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AMPLITUDE DEPENDENCE OF MODAL PROPERTIES IN LATERAL VIBRATION OF TIMBER BUILDINGS

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In-situ modal analysis of building structures often does not consider amplitude dependence of modal properties. In multi-storey timber buildings, such a consideration may be particularly important. A growing number of mid-rise timber buildings have been built in Europe over the past decade, up to 14 storeys and 45m in height. A design challenge for the engineers of some of these buildings has been the assessment of wind-induced lateral vibration. Prediction of both natural frequency and damping are difficult due to non-linear connection stiffness, and the shortage of precedent buildings for damping measurement. The practical constraints of testing occupied buildings have meant that it has not been possible to excite the buildings artificially, so output-only modal analysis techniques have been applied. The random decrement technique is of particular interest, since it has been used to assess the variation of natural frequency and damping with amplitude of acceleration, and over recent years, researchers presented a variety of ways to do this. This paper presents a comparison of contemporary methods to assess amplitude dependence of modal properties from output-only data, using synthesised data. Appropriate methods are then applied to data measured from a multi-storey timber building, to comment on the consequences of the variation of modal properties with amplitude for design. It is shown that the damping ratios of these buildings should be expressed as their variation over a range of amplitude, and that long-term monitoring has the potential to bring out a more complete picture of this behaviour.

Keywords: operational modal analysis, building structures

1. Introduction

Multi-storey timber construction has reached new heights over the past decade. Since the completion of the Murray Grove building in 2008 [1], at least 16 buildings of 7 storeys and more have been constructed with timber as a primary structural material [2]. Various developments have helped to take timber construction outside of the small residential structures it has conventionally been used for. Engineered wood products, primarily glued-laminated timber (glulam), cross-laminated timber (CLT), and laminated veneer lumber (LVL), have allowed production of large, dimensionally stable structural elements with homogenised material properties.

This material can create lightweight structures which are, at least on the basis of design calculations, prone to discomforting vibration. This study addresses a promising method to gain reliable data on the in-situ dynamic properties of timber buildings, and in particular to characterise the damping in structures which exhibit a substantial amplitude dependence of damping.

2. Literature review

The mass of material required to perform a structural function may be estimated using its specific strength or specific modulus of elasticity, where the specific properties are the respective properties normalised by density. Structural timber has both a specific strength and specific elastic modulus comparable to that of steel, and both properties are higher than those of reinforced concrete [3]. The result is that timber buildings are relatively light in comparison to conventional multi-storey buildings, since even steel framed buildings use substantial amounts of concrete in floors.

This low mass leads to dynamic sensitivity, and the potential for occupant discomfort due to lateral wind-induced vibration and vertical floor vibration have been apparent in the design of some of these buildings.

Output-only, or operational modal anlalysis (OMA) techniques enable unobtrusive tests to characterise the in-situ dynamic properties of buildings without artificial excitation. Jeary [4] show that OMA using the random decrement technique could enforce stationarity on data from a structure with amplitude-dependence of modal properties, since many short segments of data are collected, and grouped according to their initial amplitude.

Furthermore, Jeary [4] suggests that the random decrement technique could bring out the amplitude dependence of modal properties by plotting their variation with the chosen initial amplitude. This idea has attracted further interest, with researchers proposing alternative triggering conditions which ensure each damping estimate is specific to a particular amplitude [5, 6].

3. The Random Decrement Technique

The general solution for the acceleration $a(t)$ of a single degree of freedom system with viscous damping ratio ζ excited by an infinite sum of harmonic components with circular frequency ω_i. It includes components at the frequencies of the input force, and a component which is a sum of decaying sinusoids at the natural frequency of the system ω_n. The derivation of scalars A_m and B_m and the phase angles ψ and ϕ do not affect this argument.

$$a(t) = \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} \frac{A_m \omega_i^2}{\sqrt{\left(1-\left(\frac{\omega_n}{\omega_i}\right)^2\right)^2 + \left(2\zeta\frac{\omega_i}{\omega_n}\right)^2}} \sin\left(\omega_i t - \psi\right) + \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} B_m e^{-\zeta\omega_n t} \sin\left(\left(\omega_n\sqrt{1-\zeta^2}\right)t - \phi\right) \quad (1)$$

