# TORSIONAL VIBRATION SUPPRESSION IN AUTOMATIC TRANSMISSION POWERTRAIN USING CENTRIFUGAL PENDULUM VIBRATION ABSORBER

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There are numerous problems related to vibration in the powertrain that prevent improvements in fuel economy in vehicles. The primary vibration source is engine forced vibration. Engine torque fluctuations create engine speed fluctuations as a result of combustion. In the latest trend in engine technology, diesel engines and higher-power engines are widely used. These engines contribute to strong torsional vibrations in powertrain systems. A torque converter is an element that transfers torque from the engine to the gear train in the automatic transmission of an automobile. With the goal of improving fuel efficiency, a lock-up clutch system in the torque converter has been developed that locks the input and output sides directly from low engine rotation speeds. Therefore, the torsional vibration in powertrain systems is increased from low engine rotation speeds. In order to address this problem, a new vibration suppression method which is centrifugal pendulum absorber must be designed so as to effectively absorb the torsional vibration over a wide range of engine rotation speeds. However, a new vibration problem occurs when the centrifugal pendulum absorber is attached to a powertrain system. In the present paper, a centrifugal pendulum vibration absorber is applied in order to effectively absorb the torsional vibration. The generation mechanism and countermeasures for this unusual vibration are investigated. The centrifugal pendulum vibration absorber has a characteristic that its natural frequency changes in proportion to the engine rotation speed.
Keywords: Vibration of a rotating body, Centrifugal pendulum vibration absorber, Forced vibration, Torsional vibration, Automatic transmission powertrain

## 1. Introduction

Diesel engines, higher-power engines, and low cylinder engines are widely used in the latest engine technologies. These engines contribute to strong torsional vibration in powertrain systems. The requirement for improved fuel efficiency has led to the development of a lock-up clutch system for low engine rotation speeds. Since engine torque fluctuations are transmitted directly to the tires through the torque converter, the gear train, and the drive shaft in the lock-up condition, the desire to increase ride comfort improvement is increasing. A centrifugal pendulum vibration absorber, the natural frequency of which changes in proportion to the engine rotation speed, is also under development, and several studies have investigated its use to suppress torsional vibration [1]-[6].

Ishida clarified experimentally and analytically the dynamic behavior of a centrifugal pendulum absorber considering the nonlinear characteristics of the absorber in order to absorb the torsional vibration of a simple rotating body model [2]. The results showed that the natural frequency of the centrifugal pendulum absorber changes in proportion to the engine rotation speed, and the antiresonance frequency changes with the engine rotation speed so as to effectively absorb the torsional vibration over a wide range of engine rotation speeds.

Pfabe analytically investigated the optimum design of the centrifugal pendulum absorber in terms of mass reduction and minimizing the space occupied by the centrifugal pendulum absorber [3].

However, a new vibration problem occurs when the centrifugal pendulum absorber is attached to a powertrain system. Figure 1 shows the simulated results for an actual automobile. The figure shows the frequency response curve for the torque fluctuations of one drive shaft when the centrifugal pendulum absorber is attached to a torque converter. A large peak appears at an engine speed of approximately 1,500 rpm. An unusual vibration at approximately 1,500 rpm was also confirmed in a traveling test using an actual automobile to which a centrifugal pendulum absorber was attached. In the present study, the generation mechanism and countermeasures for this unusual vibration are investigated.

We analyzed the reduction of engine torsional vibration using a centrifugal pendulum vibration absorber. The model considered herein includes the engine, the torque converter, the gear train, and the drive shaft. We focused on the optimum design of the ratio between the natural frequency of the centrifugal pendulum vibration absorber and the engine rotation speed, as well as the mass of the centrifugal pendulum vibration absorber for suppressing the torsional vibration.

