VERY LOW FREQUENCY EMITTING SONAR ARRAY DESIGN

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INTRODUCTION

These last years, modern conception of submarines has vastly improved their acoustic quieting with the result that their long range passive detection becomes now more and more difficult. Designing active sonars for this goal necessitates that high power V.L.F. emitting arrays be developped.

Due to the limited weight and size of a towed sonar, combined with the use of very low frequencies, very complex phenomenons arise in acoustical interactions between transducers composing the array.

To design such arrays, a very precise knowledge of these near-field interactions is needed, in order to forecast both power distribution per component and radiatted directivity patterns.

To model accurately small (the order of the wavelength) arbitrary shaped radiatting bodies, and especially to obtain acoustic near-fields, numerical Boundary Elements Method is required. By using these technics, and meshing very realistic objects (see figure 1), we can model the whole array and its relevant interacting environment. As usual with Finite Elements Methods, the only limitation concerns the size of numerical algebraic systems to be solved with computer aid. In three dimensions, according to the common $\frac{\lambda}{4}$ criterion and the required precise description for geometrical details, we have usually to compute complex algebraic systems of more than 1000 equations. To reduce CPU costs, we use the Hamdi's variationnal formulation[1] based on Helmholtz Integral Equation and its normal derivative. By this way, we avoid numerical difficulties due to the so-called singular integrals appearing in such technics. Moreover, Hamdi's formulation provide symetrical algebraic systems, which are much more efficient to solve than usual non-symetrical ones obtained by common B.E.M..

In this paper, we present a complete computer aided tool for array design which couples the preceeding B.E. Method to an Equivalent Scheme modelling for each elementary piezo-electric tonpilz transducer.

The interest resides in the fact that heavy (and expensive) numerical modelling is separate from the optimization procedure. This means that after having choosen a geometrical configuration for the array (number and location of transducers, diameter of their head-mass, baffles, ...), we compute all relevant parameters:

* Mutual acoustical impedances;
* Elementary far-fields.

After that, we use these datas as starting point to optimize the transducers and the electrical supply, in order to obtain the required:

* Directivity patterns;
* Sound levels;

and satisfying technological constraints.

The last work consists in simulations of relevant real working cases, as, for example, components breaking down. This provides a complete survey of the system performance.

In section I, Hamdi's B.E.M. is presented, after what we describe its use for V.L.F. array modelling, and discuss numerical aspects.

Section II deals with the whole array design procedure. The coupled B.E.M-equivalent scheme model is presented.

In Section III, experimental and computed results are compared. Physical phenomenons are analysed.

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In Section IV, present and future prospects are presented. In particular, we are presently working on flextensional transducers modelling in the context of the presented C.A.D. tool. Our goal is to keep using equivalent schemes, for low cost computing, but taking into account different vibrating modes and their acoustic coupling.

I. B.E.M.

It has been widely prooved that the only mean to model underwater vibrating arbitrary shaped structures is Finite Element Method coupled to Integral Equations description for the unbounded fluid domain. This is due essentially to three points:

* first of all, analytical methods are limited to canonical geometries;

* secondly, technics based on asymptotic expansions are also generally limited to high frequency range;

* thirdly, F.E. description for the fluid domain is not well adapted to describe far field conditions.

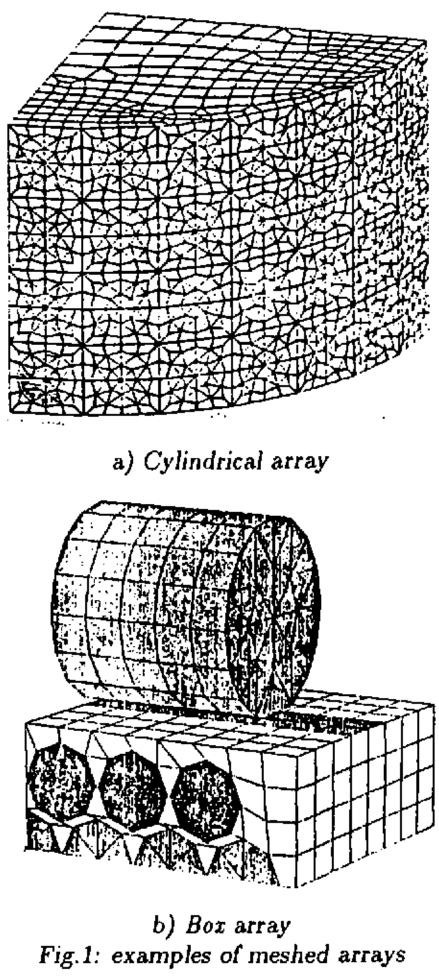
In the case of a whole array, we suppose that the environment (structure, baffles, ...) can be described by simple boundary conditions (rigid, soft or with a given local impedance). Moreover, each transducer head-mass is supposed to vibrate with a constant but unknown axial velocity V (piston). Its behavior will be describe later using an equivalent scheme relying V and P_m (mean pressure on head-mass), and electrical parameters: U and I. By this way, we avoid the use of finite elements for the structure and transducers and reduce the size of the final algebraic system to compute.