The random decrement technique, described in its general form, sets some trigger condition, and takes a sample time series of acceleration starting whenever that trigger condition is met. In this paper, any datum which meets the trigger conditions, and which is the start of a sample for averaging by the random decrement technique is termed a trigger point. Over many averages, for a sinusiodal variation of acceleration with a particular amplitude, each trigger point on a positive gradient can be assigned a pair on a negative gradient. The expected values of each of these pairs will be symmetric around the peak of the sinusoid, whatever its phase, and offset from the peak by an angle θ. If the trigger level is a_t, then the amplitude of the sinusoid is given by $a_t/\cos(\theta)$. The summation of the pair is given in Eqs. (2) to (4).

$$a_n(\tau) = \frac{a_t}{\cos\theta}\cos\left(\omega t - \theta\right) = \frac{a_t}{\cos\theta}\left(\cos\omega t\cos\theta - \sin\omega t\sin\theta\right) \quad (2)$$

$$a_{n+1}(\tau) = \frac{a_t}{\cos\theta}\cos\left(\omega t + \theta\right) = \frac{a_t}{\cos\theta}\left(\cos\omega t\cos\theta + \sin\omega t\sin\theta\right) \quad (3)$$

$$a_{sum}(\tau) = \frac{a_n(\tau) + a_{n+1}(\tau)}{2} = \frac{a_t}{\cos\theta}\cos\omega t\cos\theta = a_t\cos\omega t \tag{4}$$

The result of the random decrement technique in this form is therefore to scale any sinusoidal components to the same amplitude, that of the trigger level, and to bring them all into phase. Looking back at Eq. (1), the first summation, which contains many different frequencies ω_i, will sum to zero very soon after the first point. The second summation, in which all terms are at the natural frequency of the system ω_n, will tend to be a decaying sinusoid with initial amplitude a_t, decaying according to $e^{-\zeta\omega_n t}$. If ζ varies with amplitude, then the decay of the random decrement signature will be according to the averaging of all the decaying sinusoids with points which meet the trigger condition.

Equations (2) to (4) show that all decays are weighted equally in the summation, regardless of their amplitude, so the averaging of damping will be weighted according to the number of samples corresponding to each amplitude. The damping estimated from the random decrement signature is therefore an average of the damping in all the parts of the signal captured by the trigger condition.

4. Modeled Time-history of Building Response to Wind Load

In this study, the random decrement technique was applied to time-histories of the measured response of a building, and to the response of a numerical model of a dynamic system with a single degree of freedom.

Huang and Gu [6] propose a method to evaluate the response of a dynamic system with variable damping to a general time-history of load. They vary the damping in the system according to its amplitude envelope, quantified by its Hilbert transform, and evaluate the response by the Newmark-beta method. Since the amplitude at any time depends on the history of damping in the system up to that time, it is necessary to iterate the evaluation of the whole time history.

That method was applied here with a simulated wind time-history generated using the power spectral density for wind given in Eurocode 1 Part 1-4 [7], for a turbulence length scale of 20m and a mean wind speed varying between zero and 20m/s during the period of the time history. The amplitude and phase of each frequency component in the simulated wind force is generated by Matlab's random number generator. The time history is 100 minutes long, sampled at 100Hz. Two scenarios were modeled: a building with constant damping ratio, and one with amplitude dependence of damping ratio.

5. Monte-Carlo Simulation

Every time a wind force time-history is generated, it is different, due to the randomly generated amplitude and phase of each frequency component. The random decrement signature is also different for each realisation, as a finite number of samples are averaged, giving only an approximation of the function it tends towards. This variation has rarely been addressed in the literature, and the random decrement signature, and the parameters derived from it, are typically shown as the result of a single realisation from a particular building or time-history of synthesised data. In this study, a small monte-Carlo simulation was carried out, using 20 realisations of wind load data, the results indicating the variability of damping estimates by this family of methods.

6. Application of the Random Decrement Technique

The random decrement technique is applied here with a series of trigger conditions. What they have in common is that with sufficient samples averaged, they all will enable each point on the positive gradient of a sinusoidal curve to be paired with one on a negative gradient, so the averaging process described in Section 3 may proceed. Under the following headings, each of the trigger conditions is described.