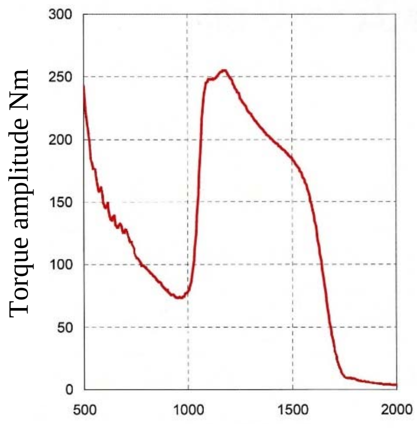
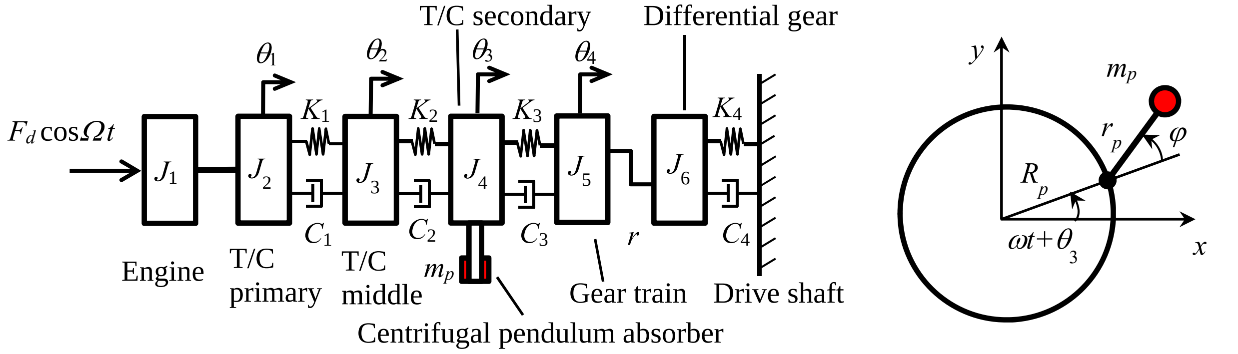



Figure 1: Frequency response curve for actual automobile simulation.

## 2. Theoretical analysis

### 2.1 Analytical model

Figure 2 shows a model of the gear train based on the actual automatic transmission of an FF vehicle. This model includes the engine (four-cylinder), T/C primary, T/C middle, T/C secondary, the gear train, the differential gear, and the centrifugal pendulum absorber. The system is modeled by a five-degree-of-freedom system. Drive shafts are modeled by rotational springs. The wheels are assumed to be fixed ends. In Fig. 2(a), $J_1,\ldots, J_6$ are the moments of inertia of the components, $K_1$, $K_2$, $K_3$, and $K_4$ are the rotational spring constants, $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$ are the angular displacements of each element, $C_1$, $C_2$, $C_3$, and $C_4$ are the damping coefficients of each damping element, $F_d$ is the dynamic torque from the engine, and $\Omega$ is the input angular frequency of the dynamic torque from engine forced vibration. The inertia of the engine $J_1$ and the inertia of the T/C primary $J_2$ are locked directly by a lock-up clutch. Moreover, $r$ is the total gear ratio of the gear train. As shown in Fig. 2(a), a centrifugal pendulum absorber is attached to T/C secondary $\theta_3$ that rotates at angular velocity $\omega$, which is also the engine speed. As shown in Fig. 2(b), $\varphi$, $m_p$, $r_p$, and $R_p$ are the angular displacement, mass, length of the centrifugal pendulum absorber, and the position at which the centrifugal pendulum




(a) Model of the gear train with centrifugal pendulum absorber (b) Centrifugal pendulum absorber

Figure 2: Analytical model.

absorber is attached, respectively.

From the rotating system of coordinates shown in Fig. 2(b), the coordinates of the position of the center of mass of the centrifugal pendulum absorber are as follows:

$$x = R_p \cos(\omega t + \theta_3) + r_p \cos(\omega t + \theta_3 + \varphi), \tag{1}$$

$$y = R_p \sin(\omega t + \theta_3) + r_p \sin(\omega t + \theta_3 + \varphi). \tag{2}$$

The velocities of the centrifugal pendulum absorber are given by the following equations:

$$v_x = -R_p(\omega + \dot{\theta}_3)\sin(\omega t + \theta_3) - r_p(\omega + \dot{\theta}_3 + \dot{\varphi})\sin(\omega t + \theta_3 + \varphi), \tag{3}$$

$$v_y = R_p(\omega + \dot{\theta}_3)\cos(\omega t + \theta_3) + r_p(\omega + \dot{\theta}_3 + \dot{\varphi})\cos(\omega t + \theta_3 + \varphi). \tag{4}$$