The general problem consists in calculating the acoustical pressure P satisfying Helmholtz equation and Sommerfeld condition in the fluid domain Ω, plus boundary conditions on the baffles: $\Gamma_{p=0}$, $\Gamma_{v=0}$, Γ_z and on active surfaces of transducers head-masses: Γ_v. The starting point of the method is the well known Helmholtz integral expression for the pressure given by:

$$c(M)\ P(M) = \iint_\Gamma \{\ P(M_0)\ \frac{\partial E(k,M,M_0)}{\partial \nu(M_0)} - \frac{\partial P(M_0)}{\partial \nu(M_0)} E(k,M,M_0)\ \}\ d\Gamma \qquad (1)$$

where c is equal to 1 if M is in Ω but not on Γ, $\frac{1}{2}$ if M is on Γ and Γ is regular, 0 if M is interior to Γ; E is the elementary solution of Helmholtz equation in three dimension: $E(k,M,M_0) = \frac{e^{ikR}}{4\pi R}$ (assuming a $e^{-i\omega t}$ time dependance); k is the wavenumber: $k = \frac{\omega}{C}$ with ω the angular frequency, and C the acoustic celerity; $\frac{\partial}{\partial \nu}$ is the (exterior) normal derivative operator.

A first method consists in discretizing Γ in surface finite elements at this stage. Then equation (1) is written at each node of the mesh, and interpolation



a) Cylindrical array

b) Box array
Fig.1: examples of meshed arrays

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in each element is used to compute integrals. This provides the following algebraic system:

$$\{\mathbf{P}\} = [\mathbf{A}]\ \{\mathbf{P}\} + [\mathbf{B}]\ \{\mathbf{V}\} \qquad (2)$$

where **P** is the vector of nodal pressures, **V** is the vector of nodal normal velocities, **A** and **B** are the matrices representing the kernel of respectively the first and second integral of equation (1).

Then applying the boundary conditions, one can obtain, with system (2), both pressure and normal velocity on Γ.

It is to underline that system (2), resulting from a list of equations, is a non-symetric one. Moreover, the singularity apearing principaly in the first integral of equation (1) has to be mentionned. To compute **A** and **B** matrix of equation (2), semi-analytical special treatments are required, and the computation becomes complex.

M.A.Hamdi[1] proposes another way of solving the problem. Before discretizing the surface, we build a symetrical variational formulation using equation (1) and its derivative form. We describe, in Appendix, how such a formulation can be obtain, when Γ is reduced to $\Gamma_v + \Gamma_z$. This case corresponds to our practical application, but it will be easy to extend equations to the general problem.

This technique presents essentially two advantages. First of all, all the integrals are regularized by the fact that the dimension of the integration space is now 4 (double surface integration). For instance, in the case of **D** matrices, the most singular ones, the kernel of the integral is singular as $\frac{1}{R^3}$ as well as the whole integrals become regular. It is then very easy to compute all the terms accurately using common numerical technics (Gauss). Secondly, the algebraic complex system given by equations (A5), (A6) is symetric. This allows us to use the most efficient numerical algorithms to solve it.