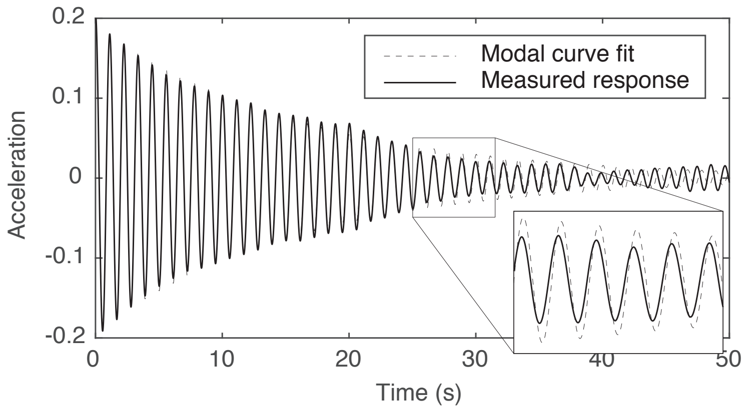

Figure 1: The breakdown of the random decrement signature, as amplitude and phase cease to follow a decaying sinusoid in the second half of the signature.

6.1 Threshold Crossing

This is the original form of the random decrement technique presented by Cole [8, 9]. A sample is triggered each time the acceleration crosses a threshold. This means that no samples are triggered with an initial amplitude below the threshold, but that all data with an amplitude above the trigger will be included in a sample.

The damping estimate by this method is therefore derived from components of vibration with initial amplitudes greater than the threshold level. If the signal consists of many small-amplitude oscillations, and progressively fewer oscillations as the amplitude increases, then the damping estimated for a particular threshold level gives an indication of the damping at that amplitude, albeit with an additional influence of the contribution from higher-amplitude components.

A key element of analysis of data by the random decrement technique, particularly when investigating amplitude dependence, is identifying the range of threshold values which give a valid random decrement signature, and therefore a reliable estimate of modal parameters. If the threshold is too high, then there will be insufficient segments averaged to eliminate the part of the signal with random frequency and phase, the first part of Eq. (1). Figure 1 shows a random decrement signature for the modeled building response time-history with approximately 100 samples averaged, and shows that the signature 'breaks down', shifting in both amplitude and phase towards the end. Since the signature no longer tends towards zero amplitude, this results in an underestimate of damping from a simple curve fit. At the other extreme, as the threshold value is reduced, noise has a greater proportionate contribution to the average, and the auto-correlation of the noise dies away very quickly, resulting in an overestimate of damping.

6.2 Peak Crossing

It is possible, however, to modify the technique to eliminate the samples with higher amplitude from the averaging process, to give a random decrement signature, and therefore an estimate of modal properties of the system, which applies to a particular amplitude, or at least a small range of amplitudes.

The peak triggering condition sets a threshold value, and starts a sample at each crossing of the threshold which is sufficiently close to the peak [5]. In this study, if the amplitude envelope, calculated by the Hilbert transform, is less than 10% higher than the threshold set, then the resulting sample is used for summation and averaging. The drawback of this technique is that it dramatically reduces the number of samples being averaged.

6.3 Range Crossing

The convergence of the random decrement signature towards a decaying sinusoid relies on sufficient samples being averaged, so that the first summation in Eq. (1) tends towards zero. One way of increasing the number of samples in the averaging process is to trigger a sample at every point which falls within a range of accelerations, so both a lower and upper threshold is set, and a sample is triggered from every point higher than the lower threshold, and lower than the upper threshold.

When damping is estimated by this method, it has the same shortcoming as the threshold crossing method, in that it effectively sets a lower limit on the amplitude of oscillation which is captured in the averaging process, but not an upper limit, so it includes a contribution to the damping estimate from components with any amplitude higher than the lower threshold.

6.4 Above Threshold

A simple extension to the range crossing method is to remove the upper threshold. This has the effect of scaling the magnitude of the random decrement signature, so that its initial value will represent all the samples averaged - it is no longer forced to be at the threshold value or within the set range. Thus both the amplitude and the damping are calculated by a similar weighted average, and may be more closely related.

6.5 Envelope Crossing

Huang and Gu [6] propose a trigger condition in which a sample is started each time the envelope of the signal, estimated by its Hilbert transform, crosses a threshold. This reduces the number of samples taken from a given time history compared with the threshold and range crossing triggers, although not so much as the peak trigger, but ensures that the random decrement signature, and therefore the damping calculated from it, is tied to the given amplitude of response.

7. Results and Discussion

7.1 Modeled Response

The estimates of damping based on each method are shown in Figs. 2 and 3. At low amplitudes, the Range, Peak and Envelope crossing methods have a large variation in damping estimates. This reflects the lower number of samples averaged, but also that for these small initial amplitudes, the part of the signal with random amplitude and phase, the first part of Eq. (1), will have a high amplitude compared to the exponential decay, and so will take more samples to average to zero. For high amplitudes, the variation increases for all methods, because of the reduced number of samples averaged. Note that the peak crossing method doesn't suffer this, because the code was set to put samples into bins of 2000 with similar amplitude.