The kinetic energy of the system is given by the following equation:

$$T = \frac{1}{2}(J_1 + J_2)(\omega + \dot{\theta}_1)^2 + \frac{1}{2}J_3(\omega + \dot{\theta}_2)^2 + \frac{1}{2}(J_4 + J_p)(\omega + \dot{\theta}_3)^2 + \frac{1}{2}m_p({v_x}^2 + {v_y}^2) + \frac{1}{2}J_5(\omega + \dot{\theta}_4)^2 + \frac{1}{2}J_6\left\{p(\omega + \dot{\theta}_4)\right\}^2. \tag{5}$$

where $p$ is the rotation speed ratio of the gear train ($p = 1/r$). The potential energy of the system is given by the following equation:

$$U = \frac{1}{2}K_1(\theta_1 - \theta_2)^2 + \frac{1}{2}K_2(\theta_2 - \theta_3)^2 + \frac{1}{2}K_3(\theta_3 - \theta_4)^2 + \frac{1}{2}K_4(p\theta_4)^2. \tag{6}$$

Lagrange's equations of motion relating to $\theta_1$ through $\theta_4$ and $\varphi$, taking system damping into consideration are given by Eq. (7) through Eq. (11):

$$(J_1 + J_2)\ddot{\theta}_1 + C_1(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2) = F_d \cos \Omega t. \tag{7}$$

$$J_3\ddot{\theta}_2 + C_1(\dot{\theta}_2 - \dot{\theta}_1) + C_2(\dot{\theta}_2 - \dot{\theta}_3) + K_1(\theta_2 - \theta_1) + K_2(\theta_2 - \theta_3) = 0. \tag{8}$$

$$(J_4 + J_p + m_p {R_p}^2 + m_p {r_p}^2 + 2m_p R_p r_p \cos\varphi)\ddot{\theta}_3 + (m_p {r_p}^2 + m_p R_p r_p \cos\varphi)\ddot{\varphi} - m_p R_p r_p \dot{\varphi}(2\dot{\theta}_3 + 2\omega + \dot{\varphi})\sin\varphi + C_2(\dot{\theta}_3 - \dot{\theta}_2) + C_3(\dot{\theta}_3 - \dot{\theta}_4) + K_2(\theta_3 - \theta_2) + K_3(\theta_3 - \theta_4) = 0. \tag{9}$$

$$(J_5 + p^2 J_6)\ddot{\theta}_4 + C_3(\dot{\theta}_4 - \dot{\theta}_3) + p^2 C_4 \dot{\theta}_4 + K_3(\theta_4 - \theta_3) + p^2 K_4 \theta_4 = 0. \tag{10}$$

$$(m_p r_p^{\,2} + m_p R_p r_p \cos\varphi)\ddot{\theta}_3 + m_p r_p^{\,2}\ddot{\varphi} + c_p\dot{\varphi} + m_p R_p r_p (\omega + \dot{\theta}_3)^2 \sin\varphi = 0. \quad (11)$$

where $c_p$ in Eq. (11) is the damping coefficient of the centrifugal pendulum absorber.

Linearizing Eqs. (9) and (11), we obtain the following:

$$(J_4 + J_p + m_p R_p^{\,2} + m_p r_p^{\,2} + 2m_p R_p r_p)\ddot{\theta}_3 + (m_p r_p^{\,2} + m_p R_p r_p)\ddot{\varphi} + C_2(\dot{\theta}_3 - \dot{\theta}_2) + C_3(\dot{\theta}_3 - \dot{\theta}_4) + K_2(\theta_3 - \theta_2) + K_3(\theta_3 - \theta_4) = 0. \quad (12)$$

$$(m_p r_p^{\,2} + m_p R_p r_p)\ddot{\theta}_3 + m_p r_p^{\,2}\ddot{\varphi} + c_p\dot{\varphi} + m_p R_p r_p \omega^2 \varphi = 0. \quad (13)$$

Here, the equation of motion of undamped free vibration of only the centrifugal pendulum absorber, its natural frequency $\omega_n$ and its order $n$ are given as follows:

$$m_p r_p^{\,2}\ddot{\varphi} + m_p R_p r_p \omega^2 \varphi = 0\,,\ \ \omega_n = n\omega\,,\ \ n = \sqrt{\frac{R_p}{r_p}}\,. \quad (14)$$

The angular frequency of the dynamic torque from the engine forced vibration $\Omega$ is twice the rotation speed $\omega$ of the four-cylinder engine ($\Omega = 2\omega$). In order to suppress the vibration component $\Omega$, the order $n$ must be set to 2 ($\omega_n = 2\omega = \Omega$). If $c_p$ is set to 0 and $n$ is set to 2 in Eq. (13), $\theta_3$ is not vibration solution, and the dynamic amplitude of $\theta_3$ is 0, because $\ddot{\theta}_3$ is 0 in Eq. (13), then the torque fluctuation is 0 in the linear analysis. Therefore, $c_p$ must be considered in the linear analysis.