Let us now consider the present application. A surface mesh is generated as close as possible to the real geometry of the array and its environment (see figure 1). Each surface is characterized by its local impedance, except radiatting head-masses. We want to compute the Z^r_{ij} term of the *in-antenna* acoustic impedance matrix between transducers i and j defined by:

$$\begin{Bmatrix} \overline{P}_1 \\ \vdots \\ \overline{P}_N \end{Bmatrix} = \begin{bmatrix} Z^r_{11} & \dots & Z^r_{1N} \\ \vdots & \ddots & \vdots \\ Z^r_{N1} & \dots & Z^r_{NN} \end{bmatrix} \begin{Bmatrix} V_1 \\ \vdots \\ V_N \end{Bmatrix} \qquad (3)$$

where $\overline{P}_i$ is the mean pressure over the radiatting surface of the transducer i, and V_i its axial velocity.

To do so, we successively impose:

$$\begin{cases} V_i = 1 \\ V_{j\neq i} = 0 \end{cases}$$

for i in $\{1, 2, \dots, N\}$, and compute every $Z^r_{ij} = \overline{P}_j$. Actually, we use the symetry of $\mathbf{Z^r}$ matrix, and only $\frac{N(N+1)}{2}$ terms have to be calculated (instead of N^2).

Furthermore, we also compute, for each transducer i, what we call the *elementary far-field* P^∞_i. This job consists — after having solved the system {(A5)-(A6)} and then knowing P and $\frac{\partial P}{\partial \nu}$ on Γ — to use equation (1) for M in Ω and is just a post-processor procedure for which CPU cost is not prohibitive. For each point of the directivity patterns, complex radiatted pressure is computed. These results will be used later to obtain directivity patterns corresponding to a given excitation for the whole array, by the way of a very simple summation:

$$D = \|\sum_{i=1}^{N} V_i P^\infty_i\|$$

Finally, radiatted sound level will be computed identically.

Ending this chapter, a few numerical points have to be discussed. First of all, the so-called *irregular frequencies* for which B.E.M. cannot provide a unique solution for the exterior problem were treated, when necessary, using aditional interior boundary elements (for example with $v_\nu = 0$). This technic was successful in all current applications.

The second remark concerns the criterion for elements size. Commonly, a $\frac{\lambda}{4}$ is adopted. Neverthe-

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less, modelling *small* objects — head-mass diameter of $\frac{\lambda}{4}$ — much more elements per λ are needed.

II. Transducers Modelling - Array design

In this section, we present the whole design process for a V.L.F. high power emitting array. This one uses common active arrays design technics, but taking into account aditional constraints due to very low frequencies.

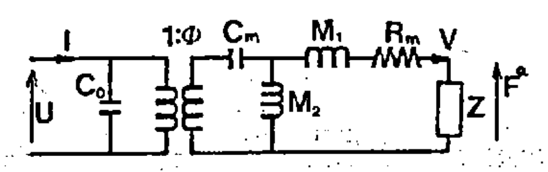
If we suppose that elementary transducer acoustic behavior and geometrical diffraction effects of the whole array are independant, then common design procedure consists in two separate steps. First of all, rough models — for instance, monopoles arrays — are used to define both sources location and relative weight, such that acceptable directivity patterns can be obtained. Secondly, elementary transducer is studied in order to satisfy central frequency, radiated power, ... It is clear that these parameters are obtained for *a priori* radiation conditions, non related to the real *in antenna* ones. Nevertheless, array directivity index calculated in the first step is used to determine each transducer power.

As we stressed in introduction, for V.L.F. arrays, transducers acoustic loads are dramatically influenced by the array geometry (head-masses diameters and locations, baffles) and the actual velocity distribution per component: acoustic mutual interactions have then to be considered when designing the elementary transducers.

The proposed design procedure introduces, in the first step described before, the use of B.E.M. to improve the choice of the array geometry. This means that we compute the directivity patterns due to optimal velocity distributions and compare them to the desired ones. In a way, it consists in a first *manual optimization* of baffles. Nevertheless, it is clear that the choosen velocities will be modified when taking into account transducers constraints — for example electrical supply grouping —. This implies that it will be neccessary to simulate the whole array behavior when it is totaly designed.

Having choosen a geometrical configuration, both mutual acoustic impedances Z^r_{ij} and elementary far-field P_i^∞, provided by the B.E.M. program, are used to design elementary transducers as follows:

In the case of a tonpilz, the following equivalent scheme is adopted:



where U and I are the electrical parameters, F^a and V the acoustic force and velocity, C_0 the static capacitance, Φ the electro-mechanical factor, C_m the motional compliance, M_1 and M_2 the head and tail masses, R_m the losses resistance and Z^a the acoustic load.