For a constant damping ratio, all the methods follow the input value, with the difference being in variability of the estimate. Once a variation in damping is introduced, the methods no longer precisely follow the theoretical value. The threshold crossing method does not accurately follow the change in gradient of the input value; it rises at a rate faster than the input damping, reflecting the fact that it includes all cycles with amplitude greater than the threshold. This effect is mitigated, though not eliminated, in both the range crossing and minimum amplitude methods, since they more accurately represent the amplitude of the sinusoids being averaged. The peak trigger eliminates this bias, by only triggering for cycles whose amplitudes are near the trigger value. The envelope crossing method achieves a greater number of samples for averaging than peak crossing, whilst still ensuring only cycles at the relevant amplitude are included, resulting in an accuracy similar to the range crossing and minimum amplitude methods, although its mean value does not appear to follow the input value so faithfully.

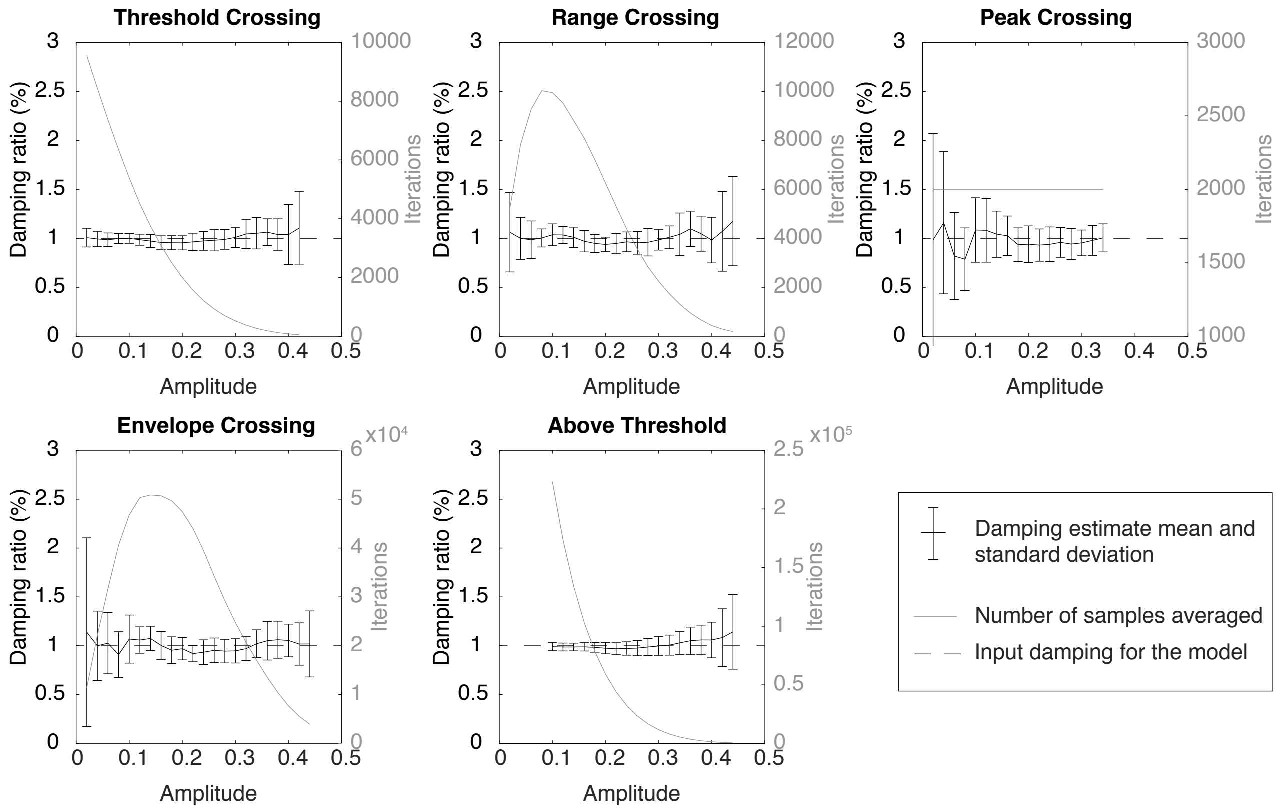

Figure 2: Amplitude dependence of damping evaluated by each method, compared with the input value for constant damping.