The Runge-Kutta-Gill method is used for numerical calculation in the linear and nonlinear analyses. As standard parameters of the centrifugal pendulum absorber, $m_p$, $r_p$, $R_p$, $n$, $c_p$, and $F_d$ are set to 1.59 kg, 0.0208 m, 0.0832 m, 2.0, 0.01 Nms/rad, and 600 Nm, respectively.

# 3. Results of numerical calculations

## 3.1 Natural frequencies and natural modes

Table 1 lists the natural frequencies and natural modes. The centrifugal pendulum absorber is not attached. In Table 1, the natural frequency for the second mode is 32.85 Hz. In the second mode, $\theta_2$, $\theta_3$, and $\theta_4$ vibrate out of phase with respect to $\theta_1$.

Table 1: Natural frequencies and natural modes

| | 1st | 2nd | 3rd | 4th |
|---|---|---|---|---|
| $f_n$ (Hz) | 9.71 | 32.85 | 81.83 | 239.66 |
| $\theta_1$ | 1.00 | 1.00 | 1.00 | 1.00 |
| $\theta_2$ | 0.42 | -5.68 | -40.44 | -354.48 |
| $\theta_3$ | 0.33 | -6.00 | -19.99 | 1562.25 |
| $\theta_4$ | 0.23 | -6.12 | 11.26 | -58.57 |

## 3.2 Torque fluctuations for one drive shaft without centrifugal pendulum absorber

This section describes numerical calculations for a system without a centrifugal pendulum absorber. In this forced vibration analysis, we focused on the torque fluctuation for one drive shaft $\Delta T_d$, which is given as follows:

$$\Delta T_d = \frac{C_4 p\Delta\dot{\theta}_4 + K_4 p\Delta\theta_4}{2}. \quad (15)$$

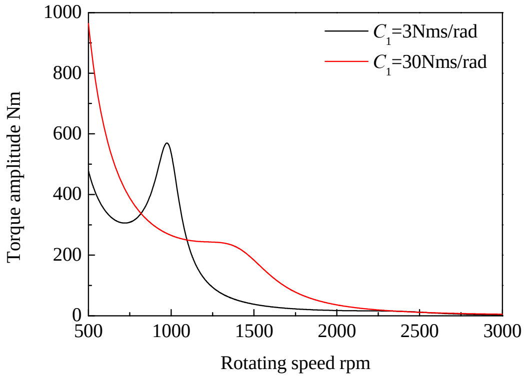


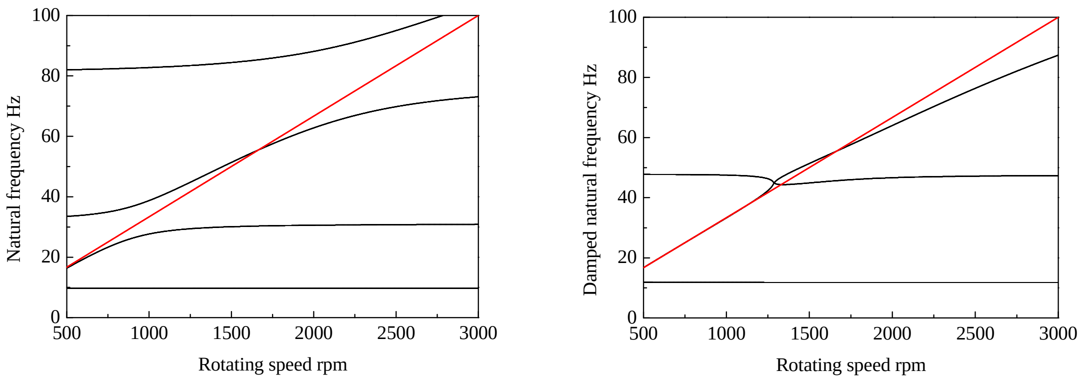

Figure 3: Torque fluctuation for one drive shaft without centrifugal pendulum absorber.

Figure 3 shows the frequency response curves for the drive shaft. The abscissa indicates the engine rotation speed, and the ordinate indicates half the peak-to-peak amplitude of the torque fluctuation for one drive shaft $\Delta T_d$ . Here, the damping coefficient $C_1$ is set to 3 and 30 Nms/rad.