The unknown parameters are Φ, C_m, M_1 and M_2. C_0 can be deduced from Φ and R_m can be estimated from the transducer technology.

Then, we adopt an iterative procedure to find optimal transducers set, such that the whole array emitted power, in its frequency range, corresponds to the desired one. On an industrial point of vue, the definition of a unique transducer will be deeply preferred. Nevertheless, when impossible — due to technological constraints: weight, dimensions, ... —, the presented general procedure remains applicable in order to design several different components. Our own experience showed that such an optimization needs computer aid.

To model the whole array, we used a modified form of the matricial system (3):

$$\{\mathbf{F^a}\} = [\mathbf{Z}]\,\{\mathbf{V}\} \qquad (4)$$

where $F_i^a = S_i\overline{P}_i$ and $Z_{ij} = S_i Z^r_{ij}$ with S_i the surface of transducer i. The equivalent scheme provides the following matrix relation for transducer i:

$$\begin{Bmatrix} U^i \\ I^i \end{Bmatrix} = \begin{bmatrix} M^i_{11} & M^i_{12} \\ M^i_{21} & M^i_{22} \end{bmatrix} \begin{Bmatrix} F^a_i \\ V_i \end{Bmatrix} \qquad (5)$$

In the case of a voltage supply, we duplicate the first

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equation of (5) for each component (the second one if a current supply):

$$\begin{Bmatrix} U^1 \\ U^2 \\ \vdots \\ U^N \end{Bmatrix} = \begin{bmatrix} M^1_{11} & 0 & \dots & 0 \\ 0 & M^2_{11} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & M^N_{11} \end{bmatrix} \begin{Bmatrix} F^a_1 \\ F^a_2 \\ \vdots \\ F^a_N \end{Bmatrix} + \begin{bmatrix} M^1_{12} & 0 & \dots & 0 \\ 0 & M^2_{12} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & M^N_{12} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix} \quad (6)$$

Combining (4) and (6), we obtain:

$$\{\mathbf{U}\} = [\mathbf{Z^e}]\{\mathbf{V}\} \quad (7)$$

with

$$\begin{cases} Z^e_{ii} = M^i_{11} Z_{ii} + M^i_{12} \\ Z^e_{ij} = M^i_{11} Z_{ij} \ , \ i \neq j \end{cases}$$

This matricial relation can then be used directly to obtain each elementary electrical supply from optimal velocities. This implies that each component is driven separately from each others. Actually, considering industrial constraints, another solution is often preferred consisting in supply grouping.

A relevant method consists in grouping transducers driven approximatively by the same voltages. Inverting (7), real velocity distribution and associated new array performances (directivity patterns, sound levels) are then calculated.

In conclusion, each step of the optimization procedure consists in:

1: calculation of $\mathbf{Z^e}$ matrix;
2: computation of U by inverting (7);
3: choice of supply grouping;
4: computation of *real* $\mathbf{V}$;
5: calculation of *real* directivity patterns and sound levels;
6: comparizon to the specifications and evaluation of technological faisability;
7: modification of Φ, C_m, M_1, M_2.

For now, the choice of the initial Φ, C_m, M_1, M_2 (and R_m), as well as step 7, needs a lot of human know-how. Nevertheless automatic processes are prospected in THOMSON SINTRA ASM laboratories. In theory the seven steps have to cover the whole frequency range. Practically, steps 1, 2 and 3 are performed only for the central frequency.

III. Experimental and theoretical results

A three staves array was modelled and measured, in order to validate the presented tool. The active pannel is about 1λ X 1.5λ in size, and composed of $\frac{\lambda}{5}$ diameter head-masses.

Directivity patterns, sound levels and electrical impedances have been mesured in the three different excitation cases: parallel, weighted and steered.

Figures 2 to 4 show a good agreement between experimental and theoretical directivity patterns in the three cases.