An important consideration in any measurement is its repeatability, and damping measurements are particularly slippery in this regard. The error in the estimates shown in Figs. 2 and 3 is made up both from the inaccuracy of the underlying method, for example the fact that the threshold crossing method includes amplitudes higher than the threshold amplitude, and from the finite averaging process. The averaging process may introduce both a random error, as shown for low amplitudes using the peak method, and a systematic error, as a low number of averages leads to a breakdown of the random decrement signature and a subsequent underestimate of damping.

7.2 Measured Response

The methods were then applied to data measured from a real building. The variations shown in Fig. 4 show that the damping estimated by each method follows some of the same patterns relative to one another described for the modeled data. The peak crossing method has a high variability for low amplitudes, reducing at higher amplitudes. The lower variability of the above threshold trigger is evident, since it gives a smooth curve, which breaks down once the number of averages drops below approximately 3000.

The variation in damping with amplitude is subtle, but the results of the peak crossing, envelope crossing and above threshold methods suggest that there is some. It would be in-keeping with the modelled behaviour of the threshold crossing and range crossing methods to reduce the apparent gradient of the amplitude dependence of damping. A longer set of measured data would enable each method to capture more samples at higher amplitudes, and gradually increase the range of amplitudes for which reliable estimates of damping are possible. It would be useful to increase this range to estimate damping at the levels of amplitude relevant to design for human comfort.

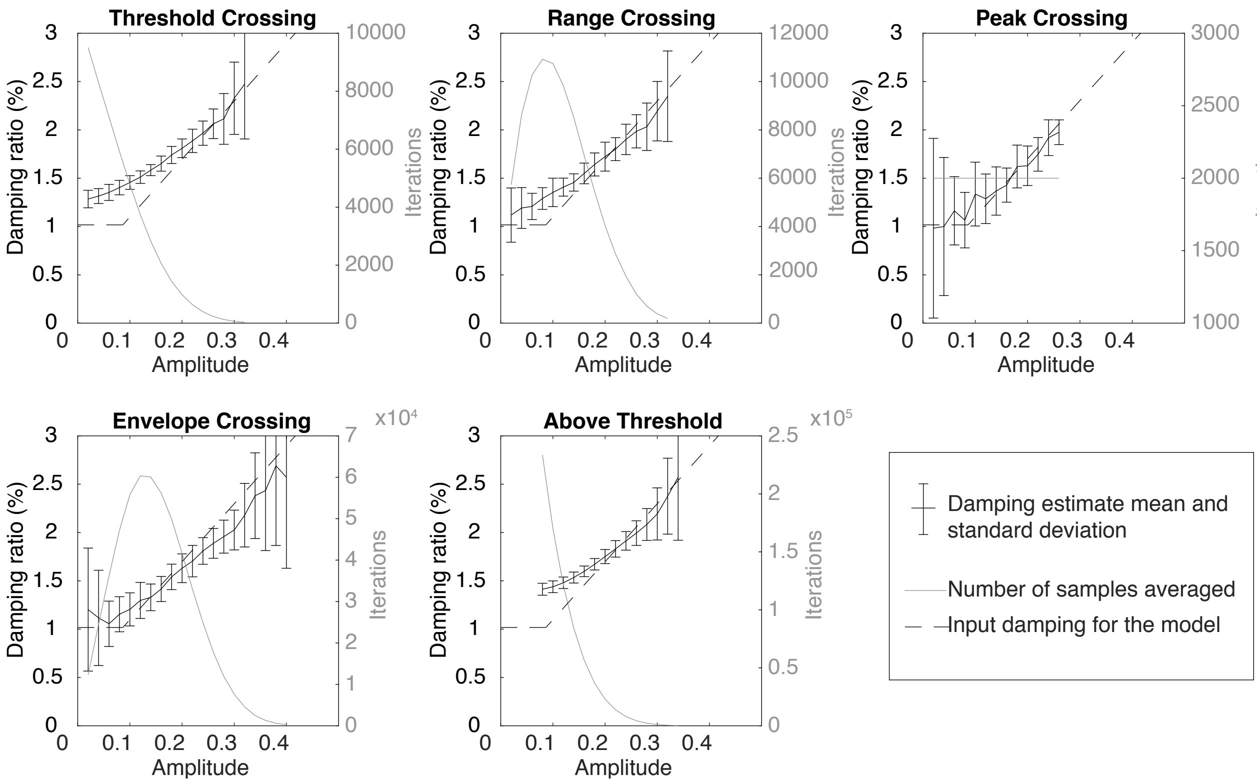

Figure 3: Amplitude dependence of damping evaluated by each method, compared with the input value for amplitude-dependent damping.