The large peak around 1,000 rpm in Fig. 3 is the main resonance for the second mode when $C_1$ is set to 3 Nms/rad. When $C_1$ is set to 30 Nms/rad, the large peak around 1,400 rpm is also the main resonance for the second mode. In the condition of $C_1$ = 30 Nms/rad, it is confirmed that the damped natural frequency of the third mode is suppressed by the over damping for $C_1$.

### 3.3 Relationship between engine rotation speed and natural frequency

In order to clarify the cause of the unusual vibration that occurs when the centrifugal pendulum absorber is attached to a torque converter, the relationship between the engine rotation speed and the natural frequency was investigated by applying eigenvalue analysis, as shown in Fig. 4. The abscissa indicates the engine rotation speed, and the ordinate indicates the undamped and damped natural frequency of a five-degree-of-freedom system in Fig. 4(a) and 4(b) ($C_1$ = 30 Nms/rad), respectively. The red line indicates the input forced vibration frequency determined by the engine rotation speed. It can be seen that the natural frequency increases with engine speed and intersects the forced vibration frequency at approximately 1,500 rpm in Fig. 4(a). The main resonance will occur in this range of engine speed. The range of engine speed that the damped natural frequencies intersect the forced vibration frequency becomes wide by damping $C_1$ in Fig. 4(b).

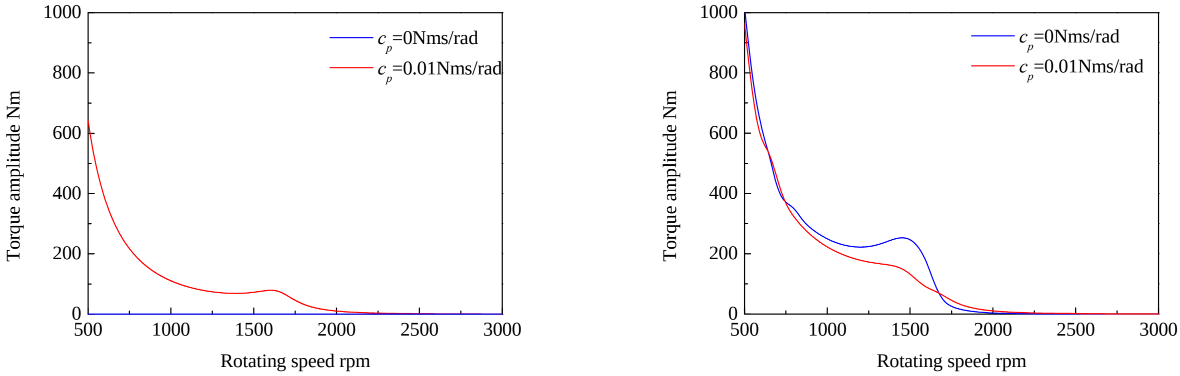



(a) Undamped natural frequency (b) $C_1$ = 30 Nms/rad

Figure 4: Natural frequency of five-degree-of-freedom system.

### 3.4 Torque fluctuation for one drive shaft with centrifugal pendulum absorber

Figures 5(a) and 5(b) show the frequency response for the torque fluctuations for one drive shaft




(a) Linear analysis (b) Nonlinear analysis

Figure 5: Torque fluctuations for one drive shaft with centrifugal pendulum absorber.

$\Delta T_d$ when the centrifugal pendulum absorber is attached, obtained through linear and nonlinear analyses, respectively. Here, the damping coefficient $C_1$ is set to 30 Nms/rad. From Fig. 5(a), the vibration amplitude of the main resonance of the second mode in Fig. 3 was reduced for a wide range of the engine speed by the centrifugal pendulum absorber. However, the large amplitude area around 1,500 rpm in Fig. 5(a) was confirmed. The main resonance occurred in this range of engine speed because the damped natural frequency intersects for a wide range of the engine speed around 1,500 rpm, as shown in Fig. 4(b). Furthermore, the large peak at around 1,500 rpm in the nonlinear analysis results shown in Fig. 5(b) is larger than that in the linear analysis results shown in Fig. 5(a) because of the nonlinearity of the centrifugal pendulum absorber. In the condition of $c_p$ = 0 Nms/rad, the nonlinearity at around 1,500 rpm in Fig. 5(b) is larger than that in the condition of $c_p$ = 0.01 Nms/rad. This is the generation mechanism for the unusual vibration caused by the centrifugal pendulum absorber, as shown in Fig. 1.