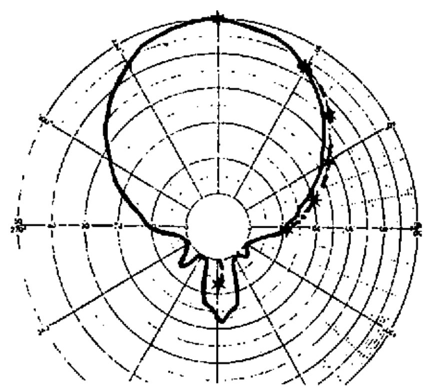

Fig.2: Directivity pattern - bearing - $ka = 0.6$
Parallel excitation case
*(* theory ; — measurement)*

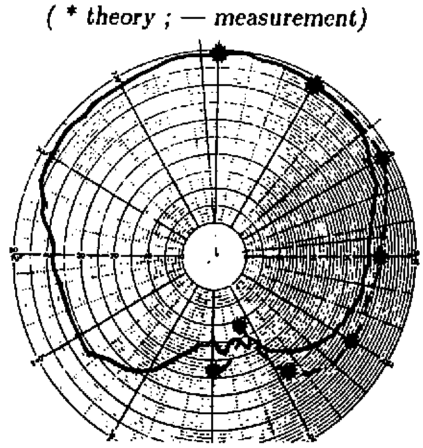

Fig.3: Directivity pattern - bearing - $ka = 0.6$
Weighted excitation case

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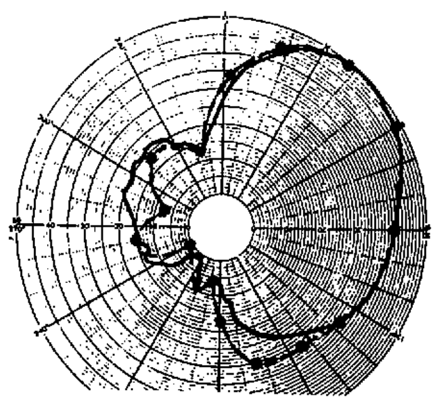

Fig.4: Directivity pattern - bearing - ka = 0.6
Steered excitation case

Due to geometrical diffraction effects, lateral and central staves have very different impedances, as we can see on figures 5 and 6 (real part of electrical impedance in the parallel excitation case).

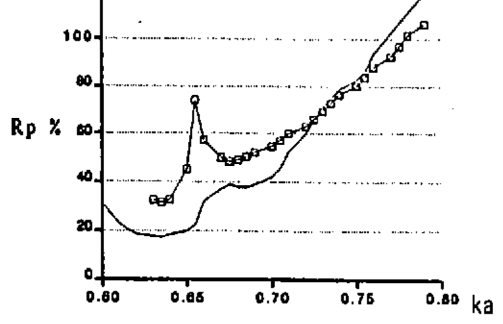

Fig.5: Central stave - resistance
(□ theory ; — measurement)

The differences appearing between theory and experiment on figure 5 are to be justified. First of all, theoretical results are strongly dependent on transducers losses R_m, and especially close to the resonance frequency. Secondly, manufacturing spreading between components impacts on acoustic characteristics around resonance frequency. Let us underline that such spreading can be taken into account by the presented program.

Finally, figure 7 give the computed and measured sound levels.

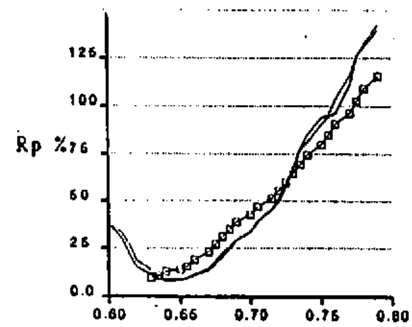

Fig.6: Lateral staves - resistance

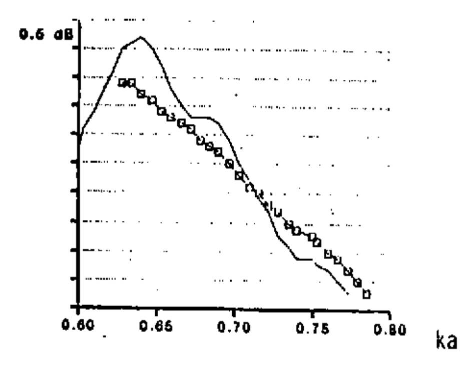

Fig.7: Sound level

IV. Future developments

Two major improvements are presently being investigated. The first one consists in introducing automatic optimization algorithms. For the moment, we are working separately on geometry and velocity distribution, and on transducers design. Knowing the geometry, and using Z_{ij}^r and P_i^∞, it is relatively easy to optimize the velocities. To do the same with the baffles (location, size, impedance) represents a much more difficult work. More, each optimization step requires a new mesh for the whole problem, and CPU costs then increase dramatically.