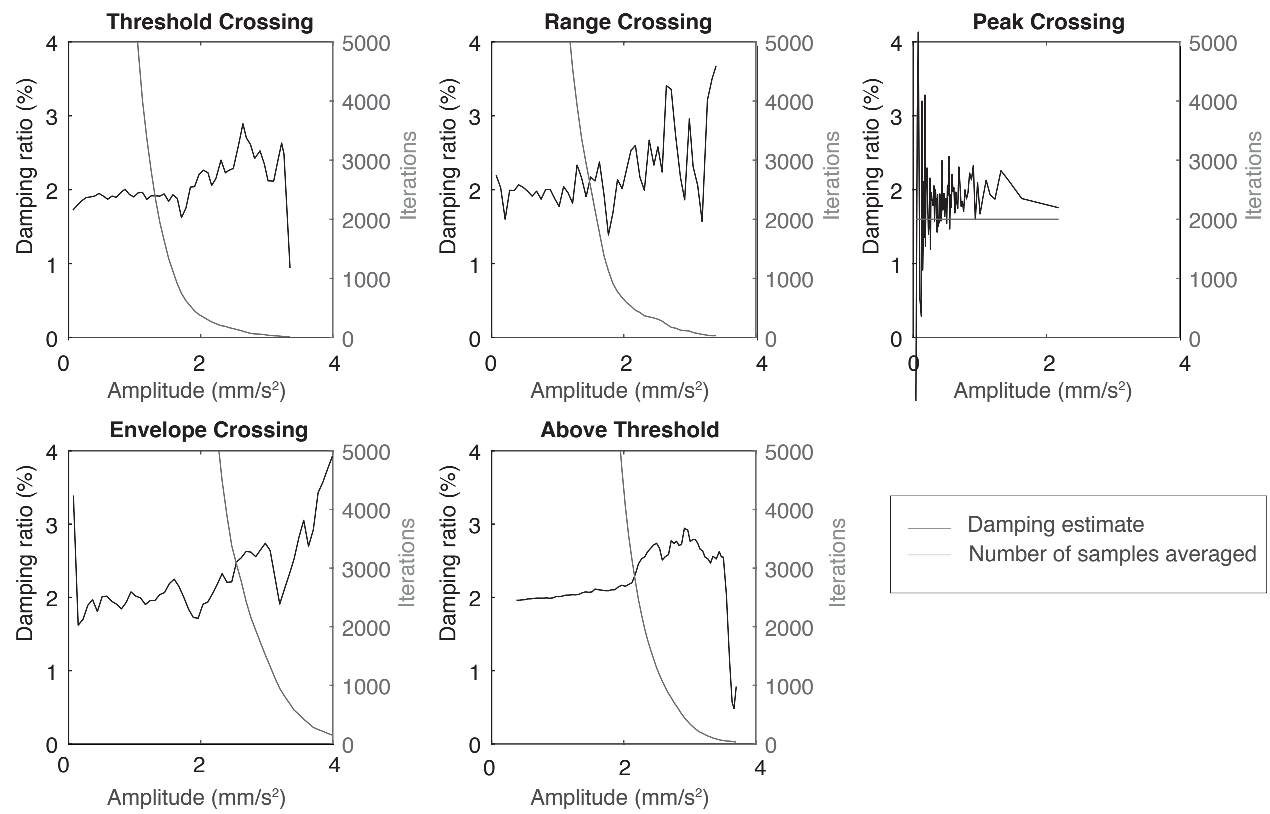

Figure 4: Amplitude dependence of damping evaluated by each method for the multi-storey building

8. Conclusion

This study has explored the variability of damping estimates made using five variants of the random decrement technique, using a Monte-Carlo simulation of wind force on a modal model of a single degree-of-freedom system. The results show that there is essentially a trade-off between methods which give results closely tied to a particular amplitude of vibration, but with relatively high variability in the estimates (the peak crossing, envelope crossing and range crossing methods) and methods which give results less well tied to amplitude, but with lower variability (the threshold crossing and above threshold methods).

Application to measured data from a multi-storey building shows the effect of the variability when identifying a singe building, with the application of each method helping to identify the range of estmates likely to be accurate. There is evidence of an increase of damping with amplitude, and longer measurement time would bring this out over a greater range of amplitudes.

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