## 3.5 Effect of mass of centrifugal pendulum absorber

The effect of the mass of the centrifugal pendulum absorber on the unusual vibration was investigated through nonlinear analysis. Figure 6 shows the frequency response curves for the torque fluctuations for one drive shaft $\Delta T_d$ for the cases in which the mass of the centrifugal pendulum absorber $m_p$ is 0.795, 1.59 (standard), and 3.18 kg. Here, the damping coefficient $C_1$ is set to 30 Nms/rad. As can be seen, the unusual vibration at around 1,500 rpm is suppressed as the mass of the centrifugal pendulum absorber increases. This is because the restoring torque of the centrifugal pendulum absorber, which is the circumferential force of the centrifugal force, increases as the mass of the centrifugal pendulum absorber increases.

Figure 7 shows the relationship between the mass of the centrifugal pendulum absorber and the vibration amplitude of the angular displacement of the centrifugal pendulum absorber. The abscissa indicates the engine rotation speed, and the ordinate indicates half the peak-to-peak amplitude of the angular displacement of the centrifugal pendulum absorber obtained in the same calculation for Fig. 6. As shown in Fig. 7, the angular displacement of the centrifugal pendulum absorber decreased as the mass of the centrifugal pendulum absorber increased for a range of the engine speed around 2,000 rpm. Figure 8 shows the relationship between the ratio of the natural frequency of the centrifugal pendulum absorber to the forced vibration frequency and the angular displacement of the centrifugal pendulum absorber for an engine speed of 1,500 rpm and an order $n$ of 2.0. The restoring force of the centrifugal pendulum absorber has the characteristic of a soft spring, whereby the natural frequency decreases with increasing angular displacement. Therefore, the decrease in the natural frequency from the order $n$ = 2.0 can be small to decrease the angular displacement of the centrifugal pendulum absorber. The unusual vibration around 1,500 rpm was suppressed, as shown in Fig. 6, as the mass of the centrifugal pendulum absorber increased to decrease the angular displacement.

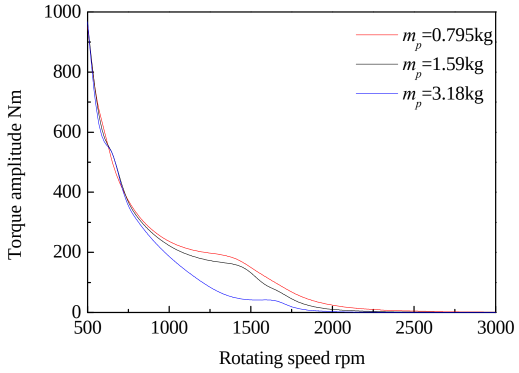



Figure 6: Torque fluctuations of one drive shaft with centrifugal pendulum absorber.