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Nevertheless, it has been shown, in our laboratories, that great improvements can be obtained by using simple passive baffles. Automatic optimization for elementary tranducers design is also of great interest, especially when strong technological constraints request more than one type of components. Now it seems to be very difficult to avoid human help for this goal.

The second research topic on array modelling concerns other kinds of transducers such as flextensional ones. The main difference with the tonpilz resides in the fact that using only two acoustic parameters ($\overline{P}$, and V) are no more sufficient to describe the whole behavior, when several flextensional transducers are located close to each others, and so interacting strongly. An original way of modelling this problem is to take into account several velocity distributions on the wet surface, as what we call *working modes*. This *modes*, completely different from the natural in-air ones, can be given, for example, by the in-water response of the transducer, with different baffle conditions (free-space, rigid or soft planes at different locations). To compute them, elastic and piezoelectric structures can be modelled by F.E.M. coupled to B.E.M. to describe acoustic radiation. This work can be done without prohibitive CPU cost, by meshing a unique transducer. After that, a similar procedure as described before is adopted. System (3) is computed but in the following form:

$$\begin{Bmatrix} P_{1_1} \\ P_{1_2} \\ \vdots \\ P_{N_n} \end{Bmatrix} = \begin{bmatrix} Z^r_{1_1 1_1} & Z^r_{1_1 1_2} & \cdots & Z^r_{1_1 N_n} \\ Z^r_{1_2 1_1} & Z^r_{1_2 1_2} & \cdots & Z^r_{1_2 N_n} \\ \vdots & \vdots & \ddots & \vdots \\ Z^r_{N_n 1_1} & Z^r_{1_2 N_n} & \cdots & Z^r_{N_n N_n} \end{bmatrix} \begin{Bmatrix} V_{1_1} \\ V_{1_2} \\ \vdots \\ V_{N_n} \end{Bmatrix} \qquad (3)$$

where P_{i_k} and V_{i_k} are the amplitudes of pressure and velocity projected on the *mode* k of the transducer i, and with the corresponding notation for $\mathbf{Z}^r$ matrix indices.

It is just the begining of this research, but first conclusions (one-dimension array of two, three or more flextensional transducers) seem to be promising.

CONCLUSION

A powerful computer aided tool has been developped and validated for the design of V.L.F. emitting sonar array design. This program, coupling an efficient numerical technic, the B.E.M., and the common and easy to handle equivalent scheme model, has already allowed us to find interesting technological solutions for controlling directivity patterns with passive baffles..

The optimization procedures seem to be more and more precisely defined and so we can hope that artificial intelligence will soon apear in our field.

REFERENCES

[1] M A HAMDI, 'Formulàtion Variationnelle Par Equations Intégrales Pour Le Calcul De Champs Acoustiques Linéaires Proches Et Lointains', Thèse de Doctorat d'Etat, Université de Technologie de Compiègne (1982)

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Appendix

Let us consider Γ reduced to $\Gamma_v + \Gamma_z$ defined as follows:

$$\begin{cases} \frac{\partial P(M)}{\partial \nu(M)} = P_\nu^0 \ , \ \forall M \in \Gamma_v \\ P(M) - Z \ \frac{\partial P(M)}{\partial \nu(M)} = 0 \ , \ \forall M \in \Gamma_z \end{cases}$$

