.59 | 17.37 | 18.88 | 19.7 | 20.15 | 20.64 | |
| Time gain | 0% | 23% | 25% | 34% | 30% | 37% | 35% | 14% |

The computation time metrics are gathered in Table 2 for both Quadratic and HCubic interpolation and for all meshes used to compute the complete frequency range. As observed in the previous applications cases, the benefit of using HCubic interpolation is more important when the $ka$ of the model increases. The benefit of using h-adaptivity, already demonstrated in [8], is important for both Quadratic and HCubic interpolation. The use of HCubic interpolation in the framework of the h-adaptivity allows to reduce further the total computation time without any compromise on the accuracy. In the framework of h-adaptivity, HCubic interpolation could also be used to extend the frequency range validity of a given mesh and therefore reduce the amount of remeshing operations.

Results of these two industrial cases confirm the conclusions drawn based on the parallelepipedic cavity modal extraction. High order PHFEM allows significant gain in performance in some cases but is not necessarily suitable for all applications. Based on the above observations, the use of HCubic interpolation elements is recommended when the model $ka$ is high or when the required accuracy is very high.

## 5. Conclusions

Third order Pseudo–Hierarchic scheme has been developed and implemented into the Actran acoustic software suite. Lobatto functions are combined to construct polynomials used by Pseudo–Hierarchical method which satisfies positivity over the element while preserving simple coupling condition between orders.

Asymptotic convergence of dispersion error on modal extraction has been demonstrated for 2D and 3D cases, and for all conventional element types: triangle, quadrangle, tetrahedron, pyramid, wedge and hexahedron.

Efficiency of the method has been investigated on two vibro-acoustic configurations involving a car firewall panel with industry accuracy standards. The use of HCubic finite elements can benefit many acoustic and vibro-acoustic applications regardless of the type of elements used. The implementation is compatible with conventional non reflecting boundary conditions (Infinite Elements and Perfectly Matched Layers) as well as incompatible mesh fluid-structure interface and h-adaptivity. Advantage of using cubic interpolation has been highlighted for models with large acoustic propagation domains, high frequencies or when a very fine accuracy is required. Reduction of computation time up to 85% was observed on a realistic vibro-acoustic case.

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