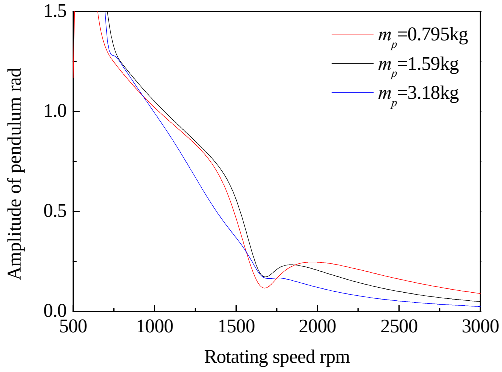



Figure 7: Amplitude of angular displacement of centrifugal pendulum absorber

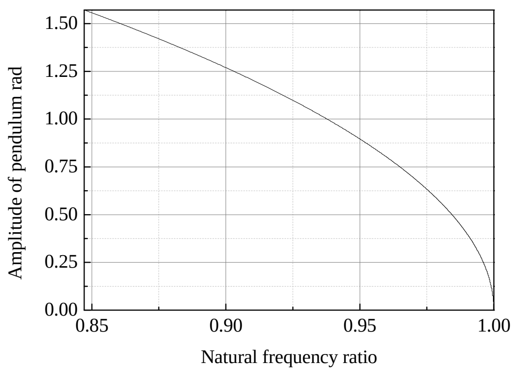
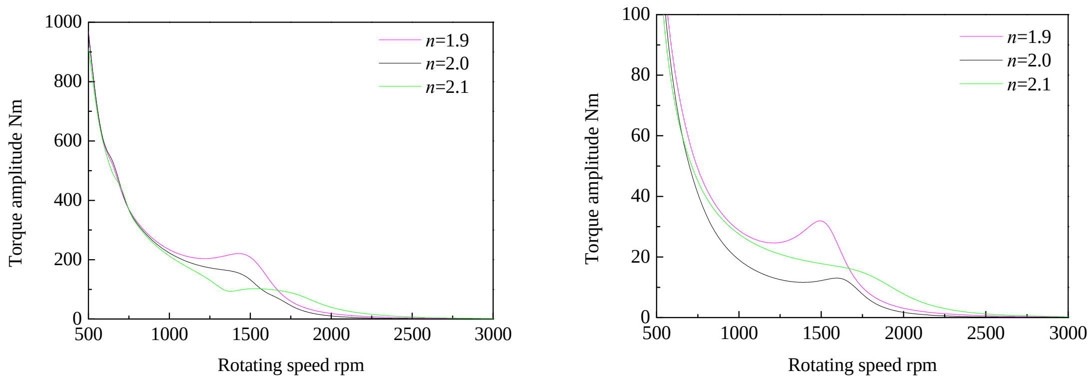



Figure 8: Relationship between natural frequency ratio and angular displacement of centrifugal pendulum absorber.

### 3.6 Effect of order of centrifugal pendulum absorber

In this section, the effects of the order of the centrifugal pendulum absorber on the unusual vibration were investigated. Figures 9(a) and 9(b) show the frequency response for the torque fluctuations for one drive shaft $\Delta T_d$ when the dynamic torque $F_d$ is set to 600 and 100 Nm, respectively, and the order $n$ of the centrifugal pendulum absorber is set to 1.9, 2.0 (standard), and 2.1 by changing the position at which the centrifugal pendulum absorber is attached $R_p$ in Eq. (14). Here, the damping coefficient $C_1$ is set to 30 Nms/rad. As shown in Fig. 9(a), the centrifugal pendulum absorber with $n$ = 2.1 is the most effective at suppressing the unusual vibration around 1,500 rpm when the dynamic torque $F_d$ is set to 600 Nm. However the centrifugal pendulum absorber with $n$ = 2.0 is the most effective when $F_d$ is set to 100 Nm, as shown in Fig. 9(b). The angular displacement of the centrifugal pendulum absorber is large enough to decrease its natural frequency when the dynamic input torque $F_d$ is 600 Nm. Therefore, the order $n$ must be set to a slightly higher value $n$ =




(a) $F_d$ = 600 Nm (b) $F_d$ = 100 Nm

Figure 9: Torque fluctuations for one drive shaft with centrifugal pendulum absorber.

2.1 in advance when taking into consideration the decrease in the natural frequency when $F_d$ is 600 Nm. The decrease in the natural frequency is small when $F_d$ is 100 Nm. Therefore, the centrifugal pendulum absorber with $n$ = 2.0 is the most effective, as shown in Fig. 9(b).

## 4. Conclusions

The present paper focused on the theoretically optimum value for the ratio between the natural frequency of the centrifugal pendulum vibration absorber and the engine rotation speed, as well as the mass of the centrifugal pendulum vibration absorber, for suppressing torsional vibration in an automatic transmission powertrain. The generation mechanism and countermeasures for the unusual vibration caused by the centrifugal pendulum absorber were also investigated. The following are the main conclusions of the present study:

(1) The main resonance occurred at an engine speed of around 1,500 rpm because the natural frequency intersects the engine speed at around 1,500 rpm due to the attachment of the centrifugal pendulum absorber. Furthermore, the resonance increased because of the nonlinearity of the centrifugal pendulum absorber. This is the generation mechanism for the unusual vibration caused by the centrifugal pendulum absorber.

(2) The unusual vibration around 1,500 rpm was suppressed as the mass of the centrifugal pendulum absorber increased because the restoring torque of the centrifugal pendulum absorber increases and its angular displacement decreases.

(3) There exist optimum values for the order $n$ of the centrifugal pendulum absorber that suppress the unusual vibration, and these values change with the dynamic input torque from the engine because the restoring force of the centrifugal pendulum absorber has the characteristic of a soft spring, whereby the natural frequency decreases with increasing angular displacement.

## ACKNOWLEDGEMENT

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