In order to form a bilinear quadratic form, we substitute equation (1) and its normal derivative by the two following equivalent ones:

$$\frac{1}{2}\iint_\gamma P\psi d\gamma - \iint_\gamma\iint_{\Gamma_0} P_0 \frac{\partial E}{\partial \nu_0}\psi d\gamma d\Gamma_0 + \iint_\gamma\iint_{\Gamma_0} \frac{\partial P_0}{\partial \nu_0} E\psi d\gamma d\Gamma_0 = 0 \ , \ \forall\psi \qquad (A1)$$

$$\frac{1}{2}\iint_\gamma \frac{\partial P}{\partial \nu}\varphi d\gamma - \iint_\gamma\iint_{\Gamma_0} P_0 \frac{\partial^2 E}{\partial \nu_0 \partial \nu}\varphi d\gamma d\Gamma_0 + \iint_\gamma\iint_{\Gamma_0} \frac{\partial P_0}{\partial \nu_0}\frac{\partial E}{\partial \nu}\varphi d\gamma d\Gamma_0 = 0 \ , \ \forall\varphi \qquad (A2)$$

where γ is a part of Γ. The subscript 0 (respectively no subscript) means that the function is taken on M_0 (respectively on M).

Then, writing (A2) for $\gamma = \Gamma_v$, and (A1)–(A2) for $\gamma = \Gamma_z$, with $\varphi = Z\psi$, and applying the boundary conditions, we obtain:

$$\frac{1}{2}\iint_{\Gamma_v} P_\nu^0 \varphi d\Gamma - \iint_{\Gamma_v}\iint_{\Gamma_{v0}} P_0 \frac{\partial^2 E}{\partial \nu_0 \partial \nu}\varphi d\Gamma d\Gamma_0 - \iint_{\Gamma_v}\iint_{\Gamma_{z0}} P_0 \left(\frac{\partial^2 E}{\partial \nu_0 \partial \nu} - \frac{1}{Z}\frac{\partial E}{\partial \nu}\right)\varphi d\Gamma d\Gamma_0$$

$$+ \iint_{\Gamma_v}\iint_{\Gamma_{v0}} P_{\nu 0}^0 \frac{\partial E}{\partial \nu}\varphi d\Gamma d\Gamma_0 = 0 \ , \ \forall\varphi \qquad (A3)$$

$$-\iint_{\Gamma_z}\iint_{\Gamma_{z0}} P_0 \left\{\frac{1}{Z}E + \left(\frac{\partial E}{\partial \nu_0} + \frac{\partial E}{\partial \nu}\right) + Z\frac{\partial^2 E}{\partial \nu_0 \partial \nu}\right\}\psi d\Gamma d\Gamma_0 - \iint_{\Gamma_z}\iint_{\Gamma_{v0}} P_0 \left(\frac{\partial E}{\partial \nu_0} - Z\frac{\partial^2 E}{\partial \nu_0 \partial \nu}\right)\psi d\Gamma d\Gamma_0$$

$$+ \iint_{\Gamma_z}\iint_{\Gamma_{v0}} P_{\nu 0}^0 \left(E - Z\frac{\partial E}{\partial \nu}\right)\psi d\Gamma d\Gamma_0 = 0 \ , \ \forall\psi \qquad (A4)$$

Let us now discretize Γ in Boundary Finite Elements, the preceeding equations (A3) and (A4) provide the following system in which matricial terms (in (A5), respectively (A6)) apear in the same order as corresponding integral terms (in (A3), respectively (A4)):

$$\underbrace{\frac{1}{2}[\mathbf{C}]\{\mathbf{P}_\nu^0\}}_{\{\mathbf{S}_v^1\}} - [\mathbf{D}_{vv}]\{\mathbf{P}_v\} - \left([\mathbf{D}_{vz}] - \frac{1}{\mathbf{Z}}[\mathbf{B}_{vz}]\right)\{\mathbf{P}_z\} + \underbrace{[\mathbf{B}_{vv}]\{\mathbf{P}_\nu^0\}}_{\{\mathbf{S}_v^2\}} = \{0\} \qquad (A5)$$

$$-\left(\frac{1}{\mathbf{Z}}[\mathbf{A}_{zz}] + [\mathbf{B}_{zz}] + [\mathbf{B}_{zz}]^t + \mathbf{Z}[\mathbf{D}_{zz}]\right)\{\mathbf{P}_z\} - ([\mathbf{B}_{zv}] - \mathbf{Z}[\mathbf{D}_{zv}])\{\mathbf{P}_z\} + \underbrace{([\mathbf{A}_{zv}] - \mathbf{Z}[\mathbf{B}_{zv}])\{\mathbf{P}_\nu^0\}}_{[\mathbf{S}_z]} = \{0\}$$

$$(A6)$$